ON CHARACTERIZATIONS OF REAL
HYPERSONFACES OF TYPE B IN
A COMPLEX HYPERBOLIC SPACE

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1. Introduction

A complex \( n \)-dimensional Kaehlerian manifold of constant holomorphic sectional curvature \( c \) is called a complex space form, which is denoted by \( M_n(c) \). A complete and simply connected complex space form consists of a complex projective space \( P_nC \), a complex Euclidean space \( C^n \) or a complex hyperbolic space \( H_nC \), according as \( c > 0 \), \( c = 0 \) or \( c < 0 \). The induced almost contact metric structure of a real hypersurface \( M \) of \( M_n(c) \) is denoted by \( (\phi, \xi, \eta, g) \).

There exist many studies about real hypersurfaces of \( M_n(c) \). One of the first researches is the classification of homogeneous real hypersurfaces of a complex projective space \( P_nC \) by Takagi [18], who showed that these hypersurfaces of \( P_nC \) could be divided into six types which are said to be of type \( A_1, A_2, B, C, D, \) and \( E \), and in [3] Cecil-Ryan and [10] Kimura proved that they are realized as the tubes of constant radius over Kahlerian submanifolds.

In differential geometry of real hypersurfaces it is very interesting to characterize homogeneous ones. There are many characterizations of homogeneous ones of type \( A_1 \) and \( A_2 \) since these two examples have a lot of beautiful geometric properties ([3],[4],[15],[16]). In particular Okumura [16] proved that a real hypersurface \( M \) of \( P_nC \) is locally congruent to one of homogeneous ones of type \( A_1 \) and \( A_2 \) if and only if the structure vector \( \xi \) is an infinitesimal isometry, that is, \( \mathcal{L}_\xi g = 0 \), where

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\( \mathcal{L} \) is the Lie derivative. Also Maeda and Udagawa [13] showed another characterization of type \( A_1 \) and \( A_2 \) in terms of \( \mathcal{L}_\xi \phi = 0 \). Motivated by this result, Ki, Kim and Lee [6] proved the fact that "\( M \) is of type \( A_1 \) or type \( A_2 \)" is equivalent to "\( \mathcal{L}_\xi A = 0 \), where \( A \) is the shape operator of \( M \)". Thus it becomes to know that \( \mathcal{L}_\xi g = 0, \mathcal{L}_\xi \phi = 0 \) and \( \mathcal{L}_\xi A = 0 \) are equivalent to each other.

From this point of view, to obtain a characterization of another types which are different from the above homogeneous ones of type \( A_1 \) or \( A_2 \) Ki, Maeda and the second author [9] have studied a characterization of type \( B \) in \( P_n C \) in terms of \( (\mathcal{L}_\xi A)^2 = -c^2 \phi^2 \) or \( (\mathcal{L}_\xi S)^2 = -c^2 \phi^2 \), where \( c \neq 0 \) constant and \( S \) denotes the Ricci tensor of \( M \).

Now we are introduced in these problems when \( c < 0 \), that is, \( M_n(c) \) is the complex hyperbolic space \( H_n C \), which admits the Bergman metric normalized so that the constant holomorphic sectional curvature \( c = -4 \). Berndt [1] showed recently that all real hypersurfaces with constant principal curvatures of a complex hyperbolic space \( H_n C \) are realized as the tubes of constant radius over certain submanifolds when the structure vector field \( \xi \) is principal. Namely, by using some results about focal sets Berndt proved the following

**Theorem A.** Let \( M \) be a connected real hypersurface of \( H_n C \) \((n \geq 2)\). Then \( M \) has constant principal curvatures and \( \xi \) is principal if and only if \( M \) is locally congruent to one of the following

1. \( (A_0) \) a horosphere in \( H_n C \),
2. \( (A_1) \) a geodesic hypersphere or a tube over a complex hyperplane \( H_{n-1} C \).
3. \( (A_2) \) a tube over a totally geodesic submanifold \( H_k C \) for \( k = 1, ..., n - 2 \).
4. \( (B) \) a tube over a totally real hyperbolic space \( H_n R \).

