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HERMITIAN METRICS IN RIZZA MANIFOLDS

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ABSTRACT. The almost Hermitian Finsler structure of a Rizza manifold is an almost Hermitian structure if a special condition satisfies. In this paper, the induced Finsler connection from Moór metric is defined and the some properties of a Kaehlerian Finsler manifold with respect to the induced Finsler connection from Moór metric are investigated.

0. Introduction

The Rizza manifold with a Finsler metric $g_{ij}(x,y)$ and an almost complex structure $f^i{}_j(x)$ was, for the first time, introduced by G. B. Rizza [7]. Afterward, it was studied by many authors as R. Miron [4], Y. Ichijyō [2], M.Fukui [1] and others. In [1], M. Fukui has proved that if $g_{ij}(x,y)$ and $f^i{}_j(x)$ satisfies the condition:

$$g_{lh}(x,y) - g_{ij}(x,y)f^i{}_l(x)f^j{}_h(x) = 0, \tag{0.1}$$

then g_{ij} is a Riemannian metric.

In the present paper we are concerned with the Hermitian metrics and curvature tensors in the Rizza manifold. We shall find that the Rizza structure is to be an almost Hermitian structure under a condition in §1. In §2, we shall introduce the Finsler connection $F\tilde{\Gamma}$ constructed by the induced Moór metric from a Rizza structure, and investigate the properties of the Finsler connection $F\tilde{\Gamma}$. In §3, we shall deal with the curvature tensors constructed by the Finsler connection $F\tilde{\Gamma}$. In a Kaehlerian Finsler manifold with respect to the Finsler connection $F\tilde{\Gamma}$,

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the analogous result as the Bochner's theorem in a Riemannian manifold is obtained in §4.

The terminology and notation in the present paper are mainly referred to M.Matsumoto's monograph [3].

1. A Rizza structure

Let M be an n-dimentional Finsler manifold with the fundamental function $L(x,y)$ positively homogeneous of the first degree in y. The Finsler metric tensor $g_{ij}(x,y)$ is given by

$$g_{ij}(x,y)=\frac{1}{2}\dot{\partial}_i\dot{\partial}_j L^2(x,y), \tag{1.1}$$

where $\dot{\partial}_i=\partial/\partial y^i$.

We assume that M admits a complex structure $f^i{}_j(x)$ which depends a point x of M and the fundamental function $L(x,y)$ satisfies the Rizza condition;

$$L(x, y cos\theta + f(x) y sin\theta) = L(x,y) \tag{1.2}$$

for any function θ.

The manifold which admits a Finsler metric $g_{ij}(x,y)$ and an almost complex structure $f^i{}_j(x)$ satisfying the Rizza condition (1.2) is called a Rizza manifold (or an almost Hermitian Finsler manifold) and the structure $(f^i{}_j(x), g_{ij}(x,y))$ is called a Rizza structure (or an almost Hermitian Finsler structure).

Differentiating (1.2) partially with respect to θ, we have

$$\dot{\partial}_i L^2(x, y cos\theta + f(x) y sin\theta)(-y^i sin\theta + f^i{}_k(x) y^k cos\theta) = 0. \tag{1.3}$$

Putting $\theta=0$, we get

$$\dot{\partial}_i L^2(x,y) f^i{}_k y^k = 0. \tag{1.4}$$

On the other hand, by Euler's theorem on homogeneous function, we have

$$\dot{\partial}_j\dot{\partial}_i L^2(x,y)\ y^j = \dot{\partial}_i L^2(x,y). \tag{1.5}$$

From (1.1), (1.5) is expressed by

$$\dot{\partial}_i L^2(x,y) = 2g_{ij}(x,y)y^j. \tag{1.6}$$

Therefore (1.4) is rewritten

$$g_{ij}(x,y)f^i{}_k(x)y^k y^j = 0, \tag{1.7}$$

that is $g(fy,y) = 0$. Differentiating (1.3) partially further with respect to θ and putting $\theta = 0$, we get

$$\dot{\partial}_j\dot{\partial}_i L^2(x,y)f^i{}_k(x)y^k f^j{}_l(x)y^l - \dot{\partial}_i L^2(x,y)y^i = 0,$$

that is

$$g_{ij}(x,y)f^i{}_k(x)f^j{}_l(x)y^k y^l = g_{ij}(x,y)y^i y^j \tag{1.8}$$

by virtue of (1.1) and (1.6). This is written as $g(fy,fy) = g(y,y)$. Differentiating (1.3) partially with respect to y^j and putting $\theta = 0$, we have

$$g_{ij}(x,y)f^i{}_k(x)f^j{}_l(x)y^k = g_{lk}(x,y)y^k, \tag{1.9}$$

that is $g(fy,fX) = g(y,X)$ for any vector field X.
Differentiating (1.9) partially further with respect to y^h, we get

$$g_{lh}(x,y) - g_{ji}(x,y)f^i{}_h(x)f^j{}_l(x) = 2C_{jih}f^i{}_k(x)f^j{}_l(x)y^k, \tag{1.10}$$

where $C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij}$.

