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STRUCTURE OF A HYPERSURFACE IMMERSED IN A PRODUCT OF TWO SPHERES

SHIN, YONG HO

0.Introduction

Submanifolds immersed in a sphere or a product of spheres have been objects of study in differential geometry. In particular, real hypersurfaces of a sphere could be found out their intrintic character under some specific conditions. Recently, many authors ([7],[10],[11],[12],[16], [23]) have researched the so-called generic submanifolds of a Riemannian manifold which are general notions real hypersurfaces of a Riemannian manifold Among them, the study on generic submanifolds of an odd-dimensional sphere or an even-dimensional Euclidean space was carried out succcessfully. But, the investigation about generic submanifolds of an even-dimensional sphere or a product of two spheres $S^n \times S^n$ has not been reported yet.

Of course, real hypersurfaces of $S^n \times S^n$ a product of two spheres have not had nice results as those of a sphere even though several geometers examined real hypersurfaces of $S^n \times S^n$ ([5],[13],[14]).

So, many geometers are desiring earnestly to suggest the epoch-making models of real hypersurfaces immersed in $S^n \times S^n$.

By the way, K.Yano and M.Okumura [20] defined the (f, g, u, v, λ)-structure induced on submanifolds of codimension 2 of an almost Hermitian manifold or real hypersurfaces of an almost contact metric manifold, which is a very useful method in studying Riemannian manifolds admiting that structure. Also, Yano[18] studied the differential geometry of $S^n \times S^n$ and prove that the (f, g, u, v, λ)-structure is naturally induced on $S^n \times S^n$ as a submanifold of codimension 2 of a $(2n+2)$-dimensional Euclidean space or a real hypersurface of $(2n+1)$-dimensional unit sphere $S^{2n+1}(1)$.

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G.D. Ludden and Okumura[13] stuided the so-called invariant hypersurface of $S^n \times S^n$, which is derived from the almost product structure defined by its projection operators on $S^n \times S^n$.

On the other hand, it is well-known that the so-called $(f, g, u, v, w, \lambda, \mu, \nu)$-sructure is naturally induced on submanifolds of codimension 3 of an almost Hermitian manifold or real hypersurfaces of a manifold with (f, g, u, v, λ)-structure (cf.[8],[9],[22]). Therefore, real hypersurfaces immersed in $S^n \times S^n$ admit the the (f, g, u, v, λ)-structure deduced from the (f, g, u, v, λ)-structure defined on $S^n \times S^n$. From this point of view, S.-S.Eum, U-H.Ki and Y.H.Kim [5] researched partially real hypersurfaces of $S^n \times S^n$ by using the concept of k-invariance.

The purpose of the present paper is devoted to study some intrinsic characters of hypersurfaces immersed in $S^n \times S^n$,characterize global properties of them by using some intergrable condition and prove that the generic submanifold of $S^n \times S^n$ with the almost contact metric structure is the real hypersurface.

In section 1, we recall the intrinsic properties of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ and have some algebraic relationships and structure equations of hypersurfaces of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$.

In section 2, we determine mainly a minimal hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ satisfying $\lambda^2 + \mu^2 + \nu^2 = 1$.

In section 3, we find the necessary and sufficient condition for a hypersurface of $S^n \times S^n$ being k-antiholomorphic and prove its global properties.

1.Structure equations of hypersurfaces of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$

Let E^{n+1} be an $(n+1)$-dimensional Euclidean space and 0 the orgin of the Cartesian coordinate system in E^{n+1}, and denote by X the position vector of point p in E^{n+1} relative to the orgin 0.

We consider a hypersurface $S^n(1/\sqrt{2})$ in E^{n+1} with center at the orgin 0 and radius $1/\sqrt{2}$. Suppose that $S^n(1/\sqrt{2})$ is covered by a system of coordinate neighborhoods $\{U; x^\alpha\}$, where here and in the sequel the indices $\alpha, \beta, \gamma, \delta, \cdots$ run over the range $\{1, 2, \cdots, n\}$. Then $X \cdot X = 1/2$ for the position vector X of the point $S^n(1/\sqrt{2})$, where the dot means the usual inner product of E^{n+1}.

Putting $X_\alpha = \partial_\alpha X$, $M_1 = -\sqrt{2}X$, $g_{\alpha\beta} = X_\alpha \cdot X_\beta$, where $\partial_\alpha = \partial/\partial x^\alpha$, and denoted by ∇_α the operator of the covariant differentiation formed with the first fundamental form $g_{\alpha\beta}$, the equations of Gauss and Weingarten are respectively given by

$$(1.1) \qquad \nabla_\alpha X_\beta = \sqrt{2} g_{\alpha\beta} M_1, \qquad \nabla_\alpha M_1 = -\sqrt{2} X_\alpha.$$

Similary, an n-dimensional sphere $S^n(1/\sqrt{2})$ is also assumed to be covered by a system of coordinate neighborhoods $\{V; y^\kappa\}$. Then the position vector Y of a point of $S^n(1/\sqrt{2})$ satisfies $Y \cdot Y = 1/2$. Here and in the sequel, the indices $\kappa, \mu, \nu, \cdots$ run over the range $\{n+1, \cdots, 2n\}$. Now, we put $Y_\kappa = \partial_\kappa Y$, $M_2 = -\sqrt{2}Y$, $g_{\kappa\mu} = Y_\kappa \cdot Y_\mu$ ($\partial_\kappa = \partial/\partial y^\kappa$) and denoted ∇_κ the operator of covariant differentiation formed with the first fundamental form $g_{\kappa\mu}$ of $S^n(1/\sqrt{2})$. Then the equations of Gauss and Weingarten are respectively given by

$$(1.2) \qquad \nabla_\kappa X_\mu = \sqrt{2} g_{\kappa\mu} M_2, \qquad \nabla_\kappa M_2 = -\sqrt{2} Y_\kappa.$$

Thus we give the differential structure to $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ naturally as a product manifold which is covered by a system of coordinate neighborhoods $\{U \times V; (x^\alpha, y^\kappa)\}$.

