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BLOW–UP OF THE GENERALIZED FRIEDMAN–GIGA SYSTEM

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1. Introduction

In this paper, we consider a parabolic system

$$\frac{\partial u_i}{\partial t} = \alpha_i \frac{\partial^2 u_i}{\partial x^2} + h_i(u_1, u_2, \cdots, u_n), \tag{1}$$

where $-a < x < a$, $t > 0$, $\alpha_i > 0$, $i = 1, 2, \cdots, n$ with

$$\begin{aligned} &u_i(\pm a, t) = 0, \ t \geq 0, \\ &u_i(x, 0) = u_{i_0}(x), \ -a < x < a, \ i = 1, 2, \cdots, n, \quad u_{i_0} \in C[-a, a]. \end{aligned} \tag{2}$$

Many papers were concerned with the blow-up of the following semilinear parabolic PDEs

$$\begin{aligned} u_t &= \Delta u + f(u) \quad \text{in} \quad \Omega \subset \mathbb{R}^n \\ u &= 0 \quad \text{on} \quad \partial\Omega \end{aligned}$$

where $f(u)$ is mostly e^u or $u^p (p > 1)$.(For reference, see J. Bebernes and D. Eberly[1] or references therein, Friedman and Kohn[4], Giga and Kohn[5] , Bressan[2], Liu[6], etc.)

However, just a few mentioned about the blow-up of the system of semilinear parabolic equations. In particular, Gang and Sleeman[10] investigated the blow-up of the system of semilinear parabolic PDEs(which we call Friedman-Giga system)using the Liapunov function.

We shall establish a blow-up for solution $u_i (i = 1, 2, \cdots, n)$ of (1) and (2) under the certain assumptions on h_i(assumption (6) which will be

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later). The main idea is based on the method of Liapunov function(see Gang and Sleeman[10]).

2. The Basic Theorem and Examples

In this section we have a general theorem concerning the nonexistence of global solutions to the pababolic system (1) and (2) and examples.

First, let $\phi(x) = \frac{\pi}{4a}\cos(\frac{\pi}{2a}x)$ be the first eigenfunction which solves the eigenvalue problem

$$\begin{aligned} &\phi_{xx}(x) + \lambda\phi(x) = 0, \ |x| < a, \\ &\phi(\pm a) = 0, \ \lambda = \frac{\pi^2}{4a^2}. \end{aligned} \tag{3}$$

Then $\phi(x)$ is positive for all $x \in (-a, a)$ and $\int_{-a}^{a} \phi(x)dx = 1$.

Suppose that $u_i(x,t)(i = 1, 2, \cdots, n)$ are local solutions to the problem(1) and (2). The existence of above solutions $u_i(x,t)$ on $[-a, a] \times [0, T)$ for some $T > 0$ is guaranteed by the standard PDE theory.(see, say, Smoller[11].)

Let

$$g_i(t) = \int_{-a}^{a} \phi(x)u_i(x,t)dx, \ i = 1, 2, \cdots, n, \ 0 \leq t < T. \tag{4}$$

Then

$$\begin{aligned} g_i'(t) &= \int_{-a}^{a} \phi(x)\frac{\partial}{\partial t}u_i(x,t)dx \\ &= \alpha_i \int_{-a}^{a} \phi(x)\frac{\partial^2}{\partial x^2}u_i(x,t)dx + \int_{-a}^{a} \phi(x)h_i(u_1, u_2, \cdots, u_n)dx. \end{aligned}$$

An integration by parts and (4) shows that

$$g_i'(t) = -\lambda_i g_i(t) + \int_{-a}^{a} \phi(x)h_i(u_1, u_2, \cdots, u_n)dx \tag{5}$$

where $\lambda_i = \alpha_i\lambda, \ i = 1, 2, \cdots, n$.