On the other hand, we remember the operators [9] O^{ij}_{hk} and ${}^*O^{ij}_{hk}$ by $O^{ij}_{hk} = \frac{1}{2}(\delta^i_h\delta^j_k - f^i{}_h f^j{}_k)$, ${}^*O^{ij}_{hk} = \frac{1}{2}(\delta^i_h\delta^j_k + f^i{}_h f^j{}_k)$ respectively. If a tensor $K_i{}^k{}_j$ satisfies $O^{ij}_{st}K_i{}^k{}_j = 0$, we say that $K_i{}^k{}_j$ is hybrid in i and j and if ${}^*O^{ij}_{st}K_i{}^k{}_j = 0$, we say that it is pure in i and j.

THEOREM 1.1. *In a Rizza manifold M with a Finsler metric g and an almost complex structure f, if $C_{kim}f^m{}_j$ is pure in i and j, then (f,g) is an almost Hermitian structure.*

PROOF. From the assumtion we have

$$C_{kim}f^m{}_j - C_{kmj}f^m{}_i = 0. \tag{1.11}$$

Transvecting (1.11) with y^j, we obtain $C_{kim}f^m{}_jy^j = 0$ by virtue of $C_{kmj}y^j = 0$. Substituting this equation into (1.10), we get (0.1). Therefore g_{ij} is a Riemannian metric, that is (f,g) is an almost Hermitian structure.

2. A Finsler connection by the induced Moór metric

The generalized Finsler metric $\tilde{g}_{ij}(x,y)$ satisfying the conditions: $\tilde{g}_{ij}(x,y) = \tilde{g}_{ij}(x,y), \tilde{g}_{ij}(x,y)\xi^i\xi^j$ is positive definite and $\tilde{g}_{ij}(x,y)$ is (0)p-homogeneous for y is called a Moór metric [5]. In a Rizza manifold with Rizza structure $(f^i{}_j(x),\ g_{ij}(x,y))$, if we put

$$\tilde{g}_{ij}(x,y) = \frac{1}{2}(g_{ij}(x,y) + g_{lk}(x,y)f^l{}_i(x)f^k{}_j(x)), \tag{2.1}$$

then $\tilde{g}_{ij}(x,y)$ is a Moór metric. This Moór metric defined by (2.1) is called the induced Moór metric from a Rizza structure $(f^i{}_j(x), g_{ij}(x,y))$. This induced Moór metric $\tilde{g}_{ij}(x,y)$ satisfies easily

$$\tilde{g}_{ij}(x,y) = \tilde{g}_{lk}(x,y)f^l{}_i(x)f^k{}_j(x), \tag{2.2}$$

from which

$$\tilde{g}_{im}(x,y)f^m{}_j(x) = -\tilde{g}_{jm}(x,y)f^m{}_i(x). \tag{2.3}$$

From (1.9), (2.1) and (2.2), we have $\tilde{g}_{ij}(x,y)y^j = g_{ij}(x,y)y^j$. Concerning the reciprocal tensor field $\tilde{g}^{ij}(x,y)$ of $\tilde{g}_{ij}(x,y)$, we could prove

$$\tilde{g}^{ij}(x,y) = \tilde{g}^{kl}(x,y)f^i{}_k(x)f^j{}_l(x), \tag{2.4}$$

from which

$$\tilde{g}^{ik}(x,y)f^j{}_k(x) = -\tilde{g}^{jk}(x,y)f^i{}_k(x). \tag{2.4$'$}$$

Let us assume that the Finsler manifold M equipped with a non-linear connection $N^i{}_j(x,y)$. We can introduce the Finsler connection $F\tilde{\Gamma} = (\tilde{\Gamma}_j{}^i{}_k, N^i{}_j, \tilde{C}_j{}^i{}_k)$ from $\tilde{g}_{ij}(x,y)$ as follows;

