Econ. Environ. Geol.
Vol. 27, No. 1, p. 41-47, 1994.

Computation of Green's Tensor Integrals in Three-Dimensional Magnetotelluric Modeling Using Integral Equations

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ABSTRACT: A fast Hankel transform (FHT) algorithm (Anderson, 1982) is applied to numerical evaluation of many Green's tensor integrals encountered in three-dimensional electromagnetic modeling using integral equations. Efficient computation of Hankel transforms is obtained by a combination of related and lagged convolutions which are available in the FHT. We express Green's tensor integrals for a layered half-space, and rewrite those to a form of related functions so that the FHT can be applied in an efficient manner. By use of the FHT, a complete or full matrix of the related Hankel transform can be rapidly and accurately calculated for about the same computation time as would be required for a single direct convolution. Computing time for a five-layer half-space shows that the FHT is about 117 and 4 times faster than conventional direct and multiple lagged convolution methods, respectively.

INTRODUCTION

In recent years numerical modeling of three-dimensional (3-D) targets in studies relating to electrical and electromagnetic (EM) prospecting has attracted a large number of workers (Hohmann, 1975; Ting and Hohmann, 1981; Lee et al., 1981; Das and Verma, 1982; Wannamaker et al. 1984; Madden and Mackie, 1989). Such studies involve the calculation of resistivity, induced polarization, magnetotelluric (MT) or EM response of a 3-D arbitrarily shaped body embedded in a conductive earth excited by a natural or an artificial source. Mathematical tools to solve these problems are an integral equation method, differential equation methods (finite difference and finite element), and a combination of aforementioned methods known as a hybrid technique. Among these techniques the integral equation method requires the smallest computer storage because unknown fields need be found only in anomalous bodies (Hohmann, 1988). However, computation cost is considerably high even in the integral equation approach and one must numerically evaluate very large sets of secondary Green's tensor integrals that are expressed as Hankel transforms of integer order in the integral equation formulation (Hohmann, 1975; Das and Verma, 1982; Wannamaker, 1984).

Since a digital linear filter method was introduced by Ghosh (1971) to estimate Hankel transform integrals, many applications of it were found in geophysical fields. Anderson (1975, 1979, 1982) presented digital filtering algorithms for efficient and accurate evaluation of complex Hankel transforms of integer order. His filters have been extensively used by many geophysicists. For example, Wannamaker et al. (1984) used subroutines ZHANK0 and ZHANK1 (Anderson, 1975), and Wannamaker (1991) employed subroutine ZHANKS (Anderson, 1979) in their 3-D MT modelings. However, although Anderson's (1982) latest filter, subroutine HANKEL, is the most powerful one because it can perform both related and lagged convolutions, its application to geophysical fields is little known.

In this paper, we solve a time-consuming problem in 3D MT modeling using Anderson's (1982) fast Hankel transform (FHT) algorithm. Layered half-space tensor integrals are expressed in a form so that FHT can be applied in an efficient manner, followed by a computer timing example for a five-layer model run three different ways (Anderson, 1975, 1979, 1982).

TENSOR INTEGRALS

Wannamaker et al. (1984) presented Green's tensor functions for an electric dipole in a layered conductive earth. Among these Green's functions, for more accurate evaluation of EM fields, secondary electric and magnetic Green's functions were partly modified by Wannamaker (1991). Unfortunately, there are many typographical errors in Wannamaker et al. (1984) and Wannamaker (1991). Therefore, for the sake of others who may wish to use our work, correct

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formulas are listed in Appendix.

Each Hankel transform tensor integral for a layered half-space is, in general, defined by integral kernels with recursive expressions in terms of many complex square-root and exponential functions. Observe that the integral kernels shown in Appendix are related each other with many common factors. Such related functions are rapidly computed by a modified FHT with an external subroutine RELFUN as described in detail by Anderson (1982, p. 365-6). Anderson (1984) presented a procedure to construct subroutine RELFUN for the homogeneos half-space tensor integrals expressed by Hohmann (1975) and for layered half-space tensor integrals derived by Wannamaker et al. (1984). In this paper, we show the revised tensor integrals listed in Appendix in a form so that the modified FHT can be interfaced with a suitably coded subprogram RELFUN. While Anderson (1984) presented only the electric tensor integrals for $l>j$, we derive both electric and magnetic tensor integrals for all cases of $l=j$, $l>j$ and $l<$j.