Therefore as a submanifold of codimension 2 in a $(2n+2)$-dimensional Euclidean space E^{2n+2},$S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ has a position vector Z of a point in $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ such that

$$Z(z^h) = \begin{pmatrix} X(x^\alpha) \\ Y(y^\kappa) \end{pmatrix},$$

where, here and in the sequel, the indices $h, i, j, k, \cdots$ run over the range $\{1, 2, \cdots, n, n+1, \cdots, 2n\}$. Then we have

$$Z \cdot Z = X \cdot X + Y \cdot Y = 1$$

and hence we see that $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ is a hypersurface of a $(2n+1)$-dimensional unit sphere $S^{2n+1}(1)$ in E^{2n+2}.

Letting $Z_i = \partial_i Z$ and $g_{ji} = Z_j \cdot Z_i$, we get

$$Z_\alpha = \begin{pmatrix} X_\alpha \\ 0 \end{pmatrix}, \qquad Z_\kappa = \begin{pmatrix} 0 \\ Y_\kappa \end{pmatrix}$$

$$(1.3) \qquad g_{ji} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & g_{\kappa\mu} \end{pmatrix}, \qquad g^{ji} = \begin{pmatrix} g^{\alpha\beta} & 0 \\ 0 & g^{\kappa\mu} \end{pmatrix} \quad .$$

g^{ji}, $g^{\alpha\beta}$ and $g^{\kappa\mu}$ are contravariant components of g_{ji}, $g_{\alpha\beta}$ and $g_{\kappa\mu}$ respectively.

Letting

$$(1.4) \qquad C = \begin{pmatrix} -X(x^\alpha) \\ -Y(y^\kappa) \end{pmatrix}, \qquad D = \begin{pmatrix} -X(x^\alpha) \\ Y(y^\kappa) \end{pmatrix},$$

we can easily see that

$$Z_i \cdot C = 0, Z_i \cdot D = 0, C \cdot D = 0, C \cdot C = 1, D \cdot D = 1$$

and hence C and D are mutually orthogonal normal vectors to $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ as a submanifold of codimension 2 in E^{2n+2}.

Let h_{ji} and k_{ji} be the components of the second fundamental tensors respectively relative to the unit normals C and D to $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. Then the equations of Gauss for $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ can be given of the form

$$\nabla_j Z_i = h_{ji} C + k_{ji} D$$

From (1.1) and (1.2), h_{ji} and k_{ji} are of the form

$$(1.5) \qquad (h_{ji}) = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & g_{\kappa\mu} \end{pmatrix}, \qquad (k_{ji}) = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & -g_{\kappa\mu} \end{pmatrix}$$

and consequently we find

$$(1.6) \qquad (h_j^i) = \begin{pmatrix} \delta_\alpha^\beta & 0 \\ 0 & \delta_\kappa^\mu \end{pmatrix}, \qquad (k_j^i) = \begin{pmatrix} \delta_\alpha^\beta & 0 \\ 0 & -\delta_\kappa^\mu \end{pmatrix},$$

where $h_j^i = h_{jh} g^{hi}$ and $k_j^i = k_{jh} g^{hi}$.

It follows from thr first equation of (1.5) and the second equation of (1.6) that

$$(1.7) \qquad h_{ji} = g_{ji}, \quad k_t^t = 0, \quad k_j^t k_t^i = \delta_j^i.$$

Hence, we see that k_j^i determines an almost product syructure on $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$.

On the other hand, as the first fundamental from g_{ji} has the form (1.3), the Chiristoffel symbols $\left\{ {h \atop j\ \ i} \right\}$ form with g_{ji} are all zero but $\left\{ {\alpha \atop \gamma\ \ \beta} \right\}$ and $\left\{ {\lambda \atop \mu\ \ \nu} \right\}$.

Using this fact and differentiating the second fundamental tensor k_i^j covariantly, we have

$$\nabla_j k_i^h = 0.$$

Denoting by l_j the third fundamental tensor relative to the normals C and D, we can write

$$(1.8) \qquad \nabla_j C = -h_j^t Z_t + l_j D, \quad \nabla_j D = -k_j^t Z_t - l_j C.$$

From (1 4),(1.6) and (1.8) it follows that (cf. [3],[19])

$$l_j = 0.$$

Consequently, the equations of Gauss and Weingarten fo $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ regared as a submanifold of codimension 2 in E^{2n+2} become respectively

$$\nabla_j Z_i = h_{ji} C + k_{ji} D, \quad \nabla_j C = -Z_j D, \quad \nabla_j D = -k_j^t Z_t.$$

Thus we can deduce the so-called equations of Gauss

$$(1.9) \qquad K_{kji}{}^h = \delta_k^h g_{ji} - \delta_j^h g_{ki} + k_k^h k_{ji} - k_j^h k_{ki},$$

K_{kji}^h being the components of the curvature tensor of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$.