$$(2.5)\qquad \begin{aligned} \tilde{\Gamma}_j{}^i{}_k &= \frac{1}{2}\tilde{g}^{ir}(\delta_k\tilde{g}_{jr} + \delta_j\tilde{g}_{rk} - \delta_r\tilde{g}_{kj}), \\ \tilde{C}^i_{jk} &= \frac{1}{2}\tilde{g}^{ir}(\dot{\partial}_k\tilde{g}_{jr} + \dot{\partial}_j\tilde{g}_{rk} - \dot{\partial}_r\tilde{g}_{kj}), \end{aligned}$$

where $\delta_k = \partial_k - N^i{}_k(x,y)\dot{\partial}_i$.

For any Finsler tensor field $K^i{}_j(x,y)$, the h- and v-covariant derivatives with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) are expressed as follows respectively

$$(2.6)\qquad K^i{}_{j|k} = \delta_k K^i{}_j + \tilde{\Gamma}_m{}^i{}_k K^m{}_j - K^i{}_m\tilde{\Gamma}_j{}^m{}_k,$$

$$(2.7)\qquad K^i{}_j|_k = \dot{\partial}_k K^i{}_j + \tilde{C}_m{}^i{}_k K^m{}_j - K^i{}_m\tilde{C}_j{}^m{}_k.$$

A straightforward calculation shows that $\tilde{g}_{ij|k} = 0$ and $\tilde{g}_{ij}|_k = 0$, that is the Finsler connection $F\tilde{\Gamma}$ is metrical. By (2.5), the $(h)h$-torsion tensor $\tilde{T}_j{}^i{}_k = \tilde{\Gamma}_j{}^i{}_k - \tilde{\Gamma}_k{}^i{}_j$ and $(v)v$-torsion tensor $\tilde{S}_j{}^i{}_k = \tilde{C}_j{}^i{}_k - \tilde{C}_k{}^i{}_j$ vanish. Consequently we have

THEOREM 2.1. *In a Rizza manifold with the induced Moór metric, the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) is metrical and $(h)h$-torsion tensor $\tilde{T}_j{}^i{}_k$ and $(v)v$-torsion tensor $\tilde{S}_j{}^i{}_k$ vanish.*

Let us put $\tilde{C}_{jik} = \tilde{g}_{il}\tilde{C}^l_{jk}$. Then the direct calculation lead us to

$$(2.8)\qquad \tilde{C}_{jik} = \frac{1}{2}(C_{jik} + C_{jpr}f^p{}_i f^r{}_k + C_{kpr}f^p{}_i f^r{}_j - C_{iqr}f^q{}_j f^r{}_k)$$

by virtue of (2.1) and (2.5). It is easily seen that $\tilde{C}_{jik} = \tilde{C}_{kij}$.
From (1.10), $C_{ijk}y^i = 0$ and (2.4)$'$, we have

$$(2.9)\qquad \tilde{C}_{jik}y^j = 0,\ \tilde{C}_{jik}y^k = 0,$$

$$(2.10)\qquad \tilde{C}_{jik}y^i = \frac{1}{2}(g_{jk} - g_{qr}f^q{}_j f^r{}_k).$$

$$(2.11)\qquad \tilde{C}_{qir}f^q{}_j f^r{}_k = -\tilde{C}_{jik},\ \tilde{C}^i_{qr}f^q{}_j f^r{}_k = -\tilde{C}^i_{jk},$$

$$(2.12)\qquad \tilde{C}_{jpr}f^p{}_i f^r{}_k = \tilde{C}_{jik},\ \tilde{C}^p_{jr}f^i{}_p f^r{}_k = -\tilde{C}^i_{jk}.$$

From the second equation of (2.12), we have $0 = \tilde{C}^i_{jm}f^m{}_l - \tilde{C}^m_{j}{}_l f^i{}_m = f^i{}_l|_j$. Thus we get

THEOREM 2.2. *In a Rizza manifold with induced Moór metric, the v-covariant derivative of an almost complex structure $f^i{}_j(x)$ with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) vanish.*