Assume that there exists a function $G_i : \mathbb{R}^n \to \mathbb{R}$ such that the inequality

$$\int_{-a}^{a} \phi(x)h_i(u_1, u_2, \cdots, u_n)dx$$
$$\geq G_i(\int_{-a}^{a} \phi(x)u_1(x,t)dx, \int_{-a}^{a} \phi(x)u_2(x,t)dx, \cdots, \int_{-a}^{a} \phi(x)u_n(x,t)dx),$$

that is,

(6)
$$\int_{-a}^{a} \phi(x)h_i(u_1, u_2, \cdots, u_n)dx \geq G_i(g_1(t), g_2(t), \cdots, g_n(t)), \ i = 1, 2, \cdots, n$$

hold. If we can choose such a function G_i then from (5) we have the differential inequality

$$\begin{aligned} g_i'(t) &\geq -\lambda_i g_i(t) + G_i(g_1(t), g_2(t), \cdots, g_n(t)), \\ g_i(0) &= \int_{-a}^{a} \phi(x)u_i(x,0)dx, \ i = 1, 2, \cdots, n. \end{aligned} \tag{7}$$

Now, we consider the system of ordinary differential equations,

$$\begin{aligned} y_i'(t) &= -\lambda_i y_i(t) + G_i(y_1(t), y_2(t), \cdots, y_n(t)) \\ y_i(0) &= g_i(0), \ i = 1, 2, \cdots, n. \end{aligned} \tag{8}$$

If above functions G_i can be chosen to be the following sense then there are several ways in which blow-up behavior of solutions to the system (8) is linked to a similar phenomenon for (7).

DEFINITION 2.1. A function $G : D \subset \mathbb{R}^n \to \mathbb{R}^n$ is said to be *quasi-monotone nondecreasing* if $\frac{\partial G_i}{\partial y_j} \geq 0$ for all $i \neq j$ where $G = (G_1, G_2, \cdots, G_n)$, $y = (y_1, y_2, \cdots, y_n)$.

Next result is needed to prove the Theorem 2.3.

LEMMA 2.2. *Suppose that $G(y) = (G_1(y), G_2(y), \cdots, G_n(y))$ is quasi-monotone nondecreasing. If there exists a time $\tau > 0$ such that the solution to the system (8) exists for $t \in [0, \tau)$ and $y_1^2(t)+y_2^2(t)+\cdots+y_n^2(t) \to \infty$ as $t \to \tau$ then the solution of (7) exists for only a finite time interval $0 \leq t < t_0 (t_0 \leq \tau)$ and $g_1^2(t) + g_2^2(t) + \cdots + g_n^2(t) \to \infty$ as $t \to t_0$.*

Proof. See Lakshmikantham and Leela[7].

THEOREM 2.3. *Suppose that the inequality (6) holds in which $G(y) = (G_1(y), G_2(y), \cdots, G_n(y))$ is quasimonotone nondecreasing and that the solution of (8) blows up in finite time. Then the solution to the system (1) and (2) blows up in finite time.*

Proof. The proof of Theorem 2.3 follows from a direct consequence of Lemma 2.2.

Here is an example.
Example 2.4. Friedman-Giga System

$$\begin{aligned} u_t - \alpha u_{xx} &= f(v) \quad (-a < x < a,\ t > 0) \\ v_t - \beta v_{xx} &= g(u) \quad (-a < x < a,\ t > 0) \\ u(\pm a, t) &= 0 = v(\pm a, t), \quad t > 0, \\ u(x,0) &= u_0(x), \quad v(x,0) = v_0(x) \quad \text{for } x \in (-a,a),\ u_0, v_0 \in C[-a,a]. \end{aligned}$$

More generally we have the following example.
Example 2.5. Generalized Friedman-Giga System

$$(9) \qquad \begin{aligned} &\frac{\partial u_i}{\partial t} = \alpha_i \frac{\partial^2 u_i}{\partial x^2} + f_i(u_{\sigma(i)}) \text{ for } (x,t) \in (-a,a) \times (0,T) \\ &u_i(\pm a, t) = 0,\ t > 0, \\ &u_i(x,0) = u_{i_0}(x), \text{ for } x \in (-a,a),\ u_{i_0} \in C[-a,a],\ i = 1,2,\cdots,n, \end{aligned}$$

where σ is the permutation of $\{1,2,\cdots,n\}$ and $f_i(u_{\sigma(i)})$ is $h_i(u_1, u_2, \cdots, u_n)$ in (1).
For the above examples we need the following definition.