On the other hand, Montiel and Romero [15] gave a characterization of the above homogeneous ones of types \( A_0, A_1 \) and \( A_2 \) by the commutativity of the structure tensor \( \phi \) and the second fundamental tensor \( A \) of \( M \) in \( H_n C \). Also Chen, Montiel and Ludden [4] gave a characterization of types \( A_0, A_1 \) and \( A_2 \) by the length of the covariant derivative of the shape operator \( A \) of \( M \) in \( H_n C \).

Now in this paper we want to investigate real hypersurfaces of type \( B \) in \( H_n C \) in terms of Lie derivatives of \( M \). Until now there are few
results about characterizations of homogeneous real hypersurfaces of type $B$ in $H_nC$. The main purpose of this paper is to characterize this manifold in the class of real hypersurfaces on which the structure vector $\xi$ is principal. We have the following:

**Theorem 1.** Let $M$ be a real hypersurface of $H_nC$ ($n \geq 2$). Suppose that $\xi$ is a principal curvature vector and $M$ satisfies $(\mathcal{L}_\xi A)^2 = c^2 \phi^2$, where $c$ is nonzero locally constant. Then $M$ is locally congruent to a homogeneous real hypersurface of type $B$, which lies on a tube over a totally real hyperbolic space $H_nR$.

**Theorem 2.** Let $M$ be a real hypersurface with constant mean curvature in $H_nC$ ($n \geq 2$). Suppose that $\xi$ is a principal curvature vector and $M$ satisfies $(\mathcal{L}_\xi S)^2 = c^2 \phi^2$, where $S$ is the Ricci tensor of $M$ and $c$ is nonzero locally constant. Then $M$ is locally congruent to a homogeneous real hypersurface of type $B$, which lies on a tube over a totally real hyperbolic space $H_nR$.

1. Preliminaries

Let $M$ be an orientable real hypersurface of $H_nC$ and let $N$ be a unit normal vector field on $M$. The Riemannian connections $\nabla$ in $H_nC$ and $\nabla$ in $M$ are related by the following formulas for any vector fields $X$ and $Y$ on $M$:

\begin{equation}
\nabla_X Y = \nabla_X Y + g(A X, Y) N,
\end{equation}

\begin{equation}
\nabla_X N = -A X,
\end{equation}

where $g$ denotes the Riemannian metric of $M$ induced from the Bergman metric $G$ of $H_nC$ and $A$ is the shape operator of $M$ in $H_nC$. An eigenvector $X$ of the shape operator $A$ is called a principal curvature vector. Also an eigenvalue $\lambda$ of $A$ is called a principal curvature. In what follows, we denote by $V_\lambda$ the eigenspace of $A$ associated with eigenvalue $\lambda$. It is known that $M$ has an almost contact metric structure induced from the complex structure $J$ on $H_nC$, that is, we define a tensor field $\phi$ of type (1,1), a vector field $\xi$ and a 1-form $\eta$ on $M$ by
\( g(\phi X, Y) = G(JX, Y) \) and \( g(\xi, X) = \eta(X) = G(JX, N) \). Then we have

\[
(1.3) \quad \phi^2 X = -X + \eta(X)\xi, \quad g(\xi, \xi) = 1, \quad \phi\xi = 0.
\]

It follows from (1.1) that

\[
(1.4) \quad (\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi,
\]

\[
(1.5) \quad \nabla_X\xi = \phi AX.
\]

Let \( \hat{R} \) and \( R \) be the curvature tensors of \( H_n C \) and \( M \), respectively. Since the curvature tensor \( \hat{R} \) has a nice form, we have the following Gauss and Codazzi equations:

\[
(1.6) \quad g(R(X, Y)Z, W)
\]
\[
= -g(Y, Z)g(X, W) + g(X, Z)g(Y, W) - g(\phi Y, Z)g(\phi X, W)
\]
\[
+ g(\phi X, Z)g(\phi Y, W) + 2g(\phi X, Y)g(\phi Z, W)
\]
\[
+ g(AY, Z)g(AX, W) - g(AX, Z)g(AY, W),
\]

\[
(1.7) \quad (\nabla_X A)Y - (\nabla_Y A)X = -\eta(X)\phi Y + \eta(Y)\phi X + 2g(\phi X, Y)\xi.
\]