3. The curvature tensors with respect to $F\tilde{\Gamma}$

The Ricci identities for an any Finsler tensor field $K^i{}_j(x,y)$ with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) are given by

$$K^h{}_{i|j|k} - K^h{}_{i|k|j} = K^r{}_i\tilde{R}_r{}^h{}_{jk} - K^h{}_r\tilde{R}_i{}^r{}_{jk} - K^h{}_i|_r\tilde{R}_j{}^r{}_k, \tag{3.1}$$

$$\begin{aligned} &K^h{}_{i|j}|_k - K^h{}_i|_{k|j} \\ &= K^r{}_i\tilde{P}_r{}^h{}_{jk} - K^h{}_r\tilde{P}_i{}^r{}_{jk} - K^h{}_{i|r}\tilde{C}_j{}^r{}_k - K^h{}_i|_r\tilde{P}_j{}^r{}_k, \end{aligned} \tag{3.2}$$

$$K^h{}_i|_j|_k - K^h{}_i|_k|_j = K^r{}_i\tilde{S}_r{}^h{}_{jk} - K^h{}_r\tilde{S}_i{}^r{}_{jk}, \tag{3.3}$$

where

$$\tilde{P}_j{}^i{}_k = \dot{\partial}_k N^i{}_j - \tilde{\Gamma}_k{}^i{}_j, \tag{3.4}$$

$$\tilde{R}_j{}^i{}_k = \delta_k N^i{}_j - \delta_j N^i{}_k, \tag{3.5}$$

$$\tilde{R}_i{}^h{}_{jk} = \delta_k\tilde{\Gamma}_i{}^h{}_j - \delta_j\tilde{\Gamma}_i^h{}_k + \tilde{\Gamma}_i{}^r{}_j\tilde{\Gamma}_r{}^h{}_k - \tilde{\Gamma}_i{}^r{}_k\tilde{\Gamma}_r{}^h{}_j + \tilde{C}_i{}^h{}_r\tilde{R}_j{}^r{}_k, \tag{3.6}$$

$$\tilde{S}_i{}^h{}_{jk} = \dot{\partial}_k\tilde{C}_i{}^h{}_j - \dot{\partial}_j\tilde{C}_i{}^h{}_k + \tilde{C}_i{}^r{}_j\tilde{C}_r{}^h{}_k - \tilde{C}_i{}^r{}_k\tilde{C}_r{}^h{}_j, \tag{3.7}$$

$$\tilde{P}_i{}^h{}_{jk} = \dot{\partial}_k\tilde{\Gamma}_i{}^h{}_j - \delta_j\tilde{C}_i{}^h{}_k + \tilde{\Gamma}_i{}^r{}_j\tilde{C}_r{}^h{}_k - \tilde{C}_i{}^r{}_k\tilde{\Gamma}_r{}^h{}_j + \tilde{C}_i{}^h{}_r\dot{\partial}_k N^r{}_j. \tag{3.8}$$

The identities (3.1) and (3.2) are obtained by $\tilde{T}^i_{jk} = 0, \tilde{S}^i_{jk} = 0$. The tensors $\tilde{R}_i{}^h{}_{jk}, \tilde{S}_i{}^h{}_{jk}$ and $\tilde{P}_i{}^h{}_{jk}$ are called the h-curvature tensor, hv-curvature tensor and v-curvature tensor of the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) respectively.

Since $\tilde{g}_{ij|k} = 0, \tilde{g}_{ij}|_k = 0$ in a Rizza manifold with the Finsler connection $F\tilde{\Gamma}$ defined by (2.5), the Ricci identities for $\tilde{g}_{ij}(x,y)$ are given by

$$\tilde{R}_{ijkl} + \tilde{R}_{jikl} = 0, \tag{3.9}$$

$$\tilde{P}_{ijkl} + \tilde{P}_{jikl} = 0, \tag{3.10}$$

$$\tilde{S}_{ijkl} + \tilde{S}_{jikl} = 0, \tag{3.11}$$

where $\tilde{R}_{ijkl} = \tilde{g}_{jh}\tilde{R}_i{}^h{}_{kl}$, $\tilde{P}_{ijkl} = \tilde{g}_{jh}\tilde{P}_i{}^h{}_{kl}$, $\tilde{S}_{ijkl} = \tilde{g}_{jh}\tilde{S}_i{}^h{}_{kl}$.