DEFINITION 2.6. A function $f(s)$ is called the *convex minimal function* of an n-pair of given functions $\{f_1(s), f_2(s), \cdots, f_n(s)\}$ where $f_i(s) > 0, f_i'(s) > 0, f_i''(s) \geq 0$ for all $s \in (0,\infty), i = 1,2,\cdots,n$ provided that $f(s)$ is positive, continuous and piecewise convex with $f_i(s) \geq f(s)$ for all $s \in (0,\infty)$ for all i and $f'(s)$ is positive and continuous in $(0,\infty)$. We will denote the convex minimal function $f(s)$ as $cmf\{f_1(s), f_2(s), \cdots, f_n(s)\}$ and also this convex minimal function is not represented uniquely.

The construction of convex minimal function

We will construct convex minimal function by induction.

1. $cmf\{\{f_1(s)\}\} = f_1(s)$

2. $cmf\{f_1(s), f_2(s)\}$ is defined as follows; if

$$\begin{aligned} &f_1(s) < f_2(s),\ 0 < s < s_0,\\ &f_1(s) = f_2(s),\ s = s_0,\\ &f_1(s) > f_2(s),\ s > s_0, \end{aligned}$$

then we can choose a line $y = C_1 s - C_2 (C_1, C_2 > 0)$ to be tangent to $f_1(s)$ at $(s_1, f_1(s_1))$ for some $s_1 \in (0, s_0)$ and $f_2(s)$ at $(s_2, f_2(s_2))$ for some $s_2 \in (s_0, \infty)$ simultaneously. In this case we let

$$cmf\{f_1(s), f_2(s)\} = f(s) = \begin{cases} f_1(s),\ 0 < s \le s_1 \\ C_1 s - C_2,\ s_1 < s \le s_2 \\ f_2(s),\ s_2 < s. \end{cases}$$

If the equation $f_1(s) = f_2(s)$ has more than one solution in $(0, \infty)$ then the convex minimal function $f(s)$ can be constructed by the repeated use of above method 2.

Note that for $f(s) = cmf\{f_1(s), f_2(s)\}$, $f(s) > 0$, $f'(s) > 0$ and $f''(s) \ge 0$ for all $s \in (0, \infty)$.

3. $$\begin{aligned} &cmf\{f_1(s), f_2(s), \cdots, f_{n+1}(s)\}\\ &= cmf\{cmf\{f_1(s), f_2(s), \cdots, f_n(s)\}, f_{n+1}(s)\} \text{ for } n \ge 2. \end{aligned}$$

Now, we consider the system (9).

Using the Jensen's inequality, we have

$$\begin{aligned} \int_{-a}^{a} \phi(x) f_1(u_{\sigma(1)}) dx &\ge f_1\Big(\int_{-a}^{a} \phi(x) u_{\sigma(1)}(x,t) dx\Big) \ (\text{since} \int_{-a}^{a} \phi(x) dx = 1)\\ &= f_1(g_{\sigma(1)}(t)). \end{aligned}$$

Similary,

$$\int_{-a}^{a} \phi(x) f_i(u_{\sigma(i)}) dx \ge f_i(g_{\sigma(i)}(t)),\ i = 2, 3, \cdots, n$$

