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THE WELL POSEDNESS OF A PARABOLIC DOUBLE FREE BOUNDARY PROBLEM

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1. Introduction

We consider the reaction-diffusion system of two-component model in one-dimensional space described by

$$u_s = d_1 u_{xx} + f(u,v) \quad v_t = d_2 v_{xx} + \gamma g(u,v) \tag{1}$$

where d_1 and d_2 are the diffusion rates of u and v, and γ is the ratio of reaction rates. It is interesting the case of that there are differences in the diffusion and reaction rates of u and v. The typical example for this is the FitzHugh-Nagumo equations which are very well-known example for a model of the nerve impulse. A free boundary (or interface) may be appeared from sharp transition when a width of layer is sufficiently small. Thus, we consider the reaction-diffusion system that the first component u reacts much faster than the second component v, although u diffuses slower than v. We define the new parameters

$$\varepsilon = \sqrt{d_1}, \ \ \tau = \gamma/\sqrt{d_1}, \ \ D = d_2/\gamma$$

and write (1) as

$$\varepsilon\tau u_t = \varepsilon^2 u_{xx} + f(u,v) \quad v_t = D v_{xx} + g(u,v) \tag{2}$$

where a new parameter $t = \gamma s$ was used. This system is defined on $(0,1)\times(0,\infty)$ and is assumed to satisfy the zero flux boundary conditions at $x = 0, 1$. The functions f and g are assumed to be of bistable

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type, i.e., the equation $f = 0$ determines u as atriple valued function of v and the curves defined by $f = 0, g = 0$ have three points of intersection, which determine all of the interactions between u and v. The term bistable refers to the fact that these points of intersection correspond to equilibria of the system (2), two of which are stable, and the third is unstable.

In 1977, Paul Fife [1] showed that the stationary solution, being smooth, exhibits an abrupt but continuously differentiable transition at the location of the limiting discontinuity when ε is small. The transition takes place with in an x-interval of length $O(\varepsilon)$. An x-interval, in which such an abrupt change takes place, is loosely called a layer. In 1981, Mimura, Tabata and Hosono [4] proved the existence of nontrival internal layer solutions to the stationary (time-independent) problem associated with (2). The question of the stability of these stationary layer solutions when ε is small was later dealt with in a pair of papers; one by Nishiura and Fujii [5] appearing in 1987 for the case where τ is large and the solution is asymptotically stable and the second in 1989 by Nishiura and Mimura [6] for the case where τ small and there is a breakdown in the stability of the stationary solutions as τ approaches 0. In the latter paper, a particularly dramatic phenomenon occurs as the stationary solutions lose stability. The loss of stability results from a Hopf bifurcation and produces a kind of periodic oscillation in the location of the internal layers. (The amplitudes of the solutions also undergo a somewhat less pronounced periodic oscillation.) These periodic solutions are called "breathers" or "breathing solutions" because of the nature of the oscillation in the position of the internal layers.

In this paper, we are interested in the singular limit $\varepsilon \downarrow 0$ of the system (2). In this case, an analysis of the layer solutions suggests that the layer of width $O(\varepsilon)$ converges to interfacial curves in x, t-space as $\varepsilon \downarrow 0$. An analysis of the dynamics of this process has been shown (see for example [3],[6]) to lead a free boundary problem consisting of the initial-boundary value problem. In 1992, Thompson, Schaaf and I showed the well-posedness of a free boundary problem with a single interfacial curve [8]. We now assume that the system (2) has a steady state with double layers on a finite interval which is more realistic and interesting case. In this case, an analysis of the layer solutions suggests that the layer of width $O(\varepsilon)$ converges to interfacial curves $x = s(t)$

and $x = m(t)$ in x,t-space as $\varepsilon \downarrow 0$ ([7]). An analysis of the dynamics of this process has been shown (see for example [3],[6]) to lead a free boundary problem consisting of the initial-boundary value problem.