But, a $(2n+2)$-dimensional Euclidean space E^{2n+2} admits a natural Kaehler structure

$$F = \begin{pmatrix} 0 & -I_1 \\ I_1 & 0 \end{pmatrix},$$

where I_1 denotes the identity matrix of degree $n+1$. It follows that $F^2 = -I$, $FU \cdot FV = U \cdot V$ for arbitary vectors U and V in E^{2n+2}, I being the identity transformation in E^{2n+2}. Linear transformation of Z_j, C and D by f give respectively

$$(1.10) \qquad FZ_j = f_j^t Z_t + u_j C + v_j D, \quad FC = -u^t Z_t + \lambda D, \\ FD = -v^t Z_t - \lambda C,$$

where f_i^h are components of a tensor field of type (1.1), u_i and v_i those of 1-forms and λ a function on $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$, and u^h and v^h are the associated vector fields with u_i and v_i respectively given by $u^h = u_i g^{ih}$ and $v^h = v_i g^{ih}$.

Applying F to (1.10) respectively, we get the so-called (f, g, u, v, λ)-structure given by ([1], [2], [6], [17], [18], [20],)

$$(1.11) \qquad \begin{aligned} & f_j^t f_t^i = -\delta_j^i + u_j u^i + v_j v^i, \\ & u_t f_j^t = \lambda v_j, \quad f_t^h u_t = -\lambda v^h, \quad v_t f_j^t = -\lambda u_j, \\ & f_t^h v^t = \lambda u^h, \quad u_t u^t = v_t v^t = 1 - \lambda^2, \quad u_t v^t = 0, \\ & f_j^t f_i^s g_{ts} = g_{ji} - u_j u_i - v_j v_i. \end{aligned}$$

It is easily verified that $f_{ji} = f_j^t g_{it}$ is skew-symmetric in j and i.

By letting $j = \alpha$ and $j = \kappa$ in (1.10), we find respectively

$$(1.12) \qquad f_\alpha^\beta = 0, \quad u_\alpha + v_\alpha = 0, \quad X_\alpha = f_\alpha^\kappa Y_\kappa - 2u_\alpha Y$$

and

$$f_\kappa^\mu = 0, \quad u_\kappa = v_\kappa, \quad Y_\kappa = -f_\kappa^\alpha X_\alpha - 2u_\kappa X. \tag{1.13}$$

Consequently, f_i^h, u_i, u^h, v_i and v^h are respectively of the form

$$(f_i^h) = \begin{pmatrix} 0 & f_\kappa^\alpha \\ f_\alpha^\nu & 0 \end{pmatrix} \tag{1.14}$$

(1.15)

$$u_i = (u_\alpha, u_\kappa), \quad u^h = \begin{pmatrix} u^\alpha \\ u^\kappa \end{pmatrix}, \quad v_i = (u_\alpha, u_\kappa), \quad v^h = \begin{pmatrix} -u^\alpha \\ u^\kappa \end{pmatrix},$$

where $u^\alpha = u_\beta g^{\alpha\beta}, u^\kappa = u_\mu g^{\kappa\mu}$.

Then, (1.6) and (1.14) imply that

$$k_t^h f_j^t + f_t^h k_j^t = 0, \tag{1.16}$$

that is, K_j^h and f_j^h anticommute each other.

We also find from (1.6) and (1.15)

$$k_j^h u^j = -v^h, \quad k_j^h v^j = -u^h. \tag{1.17}$$

If we differentiate (1.10) covariantly and make use of $\nabla F = 0$, then we have ([2], [21])

$$\begin{aligned} &\nabla_j f_i^h = -g_{ji} u^h + \delta_j^h u_i - k_{ji} v^h + k_j^h v_i, \\ &\nabla_j u_i = f_{ji} - \lambda k_{ji}, \quad \nabla_j v_i = -k_{jt} f_i^t + \lambda g_{ji}, \\ &\nabla_j \lambda = -2v_j. \end{aligned} \tag{1.18}$$

Let M be a hypersurface immersed isometrically in $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ and suppose that M is covered by the system of coordinate neighborhoods $\{\bar{V}; \bar{x}^a\}$, where here and in the sequel, the indices $a, b, c, d, \cdots$ run over the range $\{1, 2, \cdots, 2n-1\}$.

We put $B_c^h = \partial_c x^h(\partial/\partial \bar{x}^c)$. Then B_c^h are $2n-1$ linearly independent vectors of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ tangent to M and consequently B_c^h span the tangent space at each point in M. Denote by N^h the unit normal vector field to M and hence $\{B_c^h, N^h\}$ generates the tangent space at each point in $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$.

Since the immersion $i : M \to S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ is isometric, the induced metric g_{cb} on M is given by

$$g_{cb} = g_{ji} B_c^j B_b^i.$$

Next transforming B_c^j and N^j by f_j^h, we can express them respectively as follows

$$(1.19) \qquad f_j^h B_c^j = f_c^a B_a^h + w_c N^h, \quad f_j^h N^j = -w^a B_a^h,$$

where f_c^a denote the components of tensor field of type (1.1), w_c 1-form and w^a vector field associated with w^a given by $w^a = w_c g^{ca}$, g^{ca} being the contravariant components of the induced metric tensor g^{ca}. We also express the vector field u^h and v^h respectively as follows

$$(1.20) \qquad u^h = u^a B_a^h + \mu N^h, \quad v^h = v^a B_a^h + \nu N^h,$$

where u^a and v^a are vector fields, μ and ν functions on M.