From (1.3), (1.5), (1.6) and (1.7) we get

\[
(1.8) \quad SX = -(2n + 1)X + 3\eta(X)\xi + hAX - A^2 X,
\]

\[
(1.9) \quad (\nabla_X S)Y = 3\{g(\phi AX, Y) + \eta(Y)\phi AX\} + (Xh)AY
\]
\[
+ (hI - A)(\nabla_X A)Y - (\nabla_X A)AY.
\]

where \( h = \text{trace}A, S \) is the Ricci tensor of type (1,1) on \( M \) and \( I \) is the identity map.

Now let us introduce a lemma which was proved by Ki and the second author [7].
**Lemma B.** Let $M$ be a real hypersurface of a complex hyperbolic space $H_n C$. Assume that the structure vector $\xi$ of $M$ is a principal curvature vector with the corresponding principal curvature $\alpha$. Then $\alpha$ is locally constant on $M$.

Now we suppose that the structure vector $\xi$ is a principal curvature vector of the shape operator $A$ of $M$, that is $A\xi = \alpha \xi$, where $\alpha$ is the principal curvature corresponding to $\xi$. Then the covariant derivative gives

$$(\nabla_X A)\xi = (X\alpha)\xi + \alpha \phi AX - A\phi AX,$$

where we have used the second formula of (1.2). Thus it follows that

$$g((\nabla_X A)Y, \xi) = (X\alpha)\eta(Y) + \alpha g(Y, \phi AX) - g(Y, A\phi AX),$$

for any tangent vector fields $X$, and $Y$ on $M$. From this, using the equation of Codazzi (1.7) and the fact that $X\alpha = (\xi\alpha)\eta(X)$, we have

$$A\phi A = -\phi + \frac{\alpha}{2}(A\phi + \phi A).$$

From this and the equation of Codazzi (1.7) we also get the following

$$\nabla_\xi A = -\frac{\alpha}{2}(A\phi - \phi A).$$

**2. Proof of Theorem 1**

Let us suppose that the structure vector field $\xi$ is principal with corresponding principal curvature $\alpha$. Then by Lemma B it was known that $\alpha$ is locally constant on $M$. For any $X \in TM$ from (1.5) we have

$$(\mathcal{L}_\xi A)X = \mathcal{L}_\xi (AX) - A\mathcal{L}_\xi X$$

$$= [\xi, AX] - A[\xi, X]$$

$$= \nabla_\xi (AX) - \nabla_{AX} \xi - A(\nabla_\xi X - \nabla_X \xi)$$

$$= (\nabla_X A)\xi - \phi X - \phi A^2 X + A\phi AX$$

$$= \nabla_X (A\xi) - A(\nabla_X \xi) - \phi X - \phi A^2 X + A\phi AX$$

$$= \alpha \phi AX - \phi X - \phi A^2 X,$$
where we have used the equation of Codazzi (1.7) to the fourth equality. Hence we have \((\mathcal{L}_\xi A)\xi = 0\), from which \((\mathcal{L}_\xi A)^2\xi = c^2\phi^2\xi\) holds for any \(c\). Now let us consider any \(X\) in \(TM\) orthogonal to \(\xi\) such that \(AX = \lambda X\). Substituting in (2.1), we get
\[
(\mathcal{L}_\xi A)X = (-\lambda^2 + \alpha \lambda - 1)\phi X.
\]
From this and (2.1) we have
\[
(2.2) \quad (\mathcal{L}_\xi A)^2 X = (-\lambda^2 + \alpha \lambda - 1)(\alpha \phi A \phi X - \phi^2 X - \phi A^2 \phi X).
\]
Thus by Lemma B we can consider the following two cases

Case I. \(\alpha^2 = 4\)