If we take

$$\begin{aligned} &G_1(y_1, y_2, \cdots, y_n) = f_1(y_{\sigma(1)}),\\ &G_2(y_1, y_2, \cdots, y_n) = f_2(y_{\sigma(2)}),\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &G_n(y_1, y_2, \cdots, y_n) = f_n(y_{\sigma(n)}) \end{aligned}$$

respectively then (6) is satisfied. Also, such G_i is quasimonotone non-decreasing for all $i = 1, 2, \cdots, n$. Using the inequalities (7) we have the ordinary differential inequalities

$$
\begin{aligned}
&g_1'(t) \geq -\lambda\alpha_1 g_1(t) + f_1(g_{\sigma(1)}(t)),\\
&g_2'(t) \geq -\lambda\alpha_2 g_2(t) + f_2(g_{\sigma(2)}(t)),\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&g_n'(t) \geq -\lambda\alpha_n g_n(t) + f_n(g_{\sigma(n)}(t))
\end{aligned} \tag{10}
$$

where $g_1(0) = \int_{-a}^{a} \phi(x)u_{1_0}(x)dx, g_2(0) = \int_{-a}^{a} \phi(x)u_{2_0}(x)dx, \cdots, g_n(0) = \int_{-a}^{a} \phi(x)u_{n_0}(x)dx$.

Setting $\lambda_i = \lambda\alpha_i (i = 1, 2, \cdots, n)$ we need to consider the system

$$
\begin{aligned}
&y_1'(t) = -\lambda_1 y_1 + f_1(y_{\sigma(1)}(t)),\\
&y_2'(t) = -\lambda_2 y_2 + f_2(y_{\sigma(2)}(t)),\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&y_n'(t) = -\lambda_n y_n + f_n(y_{\sigma(n)}(t))
\end{aligned} \tag{11}
$$

where $y_i(t) \geq 0$ and $y_i(0) = g_i(0)$, $i = 1, 2, \cdots, n$.

Also we introduce the functional

$$V(t) = \frac{1}{n}(y_1(t) + y_2(t) + \cdots + y_n(t)). \tag{12}$$

Let $f(s) = cmf\{f_1(s), f_2(s), \cdots, f_n(s)\}$ and $\lambda_0 = \lambda \max\{\alpha_i \,|\, i = 1, 2, \cdots, n\}$. Differentiating V with respect to t along the orbits of (11) we have

$$
\begin{aligned}
\frac{dV(t)}{dt} &= \frac{1}{n}(y_1'(t) + y_2'(t) + \cdots + y_n'(t))\\
&= -\frac{1}{n}(\lambda_1 y_1(t) + \lambda_2 y_2(t) + \cdots + \lambda_n y_n(t))\\
&+ \frac{1}{n}(f_1(y_{\sigma(1)}(t)) + f_2(y_{\sigma(2)}(t)) + \cdots + f_n(y_{\sigma(n)}(t)))\\
&\geq -\frac{1}{n}\lambda_0(y_1(t) + y_2(t) + \cdots + y_n(t))\\
&+ \frac{1}{n}(f(y_1(t)) + f(y_2(t)) + \cdots + f(y_n(t)))\\
&\geq -\frac{1}{n}\lambda_0(y_1(t) + y_2(t) + \cdots + y_n(t)) + f(\frac{y_1(t) + y_2(t) + \cdots + y_n(t)}{n})\\
&= -\lambda_0 V(t) + f(V(t)).
\end{aligned}
$$

Hence we have the differential inequality

$$V' \geq -\lambda_0 V + f(V),$$
$$V(0) = \frac{1}{n}(g_1(0) + g_2(0) + \cdots + g_n(0)). \tag{13}$$

Our first result here is the following theorem.

THEOREM 2.7. *If the differential equation*

$$w' = -\lambda_0 w + f(w),$$
$$w(0) = \frac{1}{n}(g_1(0) + g_2(0) + \cdots + g_n(0)) \tag{14}$$

has a solution which exists in finite time interval $[0, t_0)$ *and* $w(t) \to \infty$ *as* $t \to t_0$*, then the solution of the system (9) with initial data* $u_{i_0}(i = 1, 2, \cdots, n)$ *constrained by* $w(0)$ *blows up in finite time.*

Proof. By Theorem 2.3 and above results, it suffices to show that the nonnegative solutions $y_i(t)(i = 1, 2, \cdots, n)$ of the system (11) blow up in finite time, which is equivalent to prove that the solution $V(t) = \frac{1}{n}(y_1(t) + y_2(t) + \cdots + y_n(t))$ of the differential inequality (13) blows up in finite time. This is achieved by use of comparison theorem(See Lakshmikantham and Leela[7]) and finite time blow-up of the solution $w(t)$ of the differential equation (14).