Each kernel function for the electric and magnetic elements for $l=$j can be represented in terms of several common complex recursive factor defined as

$$\begin{aligned} C_1(\lambda)&={}^-R_l^{TM}[e^{-u_l(z'-d_{l-1})}+{}^xA_j^{TM}]/D,\\ C_2(\lambda)&={}^-R_l^{TE}[e^{-u_l(z'-d_{l-1})}+{}^xA_j^{TE}]/D,\\ C_3(\lambda)&={}^-R_l^{TM}[e^{-u_l(z'-d_{l-1})}+{}^zA_j^{TM}]/D, \end{aligned} \tag{1}$$

where

$$D=e^{+u_l(z-d_{l-1})},$$

and symbols containing R, A and related terms are explicitly listed in Wannamaker et al. (1984). By use of the common functions in equation (1), the generalized FHT related functions can be written for use in subprogram RELFUN as

$$\begin{aligned} F_1(\lambda)&=u_l[G_1-C_1(\lambda)]+k_l^2[G_2+C_2(\lambda)]/u_l,\\ F_3(\lambda)&=\lambda[G_2+C_2(\lambda)]/u_l,\\ F_4(\lambda)&=\lambda[G_1+C_1(\lambda)],\\ F_5(\lambda)&=\lambda^3[G_3+C_3(\lambda)]/u_l,\\ F_6(\lambda)&=\lambda^2[G_3+C_3(\lambda)],\\ F_7(\lambda)&=[G_1+C_1(\lambda)]-[G_2-C_2(\lambda)],\\ F_9(\lambda)&=\lambda[G_2+C_2(\lambda)],\\ F_{10}(\lambda)&=F_3(\lambda),\\ F_{11}(\lambda)&=F_5(\lambda)/\lambda, \end{aligned} \tag{2}$$

where

$$\begin{aligned} G_1&={}^xA_l^{TM}D,\\ G_2&={}^xA_l^{TE}D,\\ G_3&={}^zA_l^{TM}D. \end{aligned}$$

Similarly, for $l>j$, kernel functions are defined as

$$\begin{aligned} C_1(\lambda)&={}^+B_{lj}^{TM}[{}^xA_j^{TM}e^{+u_jh_j}/{}^+R_j^{TM}],\\ C_2(\lambda)&={}^+B_{lj}^{TE}[{}^xA_j^{TE}e^{+u_jh_j}/{}^+R_j^{TE}],\\ C_3(\lambda)&={}^+B_{lj}^{TM}[{}^zA_j^{TM}e^{+u_jh_j}/{}^+R_j^{TM}]. \end{aligned} \tag{3}$$

Then the related functions are expressed as

$$\begin{aligned} F_1(\lambda)&=u_lC_1(\lambda)[G_1-1/D]+k_l^2C_2(\lambda)[G_2+1/D]/u_j,\\ F_3(\lambda)&=\lambda C_2(\lambda)[G_2+1/D]/u_j,\\ F_4(\lambda)&=\lambda C_1(\lambda)[G_1+1/D]/u_j,\\ F_5(\lambda)&=\lambda^3C_3(\lambda)[G_1+1/D]/u_j,\\ F_6(\lambda)&=\lambda^2u_lC_3(\lambda)[G_1-1/D]/u_j,\\ F_7(\lambda)&=C_1(\lambda)[G_1+1/D]-wu_lC_2(\lambda)[G_2-1/D]/u_j,\\ F_9(\lambda)&=w\lambda u_lC_2(\lambda)[G_2-1/D]/u_j,\\ F_{10}(\lambda)&=wF_3(\lambda),\\ F_{11}(\lambda)&=F_5(\lambda)/\lambda, \end{aligned} \tag{4}$$

where

$$\begin{aligned} G_1&={}^+R_l^{TM}D,\\ G_2&={}^+R_l^{TE}D,\\ D&=e^{+u_l(z-d_l)},\\ w&=\hat{z}_j/\hat{z}_l. \end{aligned}$$

Finally, for $l<j$, kernel functions are defined as

$$\begin{aligned} C_1(\lambda)&={}^-B_{lj}^{TM}[e^{-u_j(z'-d_{j-1})}+{}^x\mathrm{A}_j^{TM}],\\ C_2(\lambda)&={}^-B_{lj}^{TE}[e^{-u_j(z'-d_{j-1})}+{}^x\mathrm{A}_j^{TE}],\\ C_1(\lambda)&={}^-B_{lj}^{TM}[e^{-u_j(z'-d_{j-1})}+{}^z\mathrm{A}_j^{TM}], \end{aligned} \tag{5}$$