Applying the operator f_h^k to (1.19) and (1.20) respectively, and making use of (1.10), we obtain the so-called $(f, g, u, v, w, \lambda, \mu, \nu)$-structure given by

$$(1.21) \qquad f_b^e f_e^a = -\delta_b^a + u_b u^a + v_b v^a + w_b w^a,$$

$$f_e^a u^e = -\lambda v^a + \mu w^a,$$

$$(1.22) \qquad f_e^a v^e = \lambda u^a + \nu w^a,$$

$$f_e^a w^e = -\mu u^a - \nu v^a$$

or, equivalently

(1.23)
$$u_e f^e_a = \lambda v_a - \mu w_a, \quad v_e f^e_a = -\lambda u_a - \nu w_a, \quad w_e f^e_a = \mu u_a + \nu v_a,$$
$$u_e u^e = 1 - \lambda^2 - \mu^2, \quad u_e v^e = -\mu\nu, \quad u_e w^e = -\lambda\nu,$$
$$v_e v^e = 1 - \lambda^2 - \nu^2, \quad v_e w^e = \lambda\mu,$$
$$w_e w^e = 1 - \mu^2 - \nu^2$$

where u_a, v_a and w_a are 1-forms associated with u^a, v^a and w^a respectively given by $u_a = u^b g_{ba}$, $v_a = v^b g_{ba}$ and $w_a = w^b g_{ba}$. By putting $f_{ba} = f^c_b g_{ca}$, f_{cb} is skew-symmetric because f_{ji} is skew-symmetric.

Transvecting the last equation of (1.1) with $B^j_c B^i_b$ and substituting (1.20), we get

$$f^e_c f^d_b g_{ed} = g_{cb} - u_c u_b - v_c u_b - w_c w_b.$$

We now put

$$k^h_j B^j_b = k^a_b B^h_a + k_b N^h, \qquad (1.24)$$
$$k^h_j N^j = k^a B^h_a + \alpha N^h,$$

where k^a_b are components of a tensor field of type (1,1), k_b 1-form, k^a a vector field associated with k_b and α some function on M.

If we put

$$k_{ba} = k^c_b g_{ca},$$

then k_{ba} is symmetric because k_{ji} is symmetric, also, we see easily verify that k_{ba} is a second fundamental tensor of M with respect to the unit normal D if M is regarded as a submanifold of codimension 3 in E^{2n+2}.

When the action of the tangent space is invariant under the tensor field k^i_j at every point of M, that is, k_b vanishes identically along M, we call M to be *k-invariant*. We will see (1.28) which is equivalent to $\alpha^2 = 1$.

When the action of the nomal space is antiholomorphic under k^i_j at every point of M, that is , α vanishes identically along M, we call M to be ***k-antiholomorphic***.

From the first equation of (1.24) we can see that

$$(1.25) \qquad k_e^e = -\alpha.$$

Applying k_j^h to (1.24) respectively and making use of (1.7) and these equations, we obtain

$$(1.26) \qquad k_c^e k_e^a = \delta_c^a - k_c k^a,$$

$$(1.27) \qquad k_c^e k_e = -\alpha k_c,$$

$$(1.28) \qquad k_e k^e = 1 - \alpha^2.$$

Transvecting (1.24) with f_h^k and taking account of (1.16), (1.19) and (1.24) itself, we find

$$(1.29) \qquad k_c^e f_e^a + f_c^e k_e^a = k_c w^a - w_c k^a,$$

$$(1.30) \qquad k_c^e w_e + f_c^e k_e = -\alpha w_c.$$

From (1.17), (1.20) and (1.24), it follows that

$$(1.31) \qquad k_c^e u_e = -v_c - \mu k_c, \quad k_c^e v_e = -u_c - \nu k_c.$$

$$(1.32) \qquad k_e u^e = -\nu - \alpha\mu, \quad k_e v^e = -\mu - \alpha\nu.$$

Denoting by ∇_c the operator of the van der Waerden-Bortolotti covariant differentiarion, we can write the equations of Gauss and Weingarten respectively

$$(1.33) \qquad \nabla_c B_b^h = l_{cb} N^h, \quad \nabla_c N^h = -l_c^a B_a^h,$$

where l_c^b denote the components of the second fundamental tensor with respect to the unit normal vector N^h and $l_c^a = l_{cb} g^{ba}$. Then, from (1.9) and the above equations the equations of Gauss and Codazzi are given by respectively

(1.34)
$$K_{dcb}^{a} = \delta_d^a g_{cb} - \delta_c^a g_{db} + k_d^a k_{cb} - k_c^a k_{db} + l_d^a l_{cb} - l_c^a l_{db},$$

(1.35)
$$\nabla_d l_{cb} - \nabla_c l_{db} = k_d k_{cb} - k_c k_{db}.$$

By differentiating (1.19), (1.20) and (1.24) covariantly along M and taking account of (1.18), (1.33) and the fact that $\nabla_j k_i^h = 0$, we obtain the structure equations on M as follows

(1.36)
$$\nabla_c f_b^a = -g_{cb} u^a + \delta_c^a u_b - k_{cb} v^a + k_c^a v_b - l_{cb} w^a + l_c^a w_b,$$

(1.37)
$$\nabla_c u_b = \mu l_{cb} - \lambda k_{cb} + f_{cb},$$

(1.38)
$$\nabla_c v_b = k_c^e f_{eb} - k_c w_b + \nu l_{cb} + \lambda g_{cb},$$

(1.39)
$$\nabla_c w_b = -m\mu g_{cb} - \nu k_{cb} + k_c v_b - l_{ce} f_b^e,$$

(1.40)
$$\nabla_c \lambda = -2v_c, \nabla_c \mu = w_c - \lambda k_c - l_{ce} u^e, \nabla_c \nu = k_{ce} w^e - l_{ce} v^e,$$

(1.41)
$$\nabla_c k_b^a = l_{cb} k^a + l_c^a k_b,$$

(1.42)
$$\nabla_c k_b = -k_{ba} l_c^a + \alpha l_{cb},$$

(1.43)
$$\nabla_c \alpha = -2l_{ce} k^e.$$

From these structure equations, we can easily see that the 1-form k_c is the third fundamental tensor when M is considered as a submanifold of codimension 2 immersed in $S^{2n+1}(1)$.