We shall later require the following auxiliary tool.

LEMMA 2.8. *If the algebraic equation* $f(w) = \lambda_0 w$ *has*

(a) two roots $0 \leq w_1 < w_2$ *and* $w(0) > w_2$ *then* $w(t, w_0) \to \infty$ *as* $t \to t_0 < \infty$.

(b) only one root $0 \leq w_1$ *and* $w(0) > w_1$ *then* $w(t, w_0) \to \infty$ *as* $t \to t_0 < \infty$.

(c) no real solution in $[0, \infty)$ *then* $w(t, w_0) \to \infty$ *as* $t \to t_0 < \infty$.

(d) the solution of an interval $[w_1, w_2]$, $0 \leq w_1 < w_2$, *and* $w(0) > w_2$ *then* $w(t, w_0) \to \infty$ *as* $t \to t_0 < \infty$.

Proof. It follows from the simple ODE theory.

Combining all the above results together we have the following theorem.

THEOREM 2.9. *Let* $\phi(x) = \frac{\pi}{4a}\cos(\frac{\pi}{2a})x, \lambda = \frac{\pi^2}{4a^2}, \lambda_i = \alpha_i\lambda, \lambda_0 = \lambda \max\{\alpha_i \mid i = 1, 2, \cdots, n\}$, *and* $f(s) = cmf\{f_1(s), f_2(s), \cdots, f_n(s)\}$ *and let* $g_i(0) = \int_{-a}^{a}\phi(x)u_{i_0}(x)dx, i = 1, 2, \cdots, n$. *Then if*

(a) $f(s) = \lambda_0 s$ *has two solutions* s_1 *and* s_2 *with* $0 < s_1 < s_2$ *and if* $g_1(0) + g_2(0) + \cdots + g_n(0) > ns_2$ *then the solutions* s *to the system (9) blow up in finite time.*

(b) $f(s) = \lambda_0 s$ *has only one solution* s_1 *and if* $g_1(0)+g_2(0)+\cdots+g_n(0) > ns_1$ *then the solutions* s *to the system (9) blow up in finite time.*

(c) $f(s) \neq \lambda_0 s$ *for any* $s > 0$ *and if* $g_1(0) + g_2(0) + \cdots + g_n(0) > 0$ *then the solutions* s *to the system (9) blow up in finite time.*

(d) $f(s) = \lambda_0 s$ *on* $[s_1, s_2]$ *and* $f(s) > \lambda_0 s$ *on* $(0, s_1) \cup (s_2, \infty)$ *and if* $g_1(0) + g_2(0) + \cdots + g_n(0) > ns_2$ *then the solutions* s *to the system (9) blow up in finite time.*

We now specilize our results and analyze the system

$$(15) \qquad \begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial^2 u_1}{\partial x^2} + u_{\sigma(1)}^{p_1} \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial^2 u_2}{\partial x^2} + u_{\sigma(2)}^{p_2} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \dfrac{\partial u_n}{\partial t} = \dfrac{\partial^2 u_n}{\partial x^2} + u_{\sigma(n)}^{p_n} \end{cases}$$

with initial boundary conditions (2) and exponents $p_i > 1$, $i = 1, 2, \cdots, n$.

Here, we shall choose the convex minimal function.