The related functions are expressed as

$$\begin{aligned} F_1(\lambda)&=u_lC_1(\lambda)[D-G_1]+k_l^2C_2(\lambda)[D+G_2]/u_j,\\ F_3(\lambda)&=\lambda C_2(\lambda)[D+G_2]/u_j,\\ F_4(\lambda)&=\lambda C_1(\lambda)[D+G_1],\\ F_5(\lambda)&=\lambda^3C_3(\lambda)[D+G_1]/u_j,\\ F_6(\lambda)&=\lambda^2u_lC_3(\lambda)[D-G_1]/u_j,\\ F_7(\lambda)&=C_1(\lambda)[D+G_1]-wu_lC_2(\lambda)[D-G_2]/u_j,\\ F_9(\lambda)&=w\lambda u_lC_2[D-G_2]/u_j,\\ F_{10}(\lambda)&=wF_3(\lambda),\\ F_{11}(\lambda)&=F_5(\lambda)/\lambda, \end{aligned} \tag{6}$$

where

$$\begin{aligned} G_1&={}^-R_l^{TM}/D,\\ G_2&={}^-R_l^{TE}/D,\\ D&=e^{+u_l(z-d_l)},\\ w&=\hat{z}_j/\hat{z}_l. \end{aligned}$$

A general form of elecric and magnetic Hankel transforms of order n for the layered half-space is denoted by

$$\int_0^\infty F_m(\lambda)J_n(\lambda r)d\lambda=\begin{cases} {}^S\gamma_{ml}^E & \text{for } m=1,\ 3\text{-}6,\\ {}^S\gamma_{(m-6)l}^H & \text{for } m=7,\ 9\text{-}11. \end{cases}$$

where

$$n=\begin{cases} 1 & \text{for } m=1,\ 6,\ 7,\ 11,\\ 0 & \text{for } m=3\text{-}5,\ 8,\ 10. \end{cases}$$

COMPARISON OF COMPUTATION TIME

Examination of the set of kernels in equations (2), (4) or (6) reveals that other common algebraic expressions can be saved and reused again in subprogram RELFUN, such as ${}^{x}A_j^{TM}$ and all exponential factors. Then, the modified FHT can be called once for a fixed z and z', and for any desired range of r, by interfacing with RELFUN, which returns $F_m(\lambda)$ for all m. The dimension of the FHT output complex array is now (N, 5) and (N, 9) for only electric tensor integrals and for both electric and magnetic tensor integrals, respectively, where

$$N=5\ln(r_{max}/r_{min})+1,$$

and

$$0<r_{min}\leq r\leq r_{max}.$$

A computation time comparison between direct convolution, lagged convolution and the FHT for only the electric tensor integrals was run on an IBM compatible (80386) system with mathematical coprocessor using Microsoft FORTRAN 5.0 Optimizing compiler. For a five-layer half-space with $l=4$ and $j=2$, and $N=150$, which covers a very large range of r, $10^{-6}<r\leq 10^7$, a small-sized output complex array of dimension (150, 5), or 750 related and lagged convolution integrals is required. The results are summerized in Table 1.

Table 1 shows that the FHT run nealy 117 and 4 times faster than using direct and lagged convolution, respectively, to compute all 750 integrals, and 382 and 5 times fewer function evaluations. The total number of FHT function calls is very small (239), and is about the same as for a single direct convolution. The direct and lagged convolution algorithms used in comparision were ZHANKS (Anderson, 1979) and HANKEL (Anderson, 1982), respectively. From Table 1, we can see that multiple lagged convolutions are much faster than direct convolution, but are not as fast as with the FHT using both lagged and related convolutions.

The range or r chosen $[10^{-6}, 10^7]$ for this example may not occur in practical situations. Nevertheless, Table 1 indicates the constant time and function ratios expected for a five-layer model, regardless of the range of r selected. Only reducing the number of layers would significantly reduce these ratios. The excluded cases, $l\leq j$ and all magnetic elements, are also required in the complete MT integral equation solution, and would add further to the time and function evaluations shown in Table 1.

Table 1. Comparison between direct, lagged and FHT convolutions for 750 integrals for a five-layer half-space.

Method	Function calls	Ratio to FHT calls	CPU time (s)	Ratio to FHT time
Direct	90497	382	1299.13	116.99
Lagged	1115	5	42.07	3.79
FHT	239	1	11.10	1

CONCLUDING REMARKS

Wannamaker (1991) used a modified direct convolution subprogram ZHANKS (Anderson, 1979) to incorporate lagged convolutions in evaluating their base point grids in the space of z and z'. The multiple lagged convolution method reduces the Hankel transformation phase of MT modeling to only a small part, about 10% for a three-layer half-space. By use of the FHT, a futher reduction of this phase is expected, about 4%. Subsequent phases of interpolations for transform arguments within columns of the output transform array will produce the required integrals, and can be substituted in the appropriate Green' s tensor functions.

In 3-D EM modeling in layered media, use of the FHT can significantly reduce the initial Hankel transform evaluation phase compared to direct convolution, or even as compared to multiple lagged convolutions done for each column vector of the base point grids. The possible FHT savings should be an incentive to seek still further improvements in subsequent phases of the over all 3-D EM modeling.

ACKNOWLEDGMENTS

This research was partly supported by the Korean Science and Engineering Foundation.

