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Using Fuzzy Set Theory in Project Planning Problem†

Seung Hun Park* · In Seong Chang**

Abstract

This paper presents a method for solving project planning problem which the opinions of experts greatly disagree in each activity processing time. Triangular fuzzy numbers(TFNs) are used to represent activity times of experts. And we introduce a pessimistic project planning with major TFNs and an optimistic project planning with minor TFNs, and illustrate the approach using a combination of the existing composite and the comparison methods, which is used to solve the complex project planning problem.

1. Introduction

Project network analysis represents an effective tool frequently used whenever complex and structured technological process are to be planned. Occasionally, a project manager is challenged by a project with which she/he has had no prior experience. Traditionally, the Project Evaluation and Review Technique (PERT) has been used to allow for uncertain activity times by modeling them as random variables with Beta distributions[8]. However, this approach is theoretically valid only when there is some reason to assume an underlying Beta distribution : experience, information and so on.

If the uncertainty is due to a lack of them, then the activity times should be modeled as fuzzy numbers. One of the problems in solving project planning problem by considering fuzzy numbers is comparing the fuzzy numbers to perform the forward and backward passes. By the

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* Department of Industrial Engineering, Inha University.
** Utsunomiya University, Japan.

use of the composite and comparison methods[5, 15], the project planning problems with fuzzy activity times can be solved and various method have been proposed[2, 3, 4, 5, 7, 9, 11, 14, 15, 16, 17].

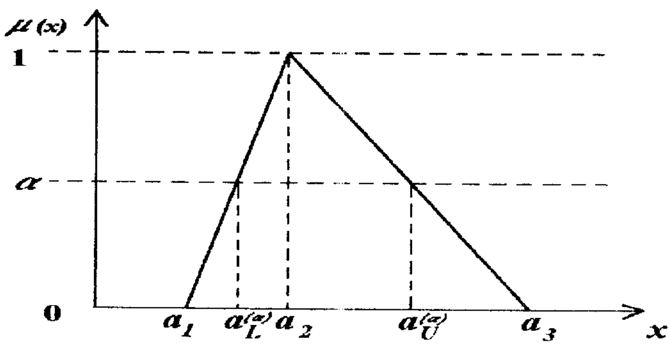
Occasionally, the opinions of many experts are required to represent uncertain activity times in a large scale project. In such cases, for each activity time, many experts are interviewed and they can express activity times as fuzzy numbers.

In this paper, we illustrate a method for solving project planning problem which the opinions of experts greatly disagree in each activity time. TFNs are used to represent activity times of experts. If the activity times are fuzzy numbers and the ordinary fuzzy operations on fuzzy numbers are used, then the project completion time is a fuzzy number. Furthermore, a unique critical path is not identified. Thus, the goal of this paper is to estimate the fuzzy project time and the degree of criticality for each path in a project network. A TFN can be defined by a triplet (a_1, a_2, a_3)[10, 18]. The TFN which is a special kind of fuzzy number can represent the estimated activity time naturally. For example, an expert may say that the activity time for activity A is generally a_2 minutes. But, due to other factors which cannot be controlled, the activity time may be occasionally as slow as a_3 minutes or as fast as a_1 minutes. This result is naturally a TFN. The membership function of a TFN $\widetilde{A}$ is defined as

$$\mu_{\widetilde{A}}(x)=\begin{cases} 0 & x<a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1\le x\le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2\le x\le a_3 \\ 0 & x>a_3 \end{cases} \tag{1}$$

where $\mu_{\widetilde{A}}(x)$ is the degree of membership or membership function value of x in $\widetilde{A}$

The TFN $\widetilde{A}$ is shown in Figure 1. An alternative



[Figure 1] A triangular fuzzy number $\widetilde{A}$

definition of the TFN can be obtained by the notion of the interval of confidence. The interval of confidence at level α of the TFN $\tilde{A}$ is (see the Figure 1)

$$A_\alpha=[a_l^{(\alpha)}, a_u^{(\alpha)}] = [(a_2-a_1)\alpha+a_1, (a_2-a_3)\alpha+a_3], \alpha\in[0, 1] \quad (2)$$

where $a_l^{(\alpha)}$ and $a_u^{(\alpha)}$ are lower boundary and upper boundary respectively

Using these fuzzy data, we will introduce a pessimistic project planning with major TFNs (see the next section) and an optimistic project planning with minor TFNs (see the next section), and the combination of the composite and comparison methods is used to determine the fuzzy project completion time and the degree of criticality for each path effectively.