LEMMA 2.10. *Let* $q = \max\{p_i \mid i = 1, 2, \cdots, n\}$ *and let* $r = \min\{p_i \mid i = 1, 2, \cdots, n\} > 1$. *Then the convex minimal function* $cmf\{s^{p_1}, s^{p_2}, \cdots, s^{p_n}\}$ *of an* n*-pair* $\{s^{p_1}, s^{p_2}, \cdots, s^{p_n}\}$ *in* $(0, \infty)$ *is given by*

$$cmf\{s^{p_1}, s^{p_2}, \cdots, s^{p_n}\} = \begin{cases} s^q, & 0 \le s < s_1, \\ C_1 s - C_2, & s_1 \le s < s_2, \\ s^r, & s_2 \le s \end{cases}$$

where

$$s_1 = (\frac{r}{q})^{\frac{r}{q-r}}(\frac{q-1}{r-1})^{\frac{r-1}{q-r}},$$

$$s_2 = (\frac{r}{q})^{\frac{q}{q-r}}(\frac{q-1}{r-1})^{\frac{q-1}{q-r}}, \quad s_1 < 1 < s_2,$$

$$C_1 = \frac{r^{\frac{r(q-1)}{q-r}}}{q^{\frac{q(r-1)}{q-r}}}(\frac{q-1}{r-1})^{\frac{(q-1)(r-1)}{q-r}},$$

$$C_2 = (\frac{r}{q})^{\frac{qr}{q-r}}\frac{(q-1)^{\frac{r(q-1)}{q-r}}}{(r-1)^{\frac{q(r-1)}{q-r}}}, \quad C_2 < C_1 < C_2 + 1.$$

Proof. The proof of the above lemma follows from the definition of the convex minimal function and a manipulative calculation.

THEOREM 2.11. *Let $q = \max\{p_i \mid i = 1, 2, \cdots, n\}$ and let $r = \min\{p_i \mid i = 1, 2, \cdots, n\} > 1$. Suppose that $\phi(x) = \frac{\pi}{4a}\cos(\frac{\pi}{2a})x, x \in [-a, a], \lambda_0 = \frac{\pi^2}{4a^2}\max\{\alpha_i | i = 1, 2, \cdots, n\}$, and let*

$$\xi(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, \alpha) \equiv \int_{-a}^{a} \phi(x)(u_{1_0} + u_{2_0} + \cdots + u_{n_0})dx - n\lambda_0^{\frac{1}{\alpha-1}},$$

$$\zeta(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, C_1, C_2) \equiv \int_{-a}^{a} \phi(x)(u_{1_0} + u_{2_0} + \cdots + u_{n_0})dx - \frac{nC_2}{C_1 - \lambda_0}.$$

If

(a) $q = r > 1$ and $\xi(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, q) = \xi(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, r) > 0$ then the solutions to the system (15) blow up in finite time.

(b) $q > r > 1$ and

(i) $s_1^{q-1} \geq \lambda_0$ then $\xi(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, q) > 0$ implies that the solutions of the system (15) blow up in finite time.

(ii) $s_1^{q-1} < \lambda_0 \leq s_2^{r-1}$ then $\zeta(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, C_1, C_2) > 0$ implies that the solutions of (15) blow up in finite time.

(iii) $\lambda_0 > s_2^{r-1}$ then $\xi(u_{1_0}, u_{2_0}, \cdots, u_{n_0}, \lambda_0, r) > 0$ implies that the solutions of (15) blow up in finite time.

Proof. Since $f(s) = cmf\{s^{p_1}, s^{p_2}, \cdots, s^{p_n}\}$ and $f(s) = \lambda_0 s$ have at most two intersection points $s_1 = 0, s_2 > 0$, the proof of Theorem 2.11 follows directly from the Theorem 2.9.

REMARK. *1. Instead of the functions* $s^{p_1}, s^{p_2}, \cdots, s^{p_n}$, *we could take* $f_1(s) = A_1(s+\eta_1)^{p_1}, f_2(s) = A_2(s+\eta_2)^{p_2}, \cdots, f_n(s) = A_n(s+\eta_n)^{p_n}$, *where* $A_i > 0, \eta_i \geq 0, p_i > 1, i = 1, 2, \cdots, n$.

2. If $f_i(s) = \exp(p_i s), p_i > 0, i = 1, 2, \cdots, n$, *then we have also the same results trivially.*

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