By streching both space and time at the layer positions with respect to ε, we obtain the following free boundary problem with double layers

$$(3)\qquad \begin{cases} v_t = Dv_{xx} - c^2 v + H(x - s(t)) - H(x - m(t)) \\ \qquad\qquad \text{for } (x,t) \in \Omega^- \cup \Omega^+, \\ v_x(0,t) = 0 = v_x(1,t) \quad \text{for } t > 0, \\ v(x,0) = v_0(x) \quad \text{for } 0 \le x \le 1, \\ \tau \dfrac{ds}{dt} = C(v(s(t),t)) \quad \text{for } t > 0, \\ \tau \dfrac{dm}{dt} = -C(v(m(t),t)) \quad \text{for } t > 0, \\ s(0) = s_0, \quad 0 < s_0 < 1 \\ m(0) = m_0, \quad 0 < m_0 < 1 \end{cases}$$

where $v(x,t)$ and $v_x(x,t)$ are assumed continuous in Ω. Here $H(y)$ is the Heaviside function, $\Omega = (0,1) \times (0,\infty)$, $\Omega^- = \{(x,t) \in \Omega : 0 < x < s(t), m(t) < x < 1\}$ and $\Omega^+ = \{(x,t) \in \Omega : s(t) < x < m(t)\}$ (see the Figure 1 below).

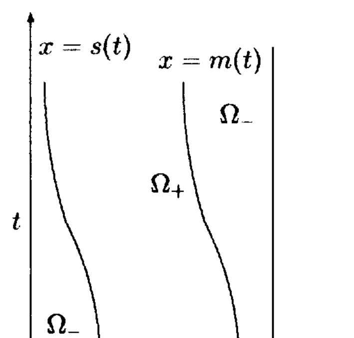

Figure 1: The (x,t)-domain for problem(3).

In this problem, we do not know a regular form of (3) for finding an existence and uniqueness of solutions. It motivates our methods of proof, which is based on Green's function and the semigroup theory. In the next section , a change of variable will be given in order to regularize problem (3) and the theory of nonlinear evolution equations can be applied. In this way, we give a proof of well-posedness. This method can be adapted to give an alternative proof for similar problems.

2. Regularization, existence, uniqueness and dependence on initial conditions

We rewrite (3) as an abstract evolution equation.

$$\frac{d(v,s,m)}{dt}+\widetilde{A}(v,s,m)=F(v,s,m),\quad (v,s,m)(0)=(v_0(\cdot),s_0,m_0)$$

of a differential equation in a space $\widetilde{X}$ of the form $\widetilde{X}=X\times \mathbf{J}\times \mathbf{J}$, where X is a Banach space of functions and J is a real interval. Here, $\widetilde{A}$ is a differential operator, represented in matrix form, as

$$\widetilde{A}:=\begin{pmatrix} -D\dfrac{d^2}{dx^2}+c^2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

and the nonlinear operator F by

$$F(v,s)=\begin{pmatrix} F_1(v(\cdot,t),s(t),m(t))\\ F_2(v(\cdot,t),s(t),m(t))\\ F_3(v(\cdot,t),s(t),m(t))\end{pmatrix}:=\begin{pmatrix} H(\cdot-s(t))-H(\cdot-m(t))\\ \dfrac{1}{\tau}C(v(s(t),t))\\ -\dfrac{1}{\tau}C(v(m(t),t)).\end{pmatrix}$$

The Neumann boundary conditions are incorporated in the definition of the Banach space X.

Since the nonlinear forcing term $F(v,s,m)$ contains a Heaviside function in its first component, the combination of this jump discontinuity and the nature of the dependence of v on s and m in the second and third components of F makes it impossible to find a function space of the form $X=L_p, 1\le p\le\infty$ such that F satisfies a Lipschitz condition on $\widetilde{X}\subset X\times\mathbf{R}\times\mathbf{R}$. Therefore, we need to make a regular problem for this one.