2. A Method for Solving Project Planning Problem

For solving project planning problem which the opinions of experts greatly disagree in each activity time, we determine both the major TFN and the minor TFN for each activity time. To determine the major and minor TFNs, we propose a ranking method based on the dominance property[10, 18]. Let us consider a sheaf G composed of m TFNs $\tilde{A}_i=(a_1, a_2, a_3)$, $i=1, 2, \cdots, m$. We define $\tilde{A}^*$ as a major TFN if this TFN dominates all the others $\tilde{A}_i$ in the sheaf G. The criterion for dominance is one of the following three in the order given below.

C1 : The fuzzy number with the higher associated ordinary number is then ranked higher than the fuzzy number with the lower associated ordinary number and the fuzzy number with the greatest associated ordinary number dominates all the others in the sheaf G. The associated ordinary number is calculated as

$$\hat{\tilde{A}} = \left(\frac{a_1+2a_2+a_3}{4}\right) \quad (3)$$

C2 : If the associated ordinary numbers happen to be equal, the mode a_2 is used as a second criterion and the one with the higher mode is judged the higher.

C3 : If the second criterion is also not sufficient, the divergence (the distance between two end points) is used as a third criterion and the one with the higher divergence is judged the higher.

On the contrary, we call TFN $\tilde{A}_*$ a minor if the TFN is dominated by all the others in the sheaf G using respectively the criteria C1, C2, and C3.

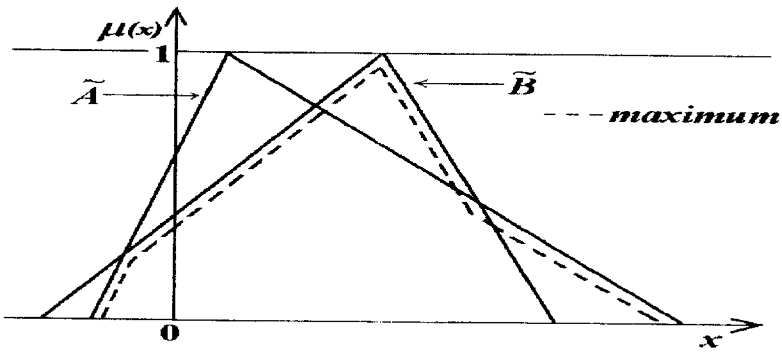
The ultimate objective of the project planning is to find the critical path that represents the shortest duration needed to complete the project. The critical path calculations include the forward and backward passes based on *max* and *min* operators, respectively. There are two methods in deciding maximum and minimum of fuzzy numbers. One is the composite method and another is the comparison method. In general, the composite method using the fuzzy maximum and minimum operators defines the maximum and minimum of fuzzy numbers[7, 10]. The composite method the maximum of two TFNs $\widetilde{A}$ and $\widetilde{B}$ as

$$\begin{aligned} \tilde{max}\{A_\alpha, B_\alpha\} &= \tilde{max}\{[a_L^{(\alpha)}, a_U^{(\alpha)}], [b_L^{(\alpha)}, b_L^{(\alpha)}]\} \\ &= [max\{a_L^{(\alpha)}, b_L^{(\alpha)}\}, max\{a_U^{(\alpha)}, b_U^{(\alpha)}\}] \end{aligned} \quad (4)$$

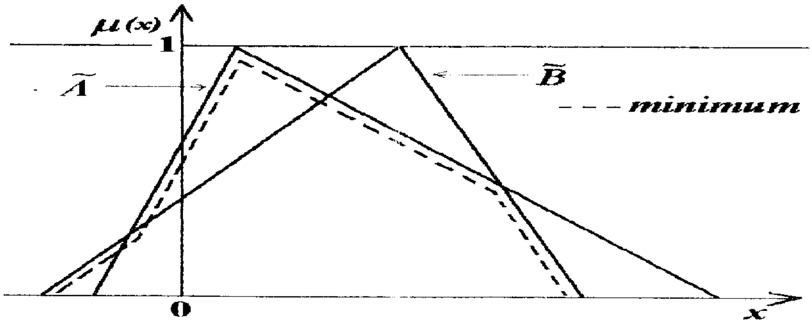
and the minimum of two TFNs $\widetilde{A}$ and $\widetilde{B}$ as

$$\begin{aligned} \tilde{min}\{A_\alpha, B_\alpha\} &= \tilde{min}\{[a_L^{(\alpha)}, a_U^{(\alpha)}], [b_L^{(\alpha)}, b_L^{(\alpha)}]\} \\ &= [min\{a_L^{(\alpha)}, b_L^{(\alpha)}\}, min\{a_U^{(\alpha)}, b_U^{(\alpha)}\}] \end{aligned} \quad (5)$$

where $a_L^{(\alpha)}$ and $b_L^{(\alpha)}$ are their respective lower boundaries and $a_U^{(\alpha)}$ and $b_U^{(\alpha)}$ are their respective lower boundaries. Figure 2 and 3 illustrate examples of the fuzzy maximum and minimum.