Now we consider for the case \(\alpha = 2\). Then for any \(X\) in \(\xi^\perp\) such that \(AX = \lambda X\) (1.10) gives
\[
(\lambda - 1)A \phi X = (\lambda - 1)\phi X.
\]
Then let us consider an open set \(M_0 = \{x \in M | \lambda(x) \neq 1\}\). Thus \(A \phi X = \phi X\) on \(M_0\). From this and (2.2) we know \((\mathcal{L}_\xi A)^2 X = 0\), which makes a contradiction to the hypothesis of our Theorem 1. Thus the set \(M_0\) should be empty. Then \(\lambda = 1\) on \(M\). Accordingly, by Theorem A \(M\) is locally congruent to a horosphere. But in this case its principal curvatures \(\alpha = 2, \lambda = 1\) also satisfies a quadratic equation of \(\lambda^2 - \alpha \lambda + 1 = 0\) so that the right side of (2.2) vanishes, which makes a contradiction.

Similarly, for the case \(\alpha = -2\) we also get the same conclusion. Accordingly, the Case I can not occur.

Case II. \(\alpha^2 - 4 \neq 0\).

Then we have \(2\lambda - \alpha \neq 0\). In fact, if we suppose \(2\lambda - \alpha = 0\), then (1.10) gives \(\alpha \lambda = 2\). Together with this fact we have \(\alpha^2 - 4 = 0\), a contradiction. Thus from (1.10) it follows \(A \phi X = \frac{\alpha \lambda^2}{2\lambda - \alpha} \phi X\). Substituting this into (2.2), we have
\[
(2.3) \quad (\mathcal{L}_\xi A)^2 X = (-\lambda^2 + \alpha \lambda - 1)\left\{-\left(\frac{\alpha \lambda + 2}{2\lambda - \alpha}\right)^2 + \alpha \frac{\alpha \lambda + 2}{2\lambda - \alpha} - 1\right\}\phi^2 X
\]
for any \(X\) in \(V_\lambda = \{X : AX = \lambda X, X \perp \xi\}\). From this, together with the hypothesis, we become to know that \(\lambda\) is constant. Therefore by
Theorem A the manifold $M$ satisfying the hypothesis of our Theorem 1 is locally congruent to one of types $A_1, A_2$ or $B$. Thus the rest of the proof is to check $(\mathcal{L}_\xi A)^2 = c^2 \phi^2, c \neq 0$ for these homogeneous types one by one.

For a case where $M$ is congruent to one of types $A_1$ or $A_2$ it was known that its principal curvatures are given by $\alpha = 2\coth 2r$, $\lambda = \tanh r$, and $\mu = \coth r$ with the multiplicities $1, 2k$, and $2(n - k - 1)$ respectively. Then these principal curvatures $\coth r, -\tanh r$ are solutions for the quadratic equation $\lambda^2 - \alpha \lambda + 1 = 0$, because it was known that $\alpha = 2\cot 2hr = \coth r + \tanh r$(cf.[1],[2]). Accordingly, from (2.2) we know that each of them does not satisfy $(\mathcal{L}_\xi A)^2 = c^2 \phi^2, c \neq 0$.

Next let us consider for a case where $M$ is of type $B$. In this case its principal curvatures are given by $\alpha = 2\tanh 2r$, $\lambda_1 = \coth r$ and $\lambda_2 = \tanh r$ with the multiplicities $1, n - 1$, and $n - 1$ respectively. Let $x = \coth r$. Then $\alpha = \frac{4x}{1+x^2}$. From this we can calculate the following

$$-\lambda_1^2 + \alpha \lambda_1 - 1 = -\frac{(x^2 - 1)^2}{1 + x^2}$$

and

$$-\lambda_2^2 + \alpha \lambda_2 - 1 = -\frac{(x^2 - 1)^2}{x^2(1 + x^2)}.$$ 

And hence a computation yields that $(\mathcal{L}_\xi A)^2 = c^2 \phi^2$ holds, when $c^2 = \left\{ \frac{(x^2 - 1)^2}{x(1 + x^2)} \right\}^2 \neq 0$. So we can conclude that a real hypersurface of type $B$ satisfies the assumption of Theorem 1. This completes the proof of Theorem 1.