We now examine a free boundary value problem depending on a new parameter $\mu \in \mathbf{R}$, $\mu = 1/\tau$ of the form

$$(F)\begin{cases} v_t + Av = H(x-s) - H(x-m), & (x \in (0,1) \setminus \{s,m\}, t > 0) \\ s'(t) = \mu C(v(s(t),t)), & (t > 0) \\ m'(t) = -\mu C(v(m(t),t)), & (t > 0) \\ v(x,0) = v_0(x), \ s(0) = s_0 \ m(0) = m_0 . \end{cases}$$

Here A is the operator $Av = -v_{xx} + c^2 v$ together with Neumann boundary conditions $v_x(0) = v_x(1) = 0$. Note that by a rescaling of t in (3) we can always achieve that $D = 1$. For the purposes of the results in this section, A can also be any other invertible second order operator. On the function C, we assume that

$$C : I \subset_{\text{open}} \longrightarrow \mathbf{R} \text{ is continuously differentiable.}$$

For the application of semigroup theory to (F), we choose the space

$$X := L_2((0,1)) \text{ with norm } \|\cdot\|_2 .$$

DEFINITION 2.1. We call (v,s,m) a solution of (F), if it satisfies the following natural properties: There exists $T > 0$ such that $v(x,t)$ is defined for $(x,t) \in [0,1] \times [0,T)$, $s(t) \in (0,1)$, $m(t) \in (0,1)$, $v(s(t),t) \in I$ for $t \in [0,T)$ and $v(m(t),t) \in I$ for $t \in [0,T)$,

(a) $v(\cdot,t) \in C^1([0,1])$ for $t > 0$ with $v_x(0,t) = v_x(1,t) = 0$,

(b) $s \in C^0([0,T)) \cap C^1((0,T))$ with $s(0) = s_0 \in (0,1)$ and $m \in C^0([0,T)) \cap C^1((0,T))$ with $m(0) = m_0 \in (0,1)$,

(c) $(Av)(x,t)$ and $v_t(x,t)$ exist for $x \in (0,1) \setminus \{s(t)\} \setminus \{m(t)\}$ and $t \in (0,T)$,

(d) $t \mapsto v(\cdot,t) \in C^0([0,T),X)$ with $v(\cdot,0) = v_0 \in X$ and

(e) v, s and m solve the differential equation for $t \in (0,T)$ and $x \in (0,1) \setminus \{s(t)\} \setminus \{m(t)\}$.

We obtain more regularity for the solution by semigroup methods, considering A as a densely defined operator

$$A : D(A) \subset_{\text{dense}} X \longrightarrow X$$
$$D(A) := \{v \in H^{2,2}((0,1)) \, : \, v_x(0) = v_x(1) = 0\} .$$

For fixed s and m satisfying Definition 2.1, the map $t \mapsto (H(\cdot - s(t)) - H(\cdot - m(t)))$ is locally Hölder-continuous into X on $(0, T)$, so by standard results for parabolic problems (see e.g.[2]) we obtain from the first equation in (F) that the following regularity holds for v:

PROPOSITION 2.2. *If (v, s, m) is a solution of (F) then $v(\cdot, t) \in D(A)$ and the map $t \mapsto v(\cdot, t)$ is in $C^0([0, T), X) \cap C^1((0, T), X)$.*

An existence proof for (F) can be obtained along these lines, but it is impossible to get differential dependence on initial conditions this way, because the right hand side $H(\cdot - s) - H(\cdot - m)$ is not regular enough.

We decompose v in (F) into a part u, which is a solution to a more regular problem, and a part g, which is worse, but explicitly known in terms of Green's function G of the operator A.

PROPOSITION 2.3. *Let $G : [0, 1]^2 \to \mathbf{R}$ be Green's function of the operator A. Define $g : [0, 1]^3 \longrightarrow \mathbf{R}$*

$$g(x, s, m) := \int_s^m G(x, y)\, dy = A^{-1}(H(\cdot - s) - H(\cdot - m))(x)$$

and $\gamma : [0, 1]^2 \longrightarrow \mathbf{R}$

$$\gamma(s, m) := g(s, s, m)\,,$$

$\eta : [0, 1]^2 \longrightarrow \mathbf{R}$

$$\eta(s, m) := g(m, s, m)\,.$$

Then $g(\cdot, s, m) \in D(A)$ for all s, m, $\dfrac{\partial g}{\partial s}(x, s, m) = -G(x, s)$ is in $H^{1,\infty}((0, 1) \times (0, 1))$, $\dfrac{\partial g}{\partial m}(x, s, m) = G(x, m)$ is in $H^{1,\infty}((0, 1) \times (0, 1))$, and $\gamma \in C^\infty([0, 1] \times [0, 1])$, $\eta \in C^\infty([0, 1] \times [0, 1])$.