[Figure 2] Maximum of fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$



[Figure 3] Minimum of fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$

The comparison method using the discrete maximum and minimum operators is an act of comparing fuzzy numbers to yield a totally ordered set[19]. Many people have studied the comparison of fuzzy numbers and have proposed many methods. A summary of such methods can be found in Refs.[6, 15]. In this paper, we define the operators $\widetilde{max}$ and $\widetilde{min}$ as the fuzzy maximum and minimum operators, respectively. On the other hand, the operators *max* and *min* represent the discrete maximum and minimum operators.

3. Approaches Using the Composite Method

Dubois and Prade[7], Prade[17], Chanas and Kamburowski[4] and Chanas[3] have applied the composite method to the forward and backward calculations. An algorithm for performing the forward and backward passes through the fuzzy network was first introduced by Dubois and Prade[7]. Essentially, the forward and backward passes are to be fuzzified with fuzzy components. In the composite method, the fuzzy maximum operator $\widetilde{max}$ and the fuzzy minimum operator $\widetilde{mix}$ are used at the decision nodes when performing the forward and backward passes. The composite method becomes progressively more difficult to work after $\widetilde{max}$ or $\widetilde{mix}$ is taken because the forward calculation degenerates into a non-TFN (see the Table 4 and the Figure 8) which has an additional discontinuous point. Furthermore, after performing the forward pass, a critical path cannot be discerned, since there cannot be any activities with zero slack time because the fuzzy subtraction is not the inverse of fuzzy addition. The addition and subtraction of two TFNs $\tilde{A}_1=(a_1, b_1, c_1)$ and $\tilde{A}_2=(a_2, b_2, c_2)$ are defined as

$$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1+a_2,\ b_1+b_2,\ c_1+c_2), \tag{6}$$

$$\tilde{A}_1 \ominus \tilde{A}_2 = (a_1-a_2,\ b_1-b_2,\ c_1-c_2). \tag{7}$$

where the $\oplus$ and $\ominus$ are fuzzy addition and fuzzy subtraction respectively.

Chanas and Kamburowski[4] created a method called FPERT which was used for performing the forward pass. Here, the goal is simply to determine the fuzzy project completion time. Therefore, they didn't mention performing a backward pass, nor identifying a critical path.

Chanas[3] stated that the fuzzified backward pass cannot be performed if the fuzzy project completion time is determined through the fuzzified forward pass. Also, he provided a way to estimate the degree of criticality of each pass. However, he didn't mention the restriction of selecting the fuzzy project completion time independent of the forward pass for performing the backward pass.

Consequently, the calculations by the backward pass are meaningless estimates. Therefore, we will use only the forward pass to determine the fuzzy project completion time and the degree of criticality for each path. The fuzzified forward calculations of Dubois and Prade[7] can be expressed as follows:

$$\tilde{t}_1^E = 0, \tag{8}$$

$$\tilde{t}_k^E = \underset{J(p)=k}{\tilde{max}}\{\widetilde{EF}_p\} = \underset{J(p)=k}{\tilde{max}}\{\tilde{t}_{I(p)}^E \oplus \tilde{d}_p\} \quad k=2,\ 3,\ \cdots,\ n. \tag{9}$$

$$\widetilde{ES}_p = \tilde{t}_{I(p)}^E, \tag{10}$$

$$\widetilde{EF}_p = \tilde{t}_{I(p)}^E \oplus \tilde{d}_p, \tag{11}$$

where

$I(p)$	the preceding node of activity p,
$J(p)$	the succeeding node of activity p,
$\tilde{d}_p$	the fuzzy activity time of activity p,
$\tilde{t}_K^E$	the fuzzy earliest node time of node k,
n	the last node,
$\widetilde{ES}_p$	the fuzzy earliest start time of activity p,
$\widetilde{EF}_p$	the fuzzy earliest finish time of activity p,
$\tilde{max}$	the fuzzy maximum operator.