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Manuscript received 1 September 1993

APPENDIX TENSOR GREEN'S FUNCTIONS

Symbols and related terms used in this appendix are explicitly listed in Wannamaker et al. (1984, p. 67-74). In layer j, the particular and complementary potential solutions give rise to primary and secondary tensor elements, i.e.,

$$\bar{G}_j^E = {}^P\bar{G}_j^E + {}^S\bar{G}_j^E, \tag{A-1}$$

and

$$\bar{G}_j^H = {}^P\bar{G}_j^H + {}^S\bar{G}_j^H. \tag{A-2}$$

The primary electric elements are (Wannamaker et al., 1984)

$${}^PG_{xxj}^E = \frac{1}{4\pi\hat{y}_i}\left\{\left[\frac{(x-x')^2}{R^2}\right]{}^P\gamma_{1j}^E - {}^P\gamma_{2j}^E + k_j^2\,{}^P\gamma_{3j}^E\right\}, \tag{A-3a}$$

$${}^PG_{yxj}^E = {}^PG^E{}_{xyj} = \frac{1}{4\pi\hat{y}_j}\left\{\left[\frac{(x-x')(y-y')}{R^2}\right]{}^P\gamma_{1j}^E\right\}, \tag{A-3b}$$

$${}^PG_{zxj}^E = {}^PG^E{}_{xzj} = \frac{1}{4\pi\hat{y}_j}\left\{\left[\frac{(x-x')(z-z')}{R^2}\right]{}^P\gamma_{1j}^E\right\}, \tag{A-3c}$$

$${}^PG_{yyj}^E = \frac{1}{4\pi\hat{y}_j}\left\{\left[\frac{(y-y')^2}{R^2}\right]{}^P\gamma_{1j}^E - {}^P\gamma_{2j}^E + k_j^2\,{}^P\gamma_{3j}^E\right\}, \tag{A-3d}$$

$${}^PG_{zyj}^E = {}^PG^E{}_{yzj} = \frac{1}{4\pi\hat{y}_i}\left\{\left[\frac{(y-y')(z-z')}{R^2}\right]{}^P\gamma_{1j}^E\right\}, \tag{A-3e}$$

and

$${}^PG_{zzj}^E = \frac{1}{4\pi\hat{y}_j}\left\{\left[\frac{(z-z')^2}{R^2}\right]{}^P\gamma_{1j}^E - {}^P\gamma_{2j}^E + k_j^2\,{}^P\gamma_{3j}^E\right\}, \tag{A-3f}$$

where $R=|r-r'|$ and

$${}^P\gamma_{1j}^E = \frac{e^{-ik_jR}}{R}\left[\frac{3}{R^2} + \frac{3ik_j}{R} - k_j^2\right], \tag{A-4a}$$

$${}^P\gamma_{2j}^E = \frac{e^{-ik_jR}}{R}\left[\frac{1}{R^2} + \frac{ik_j}{R}\right], \tag{A-4b}$$

and

$${}^P\gamma_{3j}^E = \frac{e^{\ ik_jR}}{R}. \tag{A-4c}$$

The secondary electric elements are (Wannamaker et al., 1984; Wannamaker, 1991)

$${}^SG_{xxj}^E = \frac{1}{4\pi\hat{y}_j}\left\{\frac{\partial}{\partial x'}\left[\frac{(x-x')}{r}\right]{}^S\gamma_{1j}^E + k_j^2\,{}^S\gamma_{3j}^E\right\}, \tag{A-5a}$$

$${}^SG_{yxj}^E = \frac{1}{4\pi\hat{y}_j}\left\{\frac{\partial}{\partial x'}\left[\frac{(y-y')}{r}\right]{}^S\gamma_{1j}^E\right\}, \tag{A-5b}$$

$${}^SG_{zxj}^E = \frac{-1}{4\pi\hat{y}_j}\left\{\frac{\partial}{\partial x'}\left[{}^S\gamma_{4j}^E\right]\right\}, \tag{A-5c}$$

$${}^SG_{xyj}^E = \frac{1}{4\pi\hat{y}_j}\left\{\frac{\partial}{\partial y'}\left[\frac{(x-x')}{r}\right]{}^S\gamma_{1j}^E\right\}, \tag{A-5d}$$

$${}^SG_{yyj}^E = \frac{1}{4\pi\hat{y}_j}\left\{\frac{\partial}{\partial y'}\left[\frac{(y-y')}{r}\right]{}^S\gamma_{1j}^E + k_j^2\,{}^S\gamma_{3j}^E\right\}, \tag{A-5e}$$

$${}^SG_{zyj}^E = \frac{-1}{4\pi\hat{y}_j}\left\{\frac{\partial}{\partial y'}\left[{}^S\gamma_{4j}^E\right]\right\}, \tag{A-5f}$$