Proof. Everything follows from the fact that G is in $H^{1,\infty}$ and C^∞ on either $\{x \leq y\}$ or $\{x \geq y\}$, and that $H(\cdot - s) - H(\cdot - m) \in L^2$. □

Using these preliminary observations, we decompose a solution (v, s, m) of (F) into two parts by defining

$$u(t)(x) := v(x, t) - g(x, s(t), m(t))\,.$$

Then

$$\begin{aligned}u'(t)(x) &= v_t(x,t) + G(x,s(t),m(t))s'(t) - G(x,m(t),m(t))m'(t) \\ &= -(Av)(x,t) + H(x-s(t)) - H(x-m(t)) \\ &\quad + G(x,s(t))\mu C(v(s(t),t)) + G(x,m(t))\mu C(v(m(t),t)) \\ &= -(Au(t))(x) + \mu C\Big(u(t)(s(t)) + \gamma(s(t),m(t))\Big)G(x,s(t)) \\ &\quad + \mu C\Big(u(t)(m(t)) + \eta(s(t),m(t))\Big)G(x,m(t)),\end{aligned}$$

$$s'(t) = \mu C\Big(u(t)(s(t)) + \gamma(s(t),m(t))\Big)$$

and

$$m'(t) = -\mu C\Big(u(t)(m(t)) + \eta(s(t),m(t))\Big).$$

This system can be written as an abstract evolution equation with a nonlinear forcing term f defined on the set $W := \{(u,s,m) \in C^1([0,1]) \times (0,1) \times (0,1) : u(s)+\gamma(s,m) \in I, u(m)+\eta(s,m) \in I\} \subset_{open} C^1([0,1]) \times \mathbf{R} \times \mathbf{R}$ as follows

$$\begin{aligned}&f : W \to X \times \mathbf{R} \times \mathbf{R} \\ &f(u,s,m) := (f_3(u,s,m)f_1(s) + f_4(u,s,m)f_2(m), \\ &\qquad\qquad f_3(u,s,m), \ -f_4(u,s,m)),\end{aligned}$$

where

$$\begin{aligned}&f_1 : (0,1) \to X, \ \ f_1(s)(x) := G(x,s), \\ &f_2 : (0,1) \to X, \ \ f_2(m)(x) := G(x,m), \\ &f_3 : W \to \mathbf{R}, \ \ f_3(u,s,m) := C(u(s) + \gamma(s,m)) \text{ and} \\ &f_4 : W \to \mathbf{R}, \ \ f_4(u,s,m) := C(u(m) + \eta(s,m)).\end{aligned}$$

We denote the space $X \times \mathbf{R} \times \mathbf{R}$ by $\widetilde{X}$ and define

$$\begin{aligned}&D(\widetilde{A}) := D(A) \times \mathbf{R} \times \mathbf{R}, \\ &\widetilde{A} : D(\widetilde{A}) \subset_{\text{dense}} \widetilde{X} \longrightarrow \widetilde{X}, : \widetilde{A}(u,s,m) := (Au,0,0).\end{aligned}$$

The initial value problem for (u,s,m) can then be written a

$$(R)\begin{cases} \frac{d}{dt}(u,s,m)+\widetilde{A}(u,s,m)=\mu f(u,s,m) \\ (u,s,m)(0)=(u(0),s(0),m(0))=(u_0,s_0,m_0)\,. \end{cases}$$

The advantage of (R) over (F) is, that the right hand side of (R) is one step more regular than that of (F), since it involves $G(x,s)$ and $G(x,m)$ instead of $H(x-s)-H(x-m)$. More precisely, we can show the following:

LEMMA 2.4. *The functions $f_1:(0,1)\to X$, $f_2:(0,1)\to X$, $f_3: W\to \mathbf{R}$, $f_4: W\to\mathbf{R}$ and $f: W\to\widetilde{X}$ are continuously differentiable with derivatives given by*

$$\begin{aligned} f_1'(s) &= -\frac{\partial G}{\partial y}(\cdot,s) \\ f_2'(m) &= \frac{\partial G}{\partial y}(\cdot,m) \\ &Df_3(U)(\hat u,\hat s,\hat m) \\ &\quad = C'\left(u(s)+\gamma(s,m)\right)\cdot\left(u'(s)\hat s+\gamma_s(s,m)\hat s+\gamma_m(s,m)\hat m+\hat u(s)\right) \\ &Df_4(U)(\hat u,\hat s,\hat m) \\ &\quad = C'\left(u(m)+\eta(s,m)\right)\cdot\left(u'(m)\hat m+\eta_s(s,m)\hat s+\eta_m(s,m)\hat m+\hat u(m)\right) \end{aligned}$$

$$Df(U)(\hat u,\hat s,\hat m)= \begin{pmatrix} f_3(U)f_1'(s)\hat s+Df_3(U)(\hat U)f_1(s)+f_4(U)f_2'(m)\hat m+Df_4(U)(\hat U)f_2(m) \\ Df_3(U)(\hat U) \\ -Df_4(U)(\hat U) \end{pmatrix}$$

where $U=(u,s,m)$ and $\hat U=(\hat u,\hat s,\hat m)$.

Proof. The function $\frac{\partial G}{\partial y}(x,s)$ exists for $x\neq s$ and is bounded independent of s as a function of x in $L^2((0,1))$. Moreover, it is continuous almost everywhere in $[0,1]\times[0,1]$. Lebesgue's theorem then implies that f_1 is continuously differentiable. In similar way we obtain f_2 is continuously differentiable.

In order to show f_3 is continuously differentiable, we consider the following function

$$\Gamma : C^1[0,1] \times (0,1) \to \mathbf{R}, \;\; \Gamma(u,s) := u(s) .$$

For perturbations $\hat{u}$, $\hat{s}$ of u, s there exists, by the mean value theorem, a $\theta \in (0,1)$ such that

$$\begin{aligned} &\Gamma(u+\hat{u}, s+\hat{s}) - \Gamma(u,s) - u'(s)\hat{s} - \hat{u}(s) \;= \\ &= u(s+\hat{s}) + \hat{u}(s+\hat{s}) - u(s) - u'(s)\hat{s} - \hat{u}(s) \\ &= \hat{s}\left(u'(s+\theta\hat{s}) - u'(s)\right) + \hat{u}'(s+\theta\hat{s})\hat{s} \\ &=: R(\hat{u},\hat{s}) \end{aligned}$$

and

$$\frac{R(\hat{u},\hat{s})}{\|\hat{u}\|_{C^1} + |\hat{s}|} \longrightarrow 0 \text{ as } |\hat{s}|,\, \|\hat{u}\|_{C^1} \to 0 .$$

As a result Γ is differentiable with derivative given by

$$D\Gamma(u,s)(\hat{u},\hat{s}) = u'(s)\hat{s} + \hat{u}(s) .$$

Furthermore, since $\|D\Gamma(u_1,s_1) - D\Gamma(u_2,s_2)\| \leq \|u_1 - u_2\|_{C^1} + |s_1 - s_2|$, the mapping $(u,s) \mapsto D\Gamma(u,s)$ is continuous.

From the relation $f_3(u,s,m) = C(\Gamma(u,s) + \gamma(s,m))$ we obtain that f_3 is continuously differentiable with derivative

$$\begin{aligned} &Df_3(u,s,m)(\hat{u},\hat{s},\hat{m}) \\ &= C'\left(\Gamma(u,s) + \gamma(s,m)\right) \cdot \Big(D\Gamma(u,s)(\hat{u},\hat{s}) + \gamma_s(s,m)\hat{s} + \gamma_m(s,m)\hat{m}\Big). \end{aligned}$$

Similary, if we define

$$\Delta : C^1[0,1] \times (0,1) \to \mathbf{R}, \;\; \Delta(u,m) := u(m) .$$

then $f_4(u,s,m)$ is also continuously differentiable with derivative

$$\begin{aligned} &Df_4(u,s,m)(\hat{u},\hat{s},\hat{m}) = \\ &C'\left(\Delta(u,m) + \eta(s,m)\right) \cdot \Big(D\Delta(u,m)(\hat{u},\hat{m}) + \eta_s(s,m)\hat{s} + \eta_m(s,m)\hat{m}\Big) \end{aligned}$$

which was to be shown.