The forward pass begins calculations from the start node and moves to the last node. The fuzzy earliest time $\tilde{t}_K^E$(the fuzzy earliest start time of all the activities emanating from node k), $\widetilde{ES}_p$ and the fuzzy earliest finish time, $\widetilde{EF}_p$ and the fuzzy project completion time $\tilde{T}$ are then calculated by the forward pass. A path is a set of activities that connect the start and end nodes of the network. If a path i from the start node to the end node is denoted as a $P(i)$, then the degree of criticality for a $P(i)$ is expressed as follows:

$$C_{p(i)} = \sup_x \left[\mu_{\tilde{T}_{p(i)}}(x) \cap \mu_{\tilde{T}}(x)\right], \qquad p(i) \in Q \tag{12}$$

where

sup	the supreme or global maximum of a set,
$\cap$	intersection,
Q	the sit of all paths from a start node to an end node,
$\tilde{T}_{p(i)}$	the fuzzy completion time of a path i or

$$\tilde{T}_{p(i)} = \sum_{p \in p(i)} \tilde{d}_p, \tag{13}$$

$\tilde{T}$	the fuzzy completion time of a project,
μ	the membership function of a fuzzy number.

4. Approaches Using the Comparison Method

In the comparison method, the discrete *max* and *min* are used at the decision nodes when performing the forward and backward passes. These operators can be performed by using any fuzzy number ranking method[1, 6, 12, 13, 15]. Recently, McCahon[15], McCahon and Lee[16] have performed the forward and backward calculations using Lee-Li's ranking method based on a generalized mean value and spread of fuzzy numbers. Also, they recommended the $\tilde{T}$ found when performing the forward pass as the starting point for the backward pass. However, they didn't mention the reason and recognize the backward calculations had no relation to the forward ones.

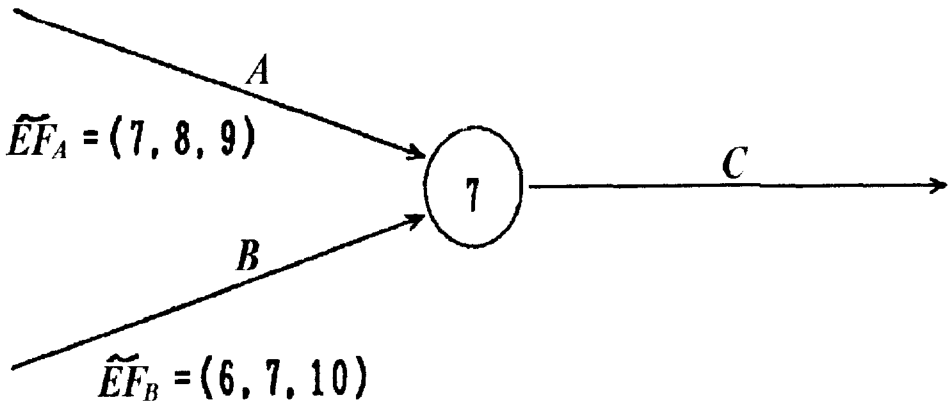
In another study, Kaufmann and Gupta[11] identified a unique critical path using Minkowski's subtraction for the backward pass and ranking method based on the dominance property. However, the major drawback to this approach is the lack of interpretation. That is, when performing the backward calculations using Minkowski's subtraction instead of ordinary fuzzy subtraction, non-TFNs (for example, (11, 7, 17)) occur in activities on non-critical paths. Furthermore, if Minkowski's subtraction is used in the composite method, a unique critical path is still not found.

5. Comparing the Composite and Comparison Methods

When comparing the composite and comparison methods, the comparison method is much easier to use than the composite method. However, the composite method is a more realistic model. For example, consider the following situation as shown Figure 4.

Activities A and B must be completed before activity C can start. If the comparison method was used to determine the $\widetilde{ES}_C$, the $\widetilde{EF}_A$ would be judged as the greater activity and $\widetilde{ES}_C$ would become the $\widetilde{EF}_A$. However, there is a small possibility that the activity B would not be complete, since the maximum possible early finish time is 10, whereas the early start time is 9. However, if the composite method was used for the $\widetilde{ES}_C$, this difficulty would not be encountered.

McCahon and Lee[16] stated that combination of the two methods can be used sequentially. One of them is that the comparison is used first to weed out activities not on highly critical paths and then the composite method is used to determine the path with the highest degree