**Remarks.**

(1) Due to the proof of Theorem 1, we get

**Corollary 1.** Let $M$ be a real hypersurface of $H_n C$. Suppose that $\xi$ is a principal curvature vector and $M$ satisfies $(\nabla_\xi A)^2 = c^2 \phi^2$, where $c$ is nonzero locally constant. Then $M$ is locally congruent to a homogeneous real hypersurface of type $B$ which lies to a tube over a totally real hyperbolic space $H_n R$.

(2) Kimura and Maeda classified real hypersurfaces $M$ satisfying $\nabla_\xi A = 0$ in $P_n C$ (cf.[11]).
3. Proof of Theorem 2

From the assumption we can put $A\xi = \alpha \xi$. Now for any $X \in TM$ let us calculate the following

\[(3.1) \quad (L_\xi S)X = L_\xi (SX) - S (L_\xi X) \]
\[= [\xi, SX] - S[\xi, X] \]
\[= \nabla_\xi (SX) - \nabla_{SX} \xi - S(\nabla_\xi X - \nabla_X \xi) \]
\[= (\nabla_\xi S)X - \phi ASX + S\phi AX.\]

Since $\xi$ is an eigenvector of $S$ and $\alpha$ is constant, we find that $(L_\xi S)\xi = L_\xi (S\xi) - S (L_\xi \xi) = 0$, from which $(L_\xi S)^2 \xi = c^2 \phi^2 \xi$ holds for any $c$.

Now let us consider any $X$ in $TM$ orthogonal to $\xi$ such that $AX = \lambda X$. Then Equation (1.8) yields the following:

\[(3.2) \quad \phi ASX = \lambda (2n + 1 + h\lambda - \lambda^2) \phi X, \]
\[S\phi AX = \lambda S\phi X \]
\[= \lambda \{- (2n + 1) \phi X + hA\phi X - A^2 \phi X\}.\]

Next we compute $(\nabla_\xi S)X$. Since we have assumed that the mean curvature of $M$ is constant, from (1.9) for any $X \in V_\lambda = \{X \in TM : AX = \lambda X\}$ it follows that

\[(3.3) \quad (\nabla_\xi S)X = (hI - A)(\nabla_\xi A)X - \lambda (\nabla_\xi A)X.\]

From (3.1),(3.2) and (3.4) we have

\[(3.4) \quad (L_\xi S)X = -\frac{\alpha}{2} \{(hI - A) - \lambda\}(A\phi - \phi A)X \]
\[- \lambda^2 (h - \lambda)\phi X + \lambda \{hA - A^2\} \phi X.\]

Now as in the proof of Theorem 1 we can consider the following two cases.

Case 1. $\alpha^2 = 4$.

If we consider for a case $\alpha = 2$, we also get $A\phi X = \phi X$ on an open subset $M_0 = \{x \in M : \lambda(x) \neq 1\}$ for any $X$ in $\xi^\perp$ such that $AX = \lambda X$. 

That is, $\phi X$ is a principal curvature vector with principal curvature $\mu = 1$ on $M_0$. Thus (3.4) implies

\[(L_\xi S)X = P(\lambda)\phi X,\]

where $P(\lambda)$ denotes the polynomial $-\frac{\alpha}{2}(1 - \lambda)(h - 1 - \lambda) - \lambda^2(h - \lambda) + \lambda(h - 1)$. Then we know $P(1) = 0$. Thus

\[(L_\xi S)^2 X = P(\lambda)(L_\xi S)\phi X = P(\lambda)P(1)\phi^2 X = 0,

which makes a contradiction to the hypothesis. Thus the open set $M_0$ should be empty so that $\lambda = 1$ on $M$. Then $M$ is locally congruent to a horosphere and its principal curvatures are given by $\alpha = 2, \lambda = 1$ with the multiplicities 1 and 2($n - 1$) respectively. Thus (3.4) reduces to

\[(L_\xi S)X = -(2n - 1)\phi X + (2n - 1)\phi X = 0.

Therefore in this case we know that our manifold $M$ does not satisfy the hypothesis of Theorem 2.