$${}^SG_{xzj}^E = \frac{-1}{4\pi\hat{y}_j}\left\{\left[\frac{(x-x')}{r}\right]{}^S\gamma_{6j}^E\right\}, \tag{A-5g}$$

$${}^SG_{yzj}^E = \frac{-1}{4\pi\hat{y}_j}\left\{\left[\frac{(y-y')}{r}\right]{}^S\gamma_{6j}^E\right\}, \tag{A-5h}$$

and

$${}^SG_{zzj}^E = \frac{1}{4\pi\hat{y}_j}\left\{{}^S\gamma_{5j}^E\right\}. \tag{A-5i}$$

with Hankel transforms

$$
{}^S\gamma^E_{1j}=\int_0^\infty\Big\{\Big[u_j\,{}^xA^{TM}_j+\frac{k_j^2}{u_j}\,{}^xA^{TE}_j\Big]e^{+u_j(z-d_{j-1})}
-\big[u_j(e^{-u_j(z'-d_{j-1})}+{}^xA^{TM}_j)\;R^{TM}_j
-\frac{k_j^2}{u_j}(e^{-u_j(z'-d_{j-1})}+{}^xA^{TE}_j)\;R^{TE}_j\big]e^{-u_j(z-d_{j-1})}\Big\}
J_1(\lambda r)d\lambda, \quad \text{(A-6a)}
$$

$$
{}^S\gamma^E_{3j}=\int_0^\infty\frac{1}{u_j}\,[{}^xA^{TE}_j]e^{+u_j(z-d_{j-1})}
+[(e^{-u_j(z'-d_{j-1})}+{}^xA^{TE}_j)^{-}R^{TE}_j]e^{-u_j(z-d_{j-1})}\}\lambda J_0(\lambda r)d\lambda, \quad \text{(A-6b)}
$$

$$
{}^S\gamma^E_{4j}=\int_0^\infty[{}^xA^{TM}_j]e^{+u_j(z-d_{j-1})}
+[(e^{-u_j(z'-d_{j-1})}+{}^xA^{TM}_j)^{-}R^{TM}_j]e^{-u_j(z-d_{j-1})}\}\lambda J_0(\lambda r)d\lambda, \quad \text{(A-6c)}
$$

$$
{}^S\gamma^E_{5j}=\int_0^\infty\frac{\lambda^2}{u_j}\,[{}^zA^{TM}_j]e^{+u_j(z-d_{j-1})}
+[(e^{-u_j(z'-d_{j-1})}+{}^zA^{TM}_j)^{-}R^{TM}_j]e^{-u_j(z-d_{j-1})}\}\lambda J_0(\lambda r)d\lambda, \quad \text{(A-6d)}
$$

and

$$
{}^S\gamma^E_{6j}=\int_0^\infty\lambda^2\{[{}^zA^{TM}_j]e^{+u_j(z-d_{j-1})}
-[(e^{-u_j(z'-d_{j-1})}+{}^zA^{TM}_j)\;R^{TM}_j]e^{-u_j(z-d_{j-1})}\}J_1(\lambda r)d\lambda. \quad \text{(A-6e)}
$$

Similarly, the primary magnetic members become

$$
{}^PG^H_{xxj}={}^PG^H_{yyj}={}^PG^H_{zz}=0, \quad \text{(A-7a)}
$$

$$
{}^PG^H_{yxj}=-{}^PG^H_{xyj}=\frac{-1}{4\pi}(z-z')\,{}^P\gamma^H_{ij}, \quad \text{(A-7b)}
$$

$$
{}^PG^H_{zxj}=-{}^PG^H_{xzj}=\frac{1}{4\pi}(y-y')\,{}^P\gamma^H_{ij}, \quad \text{(A-7c)}
$$

and

$$
{}^PG^H_{zyj}=-{}^PG^H_{yzj}=\frac{-1}{4\pi}(x-x')\,{}^P\gamma^H_{ij}, \quad \text{(A-7d)}
$$

where

$$
{}^P\gamma^H_{ij}=\frac{e^{ik_jR}}{R}\Big[\frac{1}{R^2}+\frac{ik_j}{R}\Big]. \quad \text{(A-8)}
$$