Therefore, by the product rule, the derivative of f can be calculated as indicated, and, moreover, Df is continuous. □

We can now apply semigroup theory to (R) using domains of fractional powers $\alpha \in [0,1]$ of A and $\widetilde{A}$:

$$X^\alpha := D(A^\alpha), \quad \widetilde{X}^\alpha := D(\widetilde{A}^\alpha), \quad \widetilde{X}^\alpha = X^\alpha \times \mathbf{R}.$$

For this we need to find an $\alpha \in (0,1)$ such that $X^\alpha \subset C^1([0,1])$, because then $f : W \cap \widetilde{X}^\alpha \to \widetilde{X}$ is continuously differentiable. By imbedding theorem [henry], we can find an α satisfying $\alpha > 3/4$. Standard applications of theorems for existence, uniqueness and dependence on initial conditions (cf. [2]) together with the starting regularity of solutions to (F)(Proposition 2.1), as well as the regularity of the functions g, γ and η (Proposition 2.2) then give the following result:

THEOREM 2.4. *(i) For any $1 > \alpha > 3/4$, $(u_0, s_0, m_0) \in W \cap \widetilde{X}^\alpha$ and $\mu \in \mathbf{R}$ there exists a unique solution*

$$(u,s,m)(t) = (u,s,m)(t; u_0, s_0, m_0, \mu)$$

of (R). The solution operator

$$(u_0, s_0, m_0, \mu) \mapsto (u,s,m)(t; u_0, s_0, m_0, \mu)$$

is continuously differentiable from $\widetilde{X}^\alpha \times \mathbf{R}$ into $\widetilde{X}^\alpha$ for $t > 0$. The functions $v(x,t)$

$$v(x,t) := u(t)(x) + g(x, s(t), m(t))$$

and s, m then satisfy (F) with $v(\cdot, 0) \in X^\alpha$, $v(s_0, 0) \in I$, $v(m_0, 0) \in I$.

(ii) If (v, s, m) is a solution of (F) for some $\mu \in \mathbf{R}$ with initial condition $v_0 \in X^\alpha$, $1 > \alpha > 3/4$, $s_0, m_0 \in (0,1)$, $v_0(s_0) \in I$, $v_0(m_0) \in I$, then $(u_0, s_0, m_0) := (v_0 - g(\cdot, x_0), s_0, m_0) \in \widetilde{X}^\alpha \cap W$ and

$$(v(\cdot,t), s(t), m(t)) = (u,s,m)(t; u_0, s_0, m_0, \mu) + (g(\cdot, s(t), m(t)), 0)$$

where $(u, s, m)(t; u_0, s_0, m_0, \mu)$ *is the unique solution of (R)*

(iii) For any $1 > \alpha > 3/4$, $\mu \in \mathbf{R}$,$(v_0, s_0, m_0) \in U := \{(v, s) \in X^\alpha \times (0,1) : v(s), v(m) \in I\}$ *the problem (F) has a unique solution*

$$(v(x,t), s(t), m(t)) = (v, s, m)(x, t; v_0, s_0, m_0, \mu).$$

Additionally, the mapping

$$(v_0, s_0, m_0, \mu) \mapsto (v, s, m)(\cdot, t; v_0, s_0, m_0, \mu)$$

is continuously differentiable from $X^\alpha \times \mathbf{R}^3$ *into* $X^\alpha \times \mathbf{R}^2$.

REMARK. It seems to be difficult to extend this approach for existence and uniqueness to a larger class of initial conditions. If we want the operator f to be locally Lipschitz continuous in (u, s, m), then, since it involves the map $\Gamma(u, s) = u(s)$ and $\Delta(u, m) = u(m)$, we necessarily have to take a definition set with $u \in C^{0,1}((0,1)$. In terms of the problem of finding the right exponent α for $\widetilde{X}^\alpha$ there is no difference between C^1 and $C^{0,1}$.

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