Finally, we introduce the following theorems for later use.

THEOREM A [5]. *Let M be a hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ $(n>1)$ with $(f,g,u,v,w,\lambda,\mu,\nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2=1$. If we take v^h as the unit normal vector, M as a submanifold of codimension 3 of a Euclidean space E^{2n+2} is an intersection of a complex cone with generator C and a $(2n+1)$-dimensional sphere $S^{2n+1}(1)$.*

THEOREM B [5], [13]. *Let M be a compact orientable totally geodesic invariant hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$. Then M is $S^{n-1}\times S^n$.*

THEOREM C [5]. *Let M be a compact orientable hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ $(n>1)$. If $k_c^e f_e^a + f_c^e k_e^a = 0$, $l_c^e f_e^a + f_c^e l_e^a = 0$ and $\mu(1-\lambda^2-\mu^2-\nu^2)$ does not vanish almost everywhere, then M is $S^{n-1}\times S^n$.*

2. Hypersurface with $\lambda^2+\mu^2+\nu^2=1$

In this section we assume that the $(f,g,u,v,w,\lambda,\mu,\nu)$-structure induced on M satisfies $\lambda^2+\mu^2+\nu^2=1$ and $n>1$.

Using(1.23) and the fact of $\lambda^2+\mu^2+\nu^2=1$, we can easily verify that

$$\mu u_b+\nu v_b=0, \lambda u^a+\nu w^a=0, -\lambda v^a+\mu w^a=0 \tag{2.1}$$

and hence $w_e f_b^e$, $u_e f_b^e$ and $v_e f_b^e$ vanish identically on M.

The function ν is a nonzero constant along M. In fact, if ν vanishes on some open set on M, we have $u_b=0$ because of the first equation of (1.23) and $1-\lambda^2-\mu^2=0$. Differentiating this corvariantly and using (1.37) we find

$$\mu l_{cb}-\lambda k_{cb}+f_{cb}=0,$$

which implies that $f_{cb}=0$ because and k_{cb} are symmetric and f_{cb} is skew-symmetric with respect to b and c. Contracting (1.21) with respect to a and b, we obtain $n=1$ with the aid of (1.23).

It contıadicts $n>1$. Therefore, the function ν takes nonzero value at some point of M.

If we differentiate the first equation of (2.1) covariantly and take the skew-symmetric part, then we obtain

$$(\nabla_c\mu)u_b - (\nabla_b\mu)u_c + \mu(f_{cb} - f_{bc}) + (\nabla_c\nu)u_b - (\nabla_b\nu)u_c$$
$$+ \nu(f_{eb}k_c^e - k_c w_b - f_{ec}k_b^e + k_b w_c) = 0.$$

This equation together with (1.29) becomes

$$(\nabla_c\mu)u_b - (\nabla_b\mu)u_c + 2\mu f_{cb} + (\nabla_c\nu)v_b - (\nabla_b\nu)v_c = 0,$$

from which, transvection f^{cb} gives

$$\mu f_{cb}f^{cb} = 0$$

with the aid of $f_e^a u^e = f_e^a v^e = 0$. Thus, we see that the function

$$\mu = 0 \tag{2.2}$$

on M. From (1.23), (2.2) and the assumption $\lambda^2 + \mu^2 + \nu^2 = 1$, we have

$$v_b = 0, \tag{2.3}$$

which also show that the function λ is a constant because of the first equation of (1.40).

Hence

$$\nu = \pm\sqrt{1 - \lambda^2 - \mu^2} = \pm\sqrt{1 - \lambda^2} = constant.$$

Since ν takes nonzero value at some point of M, we conclude

(2.4)) $$\nu = constant(\neq 0)$$

LEMMA 2.1. *Let M be a hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})(n > 1)$ with $(f, g, u, v, w, \lambda, \mu, \nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2 = 1$. Then M is minimal if and only if $\lambda = 0$ on M.*

Proof. From (1.32), (2.2), (2.3) and (2.4), it follows that

$$\alpha = 0, \tag{2.5}$$

that is, M is k-antiholomorphioc, which together with (1.28) implies

$$k_e k^e = 1. \tag{2.6}$$

So k^a is a unit vector. Moreover, using (1.31), we find

$$u_c = -\nu k_c \tag{2.7}$$

because of (2.3).

On the other hand, the second equation of (2.1) together with (2.4) and (2.7) yields

$$w_c = \lambda k_c. \tag{2.8}$$

Transvecting g^{cb} to (1.38) and taking account of (2.3), (2.6) and (2.8), we find

$$l = -2\lambda(n-1)/\nu \tag{2.9}$$

because of (2.4), where we have put

$$l = g^{cb} l_{cb}.$$

Therefore, we have the lemma.

REMARK 1. If $\lambda^2 + \mu^2 + \nu^2 = 1$ on the hypertsurface M, we see that

$$\mu = 0,\ \nu = constant(\neq 0),\ v_c = 0 \quad \text{and} \quad \alpha = 0.$$

And if the function λ vanishes on some open set, then we have $v_c = 0$ and $\mu = 0$. Moreover, if the 1-form u_b is zero on an open set in M, then (1.37) implies $f_{cb} = 0$, which contradicts $n > 1$ as is shown above.