The secondary elements are

$$
{}^SG^H_{xxj}=\frac{1}{4\pi}\Big\{\frac{\partial}{\partial x'}\Big[\frac{(y-y')}{r}\Big]{}^S\gamma^H_{1j}\Big\}, \quad \text{(A-9a)}
$$

$$
{}^SG^H_{yxj}=\frac{-1}{4\pi}\Big\{\frac{\partial}{\partial x'}\Big[\frac{(x-x')}{r}\Big]{}^S\gamma^H_{1j}+{}^S\gamma^H_{3j}\Big\}, \quad \text{(A-9b)}
$$

$$
{}^SG^H_{zxj}=\frac{1}{4\pi}\Big\{\frac{\partial}{\partial y'}[{}^S\gamma^H_{4j}]\Big\}, \quad \text{(A-9c)}
$$

$$
{}^SG^H_{xyj}=\frac{1}{4\pi}\Big\{\frac{\partial}{\partial y'}\Big[\frac{(y-y')}{r}\Big]{}^S\gamma^H_{1j}-{}^S\gamma^H_{3j}\Big\}, \quad \text{(A-9d)}
$$

$$
{}^SG^H_{yyj}=\frac{-1}{4\pi}\Big\{\frac{\partial}{\partial y'}\Big[\frac{(x-x')}{r}\Big]{}^S\gamma^H_{1j}\Big\}, \quad \text{(A-9e)}
$$

$$
{}^SG^H_{zyj}=\frac{-1}{4\pi}\Big\{\frac{\partial}{\partial x'}[{}^S\gamma^H_{4j}]\Big\}, \quad \text{(A-9f)}
$$

$$
{}^SG^H_{xzj}=\frac{-1}{4\pi}\Big\{\Big[\frac{(y-y')}{r}\Big]{}^S\gamma^H_{5j}\Big]\Big\}, \quad \text{(A-9g)}
$$

$$
{}^SG^H_{yzj}=\frac{1}{4\pi}\Big\{\Big[\frac{(x-x')}{r}\Big]{}^S\gamma^H_{5j}\Big]\Big\}, \quad \text{(A-9h)}
$$

and

$$
{}^SG^H_{zzj}=0, \quad \text{(A-9i)}
$$

with transforms

$$
{}^S\gamma^H_{1j}=\int_0^\infty\{[{}^xA^{TM}_j-{}^xA^{TE}_j]e^{+u_j(z-d_{j-1})}
+[(e^{-u_j(z'-d_{j-1})}+{}^xA^{TM}_j)^{-}R^{TM}_j
+(e^{-u_j(z'-d_{j-1})}+{}^xA^{TE}_j)^{-}R^{TE}_j]e^{-u_j(z-d_{j-1})}\}J_1(\lambda r)d\lambda, \quad \text{(A-10a)}
$$

$$
{}^S\gamma^H_{3j}=\int_0^\infty\{[{}^xA^{TE}_j]e^{+u_j(z-d_{j-1})}
-[(e^{-u_j(z'-d_{j-1})}+{}^xA^{TE}_j)^{-}R^{TE}_j]e^{-u_j(z-d_{j-1})}\}\lambda J_0(\lambda r)d\lambda, \quad \text{(A-10b)}
$$

$$
{}^S\gamma^H_{4j}=\int_0^\infty\frac{1}{u_j}\{[{}^xA^{TE}_j]e^{+u_j(z-d_{j-1})}
+[(e^{-u_j(z'-d_{j-1})}+{}^xA^{TE}_j)^{-}R^{TE}_j]e^{-u_j(z-d_{j-1})}\}\lambda J_0(\lambda r)d\lambda, \quad \text{(A-10c)}
$$

and

$$
{}^S\gamma^H_{5j}=\int_0^\infty\frac{\lambda^2}{u_j}\{[{}^zA^{TM}_j]e^{+u_j(z-d_{j-1})}
+[(e^{-u_j(z'-d_{j-1})}+{}^zA^{TM}_j)^{-}R^{TM}_j]e^{-u_j(z-d_{j-1})}\}J_1(\lambda r)d\lambda. \quad \text{(A-10d)}
$$

Since just complementary potential solutions exist in layers other than that containing the source, only secondary Green's function elements are defined. The forms of the secondary elements are identical to equations (A-6) and (A-10), with l substituted for j everywhere, and will not be rewritten. The pertinent Hankel transforms for the elecric elements, for $l>j$, are

$$
{}^S\gamma^E_{1l}=\int_0^\infty\{[u_l\,{}^+B^{TM}_{lj}({}^xA^{TM}_j e^{+u_jh_j}/{}^+R^{TM}_j)\,{}^+R^{TM}_l
$$

$$+\frac{k_l^2}{u_j}\,{}^{+}B_{lj}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})\,{}^{+}R_l^{TE}]e^{+u_l(z-d_l)}$$

$$-[u_l\,{}^{+}B_{lj}^{TM}\,({}^{x}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})$$

$$-\frac{k_l^2}{u_j}\,{}^{+}B_{lj}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})\,{}^{+}R_l^{TE}]e^{+u_l(z-d_l)}\}$$