By Theorem 2.2 and (3.3), the Ricci identity for $f^i{}_j(x)$ is given

$$f^r{}_i\tilde{S}_r{}^h{}_{jk} - f^h{}_r\tilde{S}_i{}^r{}_{jk} = 0, \tag{3.12}$$

from which

$$\tilde{S}_l{}^h{}_{jk} + \tilde{S}_i{}^r{}_{jk}f^h{}_r f^i{}_l = 0, \tag{3.13}$$

that is ${}^*O^{hi}_{rl}(\tilde{S}^r_i{}_{jk}) = 0$. From Theorem 2.2, the tensor O^{ij}_{sk} and ${}^*O^{ij}_{sk}$ are v-covariant constants with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5), we obtain

$$\tilde{S}^h_l{}_{jk}|_{m_1\ldots m_p} + \tilde{S}^r_i{}_{jk}|_{m_1\ldots m_p} f^h{}_r f^i{}_l = 0 \ (p = 1,2,\ldots). \tag{3.14}$$

Thus we have

THEOREM 3.1. *In a Rizza manifold with the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) the v-curvature tensor $\tilde{S}^r_i{}_{jk}$ is pure in r and i, and ${}^*O^{hi}_{rl}(\tilde{S}^r_i{}_{jk}|_{m_1\ldots m_p}) = 0$ $(p = 1,2,\ldots)$.*

4. A Kaehlerian Finsler manifold with respect to $F\tilde{\Gamma}$

In a Rizza manifold, the h-covariant derivative of an almost complex structure $f^i{}_j(x)$ for the Finsler connection $F\tilde{\Gamma}$ defined by (2.5) is expressed as

$$f^i{}_{j|k} = \partial_k f^i{}_j + \tilde{\Gamma}^i_m{}_k f^m{}_j - f^i{}_m\tilde{\Gamma}^m_j{}_k. \tag{4.1}$$

A Rizza manifold satisfying the condition $f^i{}_{j|k} = 0$ is said to be a Kaehlerian Finsler manifold with respect to a Finsler connection $F\tilde{\Gamma}$ defined by (2.5). In a Kaehlerian Finsler manifold with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5), the operators O^{ij}_{sk} and ${}^*O^{ij}_{sk}$ are h- and v-covariant constants, that is $O^{ij}_{sk|l} = 0, O^{ij}_{sk}|_l = 0, {}^*O^{ij}_{sk|l} = 0$ and ${}^*O^{ij}_{sk}|_l = 0$.

From Ricci identities for $f^i{}_j(x)$, we have

$$(4.2) \qquad f^r{}_i\tilde{R}_r{}^h{}_{jk} - f^h{}_r\tilde{R}_i{}^r{}_{jk} = 0,\ f^r{}_i\tilde{P}_r{}^h{}_{jk} - f^h{}_r\tilde{P}_i{}^r{}_{jk} = 0,$$

from which

$$(4.3) \qquad \tilde{R}_l{}^h{}_{jk} + \tilde{R}_i{}^r{}_{jk}f^h{}_rf^i{}_l = 0,\ \tilde{P}_l{}^h{}_{jk} + \tilde{P}_i{}^r{}_{jk}f^h{}_rf^i{}_l = 0,$$

that is ${}^*O^{hi}_{rl}(\tilde{R}^r_i{}_{jk}) = 0,\ {}^*O^{hi}_{rl}(\tilde{P}^r_i{}_{jk}) = 0$.

From the first equation of (4.3) we have

$$(4.4) \qquad \begin{aligned} &\tilde{R}^h_l{}_{jk|m_1\dots m_p} + \tilde{R}^r_i{}_{jk|m_1\dots m_p}f^h{}_rf^i{}_l = 0\ (p = 1,2,\dots),\\ &\tilde{R}^h_l{}_{jk}|_{m_1\dots m_p} + \tilde{R}^r_i{}_{jk}|_{m_1\dots m_p}f^h{}_rf^i{}_l = 0\ (p = 1,2,\dots). \end{aligned}$$

And the similar equations for $\tilde{P}^h_l{}_{jk}$ and $\tilde{S}^h_l{}_{jk}$ are obtained easily.

Consequentely, we have

THEOREM 4.1. *In a Kaehlerian Finsler manifold with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5), the h-curvature tensor $\tilde{R}^h_i{}_{jk}$ and the hv-curvature tensor $\tilde{P}^h_i{}_{jk}$ are pure in i and h, and*

$$\begin{aligned} &{}^*O^{hi}_{rl}(\tilde{R}^r_i{}_{jk|m_1\dots m_p}) = 0,\ {}^*O^{hi}_{rl}(\tilde{R}^r_i{}_{jk}|_{m_1\dots m_p}) = 0\ (p = 1,2,\dots),\\ &{}^*O^{hi}_{rl}(\tilde{P}^r_i{}_{jk|m_1\dots m_p}) = 0,\ {}^*O^{hi}_{rl}(\tilde{P}^r_i{}_{jk}|_{m_1\dots m_p}) = 0\ (p = 1,2,\dots),\\ &{}^*O^{hi}_{rl}(\tilde{S}^r_i{}_{jk|m_1\dots m_p}) = 0,\ {}^*O^{hi}_{rl}(\tilde{S}^r_i{}_{jk}|_{m_1\dots m_p}) = 0\ (p = 1,2,\dots). \end{aligned}$$