THEOREM 2.2. *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ with $(f, g, u, v, w, \lambda, \mu, \nu)$-structure satisfying $\lambda^2 + \mu^2 + \nu^2 = 1$. If M is a minimal hypersurface of $(f, g, u, v, w, \lambda, \mu, \nu)$-structure, then M is Sasakian C-Eienstein manifold.*

Proof. Since M is minimal, by lemma 2.1, we find

$$\nu^2 = 1$$

with the aid of (2.3), or equivalently

$$\nu = \pm 1. \tag{2.10}$$

From (2.7) and (2.8), it follows that

$$u_b = \pm k_b, \qquad w_b = 0. \tag{2.11}$$

Substituting (2.3) and (2.11) into (1,21), we find

$$f_b^e f_e^a = -\delta_b^a + k_b k^a,$$

or, equivalently

$$f_c^d f_b^e g_{de} = g_{cb} - k_c k_b,$$

so that these together with (2.6) and the fact of $f_e^a u^e = 0$ imply that the aggregate (f_c^a, g_{cb}, k_a) defines an almost contact metric structure.

But, from (2.3) and (2.10), the second equation of (1.20) means the vector fields v^h is a unit normal vector to M in the direction of N^h or the opposite direction of N^h.

We will show that M is a Sasakian C-Eienstein manifold in case of $\nu = -1$. Then (2.11) is

$$(2.12) \qquad u_b = k_b, \qquad w_b = 0.$$

Differentiating (2.12) covariantly and taking account of (1.37), (1.39), (2.2) and (2.3), we get

$$(2.13) \qquad f_{cb} = -l_{ce}k_b^e,$$

$$(2.14) \qquad k_{cb} = l_{ce}f_b^e.$$

Also, we have

$$(2.15) \qquad \nabla_c k_b = f_{cb}$$

Substituting (2.3) and (2.12) into (1.36), we obtain

$$(2.16) \qquad \nabla_c f_b^a = -g_{cb}k^a + \delta_c^a k_b$$

Thus, the aggregate (f_c^a, g_{cb}, k_a) defines a Sasakian structure.
On the other hand, if we transvect (2.13) with k_d^b, then we get

$$(2.17) \qquad l_{cb} = -f_{ce}k_b^e.$$

Transvecting this with l_d^c and making use of (2.14), we have

$$(2.18) \qquad l_{de}l_b^e = k_{de}k_b^e = g_{db} - k_d k_b$$

with the aid of (1.26).

Contracting (1.34) with respect to the indices d and a, using (1.25), (1.26),(2.5), (2.18) and the minimality of M, we have the Ricc tensor of the form

$$K_{cb} = 2(n-2)g_{cb} + 2k_c k_b,$$

that is , M is C-Einstein.

In case of $\nu = 1$, putting $'k_a = -k_a$, we can easily verify that $(f_c^a, g_{cb}, 'k_a)$ defines a Sasakian C-Einstein structure on M by the same described above. Therefore Theorem 2.2 is completely proved.

Now, let's consider the following imbeddings :

$$M \xrightarrow{\imath} S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2}) \xrightarrow{\tilde{\imath}} E^{2n+2}$$

that is, M is regarded as a submanifold of codimension 3 in E^{2n+2} by the imbedding $\tilde{\imath} \circ \imath$. Putting $Z_a = B_a^j Z_j$ and $N = N^j Z_j$, we see that Z_a is vector field on M and N a unit vector field normal to M with respect to the ambient space E^{2n+2}. Denoting by $h_{cb} = h_{ji} B_c^j B_b^i$, we see that l_{cb}, h_{cb} and k_{cb} are the second fundamental tensors with respect to the normals N, C, and D respectively. But, as h_{ji} is of the form $h_{ji} = g_{ji}$, we have

$$h_{cb} = g_{cb}. \tag{2.19}$$

Suppose that M is minimal, we can see that M is a pseudo-umbilical manifold by considering (1.25), (2.5) and (2.19) ([3], [4]). Hence, by making use of Theorem A in section 1, we have :

THEOREM 2.3. *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n > 1)$ with $(f, g, u, v, w, \lambda, \mu, \nu)$-structure satisfying $\lambda^2 + \mu^2 + \nu^2 = 1$. If M is minimal, then M as a submanifold of codimension 3 of a $(2n+2)$-dimensional Euclidean space E^{2n+2} is an intersection of a complex cone with generator C and a $(2n+1)$-dimensional unit sphere $S^{2n+1}(1)$.*

(1.38) together with (2.3) implies

$$\nu l_{cb} = k_{ce} b_b^e + \lambda(k_c k_b - g_{cb}) \tag{2.20}$$

with the aid of (2.8). From (1.43) with (2.5), it follows that

$$(2.21) \qquad l_{ce}k^e = 0.$$

Also, making use of (1.30), the first equation of (1.21), (2.2), (2.3) and (2.4), we find

$$(2.22) \qquad k_{ce}w^e = o,$$

$$(2.23) \qquad k_{ce}u^e - o.$$

Transvecting (2.20) with f_a^b and making use of (2.22) and (2.23), we have

$$(2.24) \qquad \nu l_{ce}f_b^e = -k_{cb} + \lambda f_{cb}.$$

Therefore, from (2.20), ti follows that

$$(2.25) \qquad l_{ce}l_b^a = -2\frac{\lambda}{\nu}l_{cb} + (g_{cb} - k_c k_b)$$

with the aid of (1.26),(2.4), (2.21) and (2.24), which implies

$$l_{cb}l^{cb} = 2(n-1) - 2\frac{\lambda}{\nu}l,$$

or, using (2.9)

$$(2.26) \qquad l_{cb}l^{cb} = 2(n-1) + 4\frac{\lambda^2}{\nu^2}(n-1).$$

Assuming $\|l_{cb}\|^2 - 2(n-1) \leqq 0$ at every point of M, we then have $\lambda = 0$. By Lemma 2.1, M is minimal.