$$J_1(\lambda r)d\lambda, \qquad \text{(A-11a)}$$

$${}^{S}\gamma_{3l}^{E}=\int_0^\infty \frac{1}{u_j}\{[{}^{+}B_{lj}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})]$$
$$\cdot[{}^{+}R_l^{TE}e^{+u_l(z-d_l)}+e^{-u_l(z-d_l)}]\}\lambda J_0(\lambda r)d\lambda, \qquad \text{(A-11b)}$$

$${}^{S}\gamma_{4l}^{E}=\int_0^\infty \{[{}^{+}B_{lj}^{TM}\,({}^{x}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})]$$
$$\cdot[{}^{+}R_l^{TM}e^{+u_l(z-d_l)}+e^{-u_l(z-d_l)}]\}\lambda J_0(\lambda r)d\lambda, \qquad \text{(A-11c)}$$

$${}^{S}\gamma_{5l}^{E}=\int_0^\infty \frac{\lambda^2}{u_j}\{[{}^{+}B_{lj}^{TM}\,({}^{z}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})]$$
$$\cdot[{}^{+}R_l^{TM}e^{+u_l(z-d_l)}+e^{-u_l(z-d_l)}]\}\lambda J_0(\lambda r)d\lambda, \qquad \text{(A-11d)}$$

and

$${}^{S}\gamma_{6l}^{E}=\int_0^\infty \lambda^2\frac{u_l}{u_j}\{[{}^{+}B_{lj}^{TM}\,({}^{z}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})]$$
$$\cdot[{}^{+}R_l^{TM}e^{+u_l(z-d_l)}-e^{-u_l(z-d_l)}]\}J_1(\lambda r)d\lambda. \qquad \text{(A-11e)}$$

The magnetic transforms are

$${}^{S}\gamma_{1l}^{H}=\int_0^\infty \{[{}^{+}B_{lj}^{TM}\,({}^{x}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})\,{}^{+}R_l^{TM}$$

$$-\frac{\hat{z}_ju_l}{\hat{z}_lu_j}\,{}^{+}B_{lj}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})\,{}^{+}R_l^{TE}]e^{+u_z(z-d_l)}$$

$$+[{}^{+}B_{lj}^{TM}\,({}^{x}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})$$

$$+\frac{\hat{z}_ju_l}{\hat{z}_lu_j}\,{}^{+}B_{lj}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})]e^{-u_l(z-d_l)}\}$$

$$J_1(\lambda r)d\lambda, \qquad \text{(A-12a)}$$

$${}^{S}\gamma_{3l}^{H}=\int_0^\infty \frac{\hat{z}_ju_l}{\hat{z}_lu_j}\{[{}^{+}B_{lj}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})]$$

$$\cdot[{}^{+}R_l^{TE}e^{+u_l(z-d_l)}-e^{-u_l(z-d_l)}\}\lambda J_0(\lambda r)d\lambda, \qquad \text{(A-12b)}$$

$${}^{S}\gamma_{4l}^{H}=\int_0^\infty \frac{\hat{z}_j}{\hat{z}_lu_j}\{[{}^{+}B_{ll}^{TE}\,({}^{x}A_j^{TE}e^{+u_jh_j}/{}^{+}R_j^{TE})]$$
$$\cdot[{}^{+}R_l^{TE}e^{+u_l(z-d_l)}+e^{-u_l(z-d_l)}\}\lambda J_0(\lambda r)d\lambda, \qquad \text{(A-12c)}$$

and

$${}^{S}\gamma_{5l}^{H}=\int_0^\infty \frac{\lambda^2}{u_j}\{[{}^{+}B_{lj}^{TM}\,({}^{z}A_j^{TM}e^{+u_jh_j}/{}^{+}R_j^{TM})]$$
$$\cdot[{}^{+}R_l^{TM}e^{+u_l(z-d_l)}+e^{-u_l(z-d_l)}\}J_1(\lambda r)d\lambda. \qquad \text{(A-12d)}$$

Finally, in layer $l<j$, the electric Hankel transforms are

$${}^{S}\gamma_{1l}^{E}=\int_0^\infty (\{[u_l\,{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_j-1)}+{}^{x}A_j^{TM})$$

$$+\frac{k_l^2}{u_j}\,{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_j-1)}+{}^{x}A_j^{TE})]e^{+u_l(z-d_{l-1})}$$

$$-[u_l\,{}^{-}B_{lj}^{TM}\,(e^{-u_j(z'-d_j-1)}+{}^{x}A_j^{TM})\,{}^{-}R_l^{TM})$$

$$-\frac{k_l^2}{u_j}\,{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_j-1)}+{}^{x}A_j^{TE})\,{}^{-}R_l^{TE})]e^{-u_l(z-d_{l-1})}\}$$