Case II. $\alpha^2 \neq 4$

In this case we also know $2\lambda \neq \alpha$ as in the proof of Theorem 1. Then by (2.10) we have the last formulas of (3.2) and (3.3) as follows:

\[S\phi AX = \lambda\left\{- (2n + 1) + h \frac{\alpha \lambda - 2}{2\lambda - \alpha} - \left(\frac{\alpha \lambda - 2}{2\lambda - \alpha}\right)^2\right\}\phi X,

\[(\nabla_\xi S)X = \frac{\alpha}{2} \left(\lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right) \left(h - \lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right)\phi X.

Substituting these into (3.1), we get

\[(L_\xi S)X = \left(\frac{\alpha}{2} - \lambda\right) \left(\lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right) \left(h - \lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right)\phi X.

for any $X \in V_\lambda$ so that

\[(L_\xi S)^2 X = \left(\frac{\alpha}{2} - \lambda\right) \left(\frac{\alpha}{2} - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right) \left(\lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right).

\[\left(\frac{\alpha \lambda - 2}{2\lambda - \alpha} - \lambda\right) \left(h - \lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha}\right)^2 \phi^2 X.

(3.5)
Thus from the assumption of Theorem 2 we know that (3.2) implies \( \lambda \) is constant. Accordingly, by Theorem A the manifold \( M \) satisfying the hypothesis of Theorem 2 is locally congruent to a homogeneous one of types \( A_1, A_2 \), or \( B \).

First let us consider for a case where \( M \) is of type \( A_1 \), and \( A_2 \). Then its principal curvatures are given by \( \alpha = 2 \coth 2r, \lambda = \coth r, \) and \( \mu = \tanh r \) with the multiplicities \( 1, 2k, \) and \( 2(n - k - 1) \) respectively. From this we can calculate the following

\[
\frac{\alpha \lambda - 2}{2\lambda - \alpha} = \frac{2 \coth 2r \coth r - 2}{2 \coth r - (\coth r + \tanh r)} = \frac{\coth^2 r - 1}{\coth r - \tanh r} = \coth r.
\]

This means \( \lambda - \frac{\alpha \lambda - 2}{2\lambda - \alpha} = 0 \). From this, together with (3.5) we know that our manifold \( M \) does not satisfy the hypothesis of Theorem 2.

Next it remains only to find a nonzero constant \( c \) for a case where \( M \) is of type \( B \). Then making use of the method given in the proof of Theorem 1 we can calculate the right hand side of (3.5) as follows:

\[
\frac{\alpha}{2} - \lambda_1 = -\frac{x(1 - x^2)}{1 + x^2},
\]

\[
\frac{\alpha}{2} - \lambda_2 = \frac{x^2 - 1}{x(1 + x^2)},
\]

\[
\lambda_1 - \lambda_2 = \frac{x^2 - 1}{x}, \quad \text{and}
\]

\[
h - \lambda_1 - \lambda_2 = \frac{4x}{1 + x^2} + \frac{(n - 2)(x^2 + 1)}{x}.
\]

Summing up these formulas, we know that \( (L_S \xi S)^2 = c^2 \phi^2 \) holds, when

\[
c^2 = \left(\frac{1 - x^2}{1 + x^2}\right)^2 \left(\frac{x^2 - 1}{x}\right)^2 \left\{\frac{4x}{1 + x^2} + \frac{(n - 2)(x^2 + 1)}{x}\right\}^2.
\]

**REMARKS.**

(3) By virtue of the proof of Theorem 2, we have

**COROLLARY 2.** Let \( M \) be a real hypersurface with constant mean curvature in \( H_nC \). Suppose that \( \xi \) is a principal curvature vector
and $M$ satisfies $(\nabla_\xi S)^2 = c^2 \phi^2$, where $S$ is the Ricci tensor of $M$ and $c$ is nonzero locally constant. Then $M$ is locally congruent to a homogeneous real hypersurface of type $B$ which lies a tube over a totally real hyperbolic space $H_nR$.

(4) Ki ([5]) proved that $H_nC(n \geq 3)$ does not admit a real hypersurface $M$ with parallel Ricci tensor.

(5) Maeda classified real hypersurfaces $M$ satisfying $\nabla_\xi S = 0$(in $P_nC$) under the condition that $\xi$ is a principal curvature vector (cf.[12])

References


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