Thus we have :

THEOREM 2.4. *Let M be a hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})(n > 1)$ with $(f,g,u,v,w,\lambda,\mu,\nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2=1$. If the square of length of the second fundamental form is not greater than $2(n-1)$ at every point of M, then M is the same type as that of Theorem 2.3.*

From (2.25), we have, in the consequence of (2.21),

$$l_{ce}l_b^e l_a^b = l_{ca} - (2\lambda/\nu)l_{ce}l_a^e. \tag{2.27}$$

Therefore, by using the Caley-Hamilton's Theorem, we obtain 3 constant principal curvature of M as follows

$$0, 1-\lambda/\nu, -(1+\lambda)/\nu.$$

We now let

$$X_1 = 1-\lambda/\nu, \quad X_2 = -(1+\lambda)/\nu. \tag{2.28}$$

We may assume the matrix (l_c^a) is digonal without loss of generality

$$(\ l_c^a\ ) = \begin{pmatrix} 0 & & & & & & & \\ & \ddots & & & & 0 & & \\ & & 0 & & & & & \\ & & & x_1 & & & & \\ & & & & \ddots & & & \\ & & & & & x_1 & & \\ & & 0 & & & & x_2 & \\ & & & & & & & \ddots & \\ & & & & & & & & x_2 \end{pmatrix}.$$

Let the multiplicities of x_1 and x_2 be m and s respectively. Then we have from (2.9) and above expressions;

$$mx_1 + sx_2 = -2\lambda(n-1)/\nu,$$

or, using (2.28)

$$(2.29) \qquad (1-\lambda)m - (1+\lambda)s = -2(n-1)\lambda.$$

Also, making use of (2.26), we have

$$(2.30) \qquad (1-\lambda/\nu)^2 m + (1+\lambda/\nu)^2 s = 2(n-1) + 4\lambda^2(n-1)/\nu^2.$$

Thus, from (2.29) and (2.30), it follows that

$$(2.31) \qquad m = s = n-1.$$

If $x_1 = 0$, that is, $\lambda = 1$, then the first equation of (1.23) implies

$$u_e = 0$$

because of $\lambda^2 + \mu^2 + \nu^2 = 1$. But it is impossible by the Remark 1. Thus, we have $x_1 \neq 0$. Similarly, we can see that $x_2 \neq 0$. And x_1 and x_2 are distinct by their own properties.

Hence we have ;

THEOREM 2.5. *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n > 1)$ with $(f, g, u, v, w, \lambda, \mu, \nu)$-structure satisfying $\lambda^2 + \mu^2 + \nu^2 = 1$. Then we have 3 distinct constant principal curvature with multiplicities 1, $n-1$,$n-1$ respectively.*

3. Antiholomorphic hypersurfaces satisfying $k \circ f + f \circ k = 0$

Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ such that $k_c^e f_e^a + f_c^e k_e^a = 0$ holds every point of M or, equivalently

$$(3.1) \qquad k_{ce} f_b^e = k_{be} f_c^e.$$

Then (1.29) reduces to

$$k_c w_b - k_b w_c = 0. \tag{3.2}$$

Transvecting (3.1) with f^{cb} and taking account of (1.21), (1.22), (1.23) and (1.30) –(1.32), we find

$$\alpha(\mu^2 + \nu^2) + 2\mu\nu = 0 \tag{3.3}$$

because the tensor $k_{ce} f_b^e$ is symmetric

If we transvect (3.1) with k_d^c and use (1.26), we have

$$f_{bd} - k_d(f_{be}k^e) = k_{be}k_d^c f_c^e,$$

as taking the symmetric part with respect to indices b and d,

$$k_d(f_{be}k^e) + k_b(f_{de}k^e) = 0$$

because of skew-symmetric tensor f_{cb}.

Transvecting this with w^d and making use of (1.22) and (1.32), we obtain

$$\theta f_{be}k^e + \{\alpha(\mu^2 + \nu^2) + 2\mu\nu\} = 0,$$

which together with (3.3) gives

$$\theta f_{be}k^e = 0 \tag{3.4}$$

where, here and in the sequel we have put

$$\theta = k_e w^e. \tag{3.5}$$

On the other hand, we easily verify from (3.2) that

$$(1 - \mu^2 - \nu^2)k_c = \theta w_c, (1 - \alpha^2)w_c = \theta k_c, \tag{3.6}$$

where we have used (1.23) and (1.28).

If the function $\|k_{be}f^e\|^2$ does not vanish at some point p of M, then we see from (3.4) that $\theta(p)=0$ and hence $(1-\alpha^2)w_c=0$ at the point. So we have $w_c=0$ at $p\in M$. Thus (1.30) leads to $f_{ce}k^e=0$, which is contradictory. Consequently we have

$$f_{be}k^e=0$$

on M. hence (1.30) becomes

$$k_{ce}w^e=-\alpha w_c. \tag{3.7}$$

Applying the expression $f_{be}k^e=0$ with f_a^b and using (1.21) and (1.32). we find

$$k_a=\theta w_a-(\mu+\alpha\nu)v_a-(\nu+\alpha\mu)u^a,$$

which together with (3.6) gives

$$(\mu^2+\nu^2)k_c+(\mu+\alpha\nu)v_c+(\nu+\alpha\mu)u_c=0. \tag{3.8}$$

If we transvect (3.8) with u^c and v^c sucessively and consider (1.23), (1.32) and (3.3), we get

(3.9)
$$(\nu+\alpha\mu)(1-\lambda^2-\mu^2-\nu^2)=0,(\mu+\alpha\nu)(1-\lambda^2-\mu^2-\nu^2)=0.$$

Therefore, (3.8) implies

$$(\mu^2+\nu^2)(1-\alpha^2)(1-\lambda^2-\mu^2-\nu^2)=0 \tag{3.10}$$

because of (1.28).