$$J_1(\lambda r)d\lambda \qquad \text{(A-13a)}$$

$${}^{S}\gamma_{3l}^{E}=\int_0^\infty \frac{1}{u_j}\{[{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TE})$$
$$\cdot[e^{+u_l(z-d_{l-1})}+{}^{-}R_l^{TE}e^{-u_l(z-d_{l-1})}]\}\lambda J_0(\lambda r)d\lambda \qquad \text{(A-13b)}$$

$${}^{S}\gamma_{4l}^{E}=\int_0^\infty \{[{}^{-}B_{lj}^{TM}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TM})]$$
$$\cdot[e^{+u_l(z-d_{l-1})}+{}^{-}R_l^{TM}e^{-u_l(z-d_{l-1})}]\}\lambda J_0(\lambda r)d\lambda \qquad \text{(A-13c)}$$

$${}^{S}\gamma_{5l}^{E}=\int_0^\infty \frac{\lambda^2}{u_j}\{[{}^{-}B_{lj}^{TM}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TM})]$$
$$\cdot[e^{+u_l(z-d_{l-1})}+{}^{-}R_l^{TM}e^{-u_l(z-d_{l-1})}]\}\lambda J_0(\lambda r)d\lambda \qquad \text{(A-13d)}$$

and

$${}^{S}\gamma_{6l}^{E}=\int_0^\infty \lambda^2\frac{u_l}{u_j}\{[{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_{j-1})}+{}^{z}A_j^{TM})]$$
$$\cdot[e^{+u_l(z-d_{l-1})}-{}^{-}R_l^{TM}e^{-u_l(z-d_{l-1})}]\}J_1(\lambda r)d\lambda, \qquad \text{(A-13e)}$$

with the magnetic integrals

$${}^{S}\gamma_{1l}^{H}=\int_0^\infty \{[{}^{-}B_{lj}^{TM}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TM})$$

$$-\frac{\hat{z}_ju_l}{\hat{z}_lu_l}\,{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TE})]e^{+u_l(z-d_{l-1})}$$

$$+[{}^{-}B_{lj}^{TM}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TM})\,{}^{-}R_l^{TM}$$

$$+\frac{\hat{z}_ju_l}{\hat{z}_lu_j}\,{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TE})\,{}^{-}R_l^{TE}]e^{-u_l(z-d_{l-1})}\}$$

$$J_1(\lambda r)d\lambda, \qquad \text{(A-14a)}$$

$${}^{S}\gamma_{3l}^{H}=\int_0^\infty \frac{\hat{z}_ju_l}{\hat{z}_lu_j}\{[{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TE})]$$
$$\cdot[e^{+u_j(z-d_{j-1})}-{}^{-}R_l^{TE}e^{-u_l(z-d_{l-1})}]\}\lambda J_0(\lambda r)d\lambda \qquad \text{(A-14b)}$$

$${}^{S}\gamma_{4l}^{H}=\int_0^\infty \frac{\hat{z}_j}{\hat{z}_lu_j}\{[{}^{-}B_{lj}^{TE}\,(e^{-u_j(z'-d_{j-1})}+{}^{x}A_j^{TE})]$$
$$\cdot[e^{+u_l(z-d_{l-1})}+{}^{-}R_l^{TE}e^{-u_l(z-d_{l-1})}]\}\lambda J_0(\lambda r)d\lambda \qquad \text{(A-14c)}$$

and

$${}^{S}\gamma_{5l}^{H}=\int_0^\infty \frac{\lambda^2}{u_j}\{[{}^{-}B_{lj}^{TM}\,(e^{-u_j(z'-d_{j-1})}+{}^{z}A_j^{TM})]$$
$$\cdot[e^{+u_l(z-d_{l-1})}+{}^{-}R_l^{TM}e^{-u_l(z-d_{l-1})}]\}J_1(\lambda r)d\lambda \qquad \text{(A-14d)}$$

적분방정식을 사용한 3차원 MT 모델링에서의 텐서 그린 적분의 계산

김희준 · 이동성

요 약: 적분방정식을 사용한 3차원 전자기 모델링에 나오는 많은 텐서 그린 적분의 수치계산에 신속 한켈변환 (FHT) 아르고리즘 (Anderson, 1982)을 적용하였다. 한켈변환은 FHT에서 사용가능한 연관 및 지연 중합으로 효율적으로 계산할 수 있다. 먼저 수평 층서모형에 대한 텐서 그린 적분을 보여주고 난 다음 이들을 FHT로 신속하게 계산할 수 있도록 서로 연관된 형태의 함수로 고쳐쓴다. FHT로 연관된 한켈변환의 전행열이 단일 직접 중합과 거의 비슷한 계산시간으로 신속 정확하게 구해진다. 5층 수평 층서모형에 대한 컴퓨터실험의 결과, FHT는 직접 및 지연 중합법에 비하여 각각 117 및 4배 빠르다.