On the other hand, the Nijenhuis tensor of $f^i{}_j(x)$ is expressed by

$$(4.5) \qquad N^i{}_{jk} = (\partial_rf^i{}_j)f^r{}_k - (\partial_rf^i{}_k)f^r{}_j - f^i{}_r\partial_jf^r{}_k - f^i{}_r\partial_kf^r{}_j.$$

Substituting (4.1) into (4.5), we obtain

$$\begin{aligned} N^i{}_{jk} = & (f^i{}_{j|r} - \tilde{\Gamma}^i_{m\,r} f^m{}_j + f^i{}_m \tilde{\Gamma}^m_{j\,r}) f^r{}_k \\ & - (f^i{}_{k|r} - \tilde{\Gamma}^i_{m\,r} f^m{}_k + f^i{}_m \tilde{\Gamma}^m_{k\,r}) f^r{}_j \\ & + f^i{}_r (f^i{}_{k|j} - \tilde{\Gamma}^r_{m\,j} f^m{}_k + f^r{}_m \tilde{\Gamma}^m_{k\,j}) \\ & - f^i{}_r (f^r{}_{j|k} - \tilde{\Gamma}^r_{m\,k} f^m{}_j + f^r{}_m \tilde{\Gamma}^m_{j\,k}) \\ = & f^i{}_{j|r} f^r{}_k - f^i{}_{k|r} f^r{}_j + f^i{}_r f^r{}_{k|j} - f^i{}_r f^r{}_{j|k}. \end{aligned}$$

Thus we have

THEOREM 4.2. *In a Kaehlerian Finsler manifold with respect to the Finsler connection* $F\tilde{\Gamma}$ *defined by (2.5), the almost complex structure* $f^i{}_j(x)$ *is integrable.*

In a Kaehlerian Finsler manifold with respect to the Finsler connection $F\tilde{\Gamma}$ defined by (2.5), if $\tilde{R}_i{}^h{}_{jk} = \tilde{R}(\tilde{g}_{ij}\delta^h_k - \tilde{g}_{ik}\delta^h_j)$, then the first equation of (4.2) can be rewritten as

$$f^r{}_i \tilde{R}(\tilde{g}_{rj}\delta^h_k - \tilde{g}_{rk}\delta^h_j) = f^h{}_r \tilde{R}(\tilde{g}_{ij}\delta^r_k - \tilde{g}_{ik}\delta^r_j).$$

Now, we suppose $\tilde{R} \neq 0$, then we have

$$f^r{}_i \tilde{g}_{rj}\delta^h_k - f^r{}_i \tilde{g}_{rk}\delta^h_j = f^h{}_k \tilde{g}_{ij} - f^h{}_j \tilde{g}_{ik}. \tag{4.6}$$

Contracting (4.6) with respect to h and k and using (2.3), we get $(n-2)\tilde{g}_{rj} f^r{}_i = 0$. If $n > 2$, then $\tilde{g}_{rj} f^r{}_i = 0$. Transvecting this equation with $\tilde{g}^{jk}$, we have $f^k{}_j = 0$. This is a contradiction. Cosequentely we obtain $\tilde{R} = 0$. Thus we have

THEOREM 4.3. *In an* n*-dimensional Kaehlerian Finsler manifold with respect to the Finsler connection* $F\tilde{\Gamma}$ *defined by (2.5), if* $\tilde{R}_i{}^h{}_{jk} = \tilde{R}(\tilde{g}_{ij}\delta^h_k - \tilde{g}_{ik}\delta^h_j)$*, and* $n > 2$*, then* $\tilde{R} = 0$.

REMARK. If the metric is a Riemannian one, the h-curvature tensor $\tilde{R}_i{}^h{}_{jk}$ coincide with the Riemannian-Christoffel's curvature tensor, and this theorem reduces to the well-known Bochner's theorem; If an n-dimensional Kaehlerian manifold is a constant curvature and $n > 2$, then it is of zero curvature.

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