Since we have from (3.3)

$$(\nu+\alpha\mu)^2+(\mu+\alpha\nu)^2=\mu^2+\nu^2+2\alpha\mu\nu, \tag{3.11}$$

(3.9) is turned out to be

$$(3.12) \qquad (\mu^2 + \nu^2 + 2\alpha\mu\nu)(1 - \lambda^2 - \mu^2 - \nu^2) = 0.$$

Differentiating (3.12) covariantly and considering the orginal expression, we find

$$(1 - \lambda^2 - mu^2 - \nu^2)\nabla_c(\mu^2 + \nu^2 + 2\alpha\mu\nu) = 0.$$

If we suppose that the function $\mu^2 + \nu^2 + 2\alpha\mu\nu$ is not constant at some point of M, then it means

$$\lambda^2 + \mu^2 + \nu^2 = 1$$

at this point. Hence, due to Remark.1 in section 1, we see that $\mu = 0$ and $\nu = constant$ at the point. It contradicts the fact that the function $\mu^2 + \nu^2 + 2\alpha\mu\nu$ is not constant at the point.

Developed above, we have

LEMMA 3.1. *Let M be a hypersurface satisfying (3.1) of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. Then we have*

$$\lambda^2 + \mu^2 + \nu^2 = 1 \quad or \quad \mu^2 + \nu^2 + \alpha\mu\nu = 0$$

on M.

We now prove

LEMMA 3.2. *Under the same assumptions as those stated in Lemma 3.1, M is k-antiholomorphic if and only if $\lambda^2 + \nu^2 = 1$ holds at every point of M.*

Proof. If M is k-antiholomorphic, that is , α vanishes identically, then (3.12) yields

$$(3.13) \qquad (\mu^2 + \nu^2)(1 - \lambda^2 - \mu^2 - \nu^2) = 0.$$

Also we have

$$k_e k^e = 1, \quad l_{ce} k^e = 0 \tag{3.14}$$

because of (1.28) and (1.43).

We now suppose that the function μ and ν vanish at some point p of M, then (3.6) leads to

$$k_c = \theta w_c, \quad w_c = \theta k_c. \tag{3.15}$$

Thus, it follows that $\theta^2 = 1$ because k^a is a unit vector. Since $\mu(p) = 0$, the second equation of (1.40) means

$$l_{ce} u^e = (1 - \theta\lambda) w_c$$

with the aid of (3.15).

Transvecting this with w^c and taking account of (3.14) and (3.15) and the fact that w^a is a unit vector, we find $\theta\lambda = 1$ and consequently $\theta = \lambda = constant$ on the set of such points. Hence, the first equation of (1.40) means $v_c = 0$ at the point of M. Therefore, the fact

$$v_e v^e = 1 - \lambda^2 - \mu^2 - \nu^2$$

implies that $1 - \lambda^2 = 0$ at the point. So, using $\mu(p) = 0$, we see that $u_c = 0$ at $p \in M$. Due to Remark 1, it is contrdictory. Thereby (3.13) reduces to $\lambda^2 + \mu^2 + \nu^2 = 1$ on M. So using Remark 1 again, it means $\lambda^2 + \nu^2 = 1$ on M.

Conversely, if $\lambda^2 + \nu^2 = 1$ holds on M, then we have $v_c = 0$. Thus the first expression of (1.32) gives

$$\mu + \alpha\nu = 0. \tag{3.16}$$

If $\mu^2 + \nu^2 + 2\alpha\mu\nu = 0$ holds on M, then (3.16) yields $\mu^2 = \nu^2$. So we have $\lambda^2 + \nu^2 = 1$. Hence, the first relationship of (1.23) means $u_c = 0$, which is contradictory.

Therefore, owing to Lemma 3.1, we see that

$$\lambda^2 + \mu^2 + \nu^2 = 1$$

holds on M. From Remark 1 in section 2, it follows that

$$\mu = 0, \qquad \nu = constant(\neq 0).$$

Thus (3.16) implies that the function α vanishes identically. This completes the proof of the Lemma.

According to Theorem 3.3, Theorem 3.4 of [5] and Lemma 3.2, we have

THEOREM 3.3. *Let M be a k-antiholomorphic hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})(n > 1)$ satisfying*

$$k_c^e f_e^a + f_c^e k_e^a = 0.$$

If we take v^h as the unit normal vector, then M is a minimal Sasakian C-Einstein manifold.

THEOREM 3.4. *Under the same assumptions as those statded in Theorem 3.3, M as a submanifold of codimension 3 of a Euclidean $(2n+2)$-space, is an intersection of a complex cone with generator C and a $(2n+1)$-sphere $S^{2n+1}(1)$.*

Combining Theorem 2.2, Theorem 2.3, Theorem 2.4 and Lemma 3 2, we conclude

THEOREM 3.5. *Let M be a k-antiholomorphic hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n > 1)$ satisfying $k_c^e f_e^a + f_c^e k_e^a = 0$ If M is minimal (or the square of length of the second fundamental tensor of M is not greater than $2(n-1)$ at every point of M), then M is the same type of Theorem 3.3 and Theorem 3.4.*

Combining Theorem 2.5 and Lemma 3.2, we have

THEOREM 3.6. *Let M be a k-antiholomorphic hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})(n > 1)$ satisfying $k_c^e f_e^a + f_c^e k_e^a = 0$. Then the second fundamental tensor of M has three distinct constant principal curvatures $0, (1-\lambda)/\nu, -(1+\lambda)\nu$ with multiplicities $1, n-1$, $n-1$ respectively.*

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Department of Mathematics
University of Ulsan
Ulsan 680–749, Korea