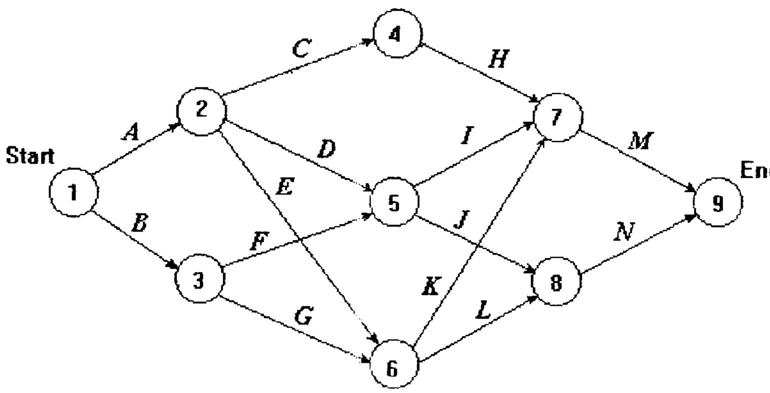


[Figure 4] Comparing the composite and comparison methods for the $\widetilde{ES}_C$

of criticality. In this paper, we propose the combination of the composite method and the comparison ranking method based on the dominance property.

6. Approach Using the Combination of the Two Methods.

Let us consider the project network shown in Figure 5. The implementation of the project has 9 nodes and it needs 14 main activities each one represented by a directed link.



[Figure 5] Project network in the example.

For each activity on this network, four experts were interviewed and these experts expressed their valuations in the form of TFNs. Table 1 gives the TFN data and shows the minor and major. When the first criterion dealing with the associated ordinary number is not sufficient, then the second if needed the third criteria are applied. For each activity in table 1, row 1 gives the TFN of expert 1, row 2 that for expert 2, etc.

〈Table 1〉 The expert's TFN and associated ordinary number for each activity time

Activity(P)	Expert's TFN($\tilde{d}_p$)	$\tilde{dp}$	Activity(P)	Expert's TFN($\tilde{d}_p$)	$\tilde{dp}$
A	(5, 7, 9)*	7.00*	H	(11, 13, 14)*	12.75*
	(6, 7, 11)	7.75		(10, 12, 17)	12.75
	(4, 8, 13)*	8.25*		(10, 10, 10)$_*$	10.00$_*$
	(3, 6, 14)	7.25		(8, 9, 18)	11.00
B	(3, 8, 8)*	6.75*	I	(7, 9, 12)	9.25
	(4, 5, 11)	6.25		(6, 9, 13)	9.25
	(2, 7, 8)	6.00		(5, 8, 15)$_*$	9.00$_*$
	(3, 4, 12)$_*$	5.75$_*$		(4, 11, 13)*	9.75*
C	(6, 6, 6)	6.00	J	(4, 4, 4)$_*$	4.00$_*$
	(6, 6, 6)	6.00		(4, 6, 7)	5.75
	(7, 7, 7)*	7.00*		(5, 5, 5)	5.00
	(5, 5, 5)$_*$	5.00$_*$		(5, 6, 7)*	6.00*
D	(3, 4, 6)$_*$	4.25$_*$	K	(5, 7, 9)*	7.00*
	(1, 5, 8)	4.75		(5, 6, 10)	6.75
	(1, 6, 8)*	5.25*		(5, 5, 5)$_*$	5.00$_*$
	(2, 4, 9)	4.75		(5, 5, 5)$_*$	5.00$_*$
E	(10, 12, 13)*	12.75*	L	(11, 13, 13)	12.50
	(5, 11, 14)	10.25		(10, 12, 17)*	12.75*
	(5, 10, 17)	10.50		(8, 8, 8)$_*$	8.00$_*$
	(3, 9, 11)$_*$	8.00$_*$		(7, 8, 10)	8.25
F	(4, 6, 7)*	5.75*	M	(8, 8, 10)	8.50
	(4, 5, 8)$_*$	5.50$_*$		(7, 9, 11)*	9.00*
	(3, 6, 7)	5.50		(5, 6, 13)$_*$	7.75$_*$
	(2, 5, 11)	5.75		(4, 9, 10)	8.00
G	(5, 11, 12)	9.75	N	(17, 19, 21)*	19.00*
	(8, 13, 15)*	12.25*		(13, 15, 28)	17.75
	(7, 10, 11)$_*$	9.50$_*$		(14, 14, 15)$_*$	14.25$_*$
	(5, 12, 13)	10.50		(12, 18, 22)	17.50

(the upper* means the major TFN for the pessimistic sequence, and the lower$_*$ means the minor TFN for the optimistic sequence)

We will show the pessimistic project planning with major TFNs mainly. With these major TFNs in Table 1, we will now compute the pessimistic planning. The comparison method based on the dominance property is first used as a first cut to determine which paths are most critical and the forward calculations are listed in Table 2. The fuzzy project completion time $\tilde{T}$ is as follows:

$$\tilde{T}=max\{\widetilde{EF}_M,\ \widetilde{EF}_N\} = \widetilde{EF}_N = (41,\ 51,\ 64)$$

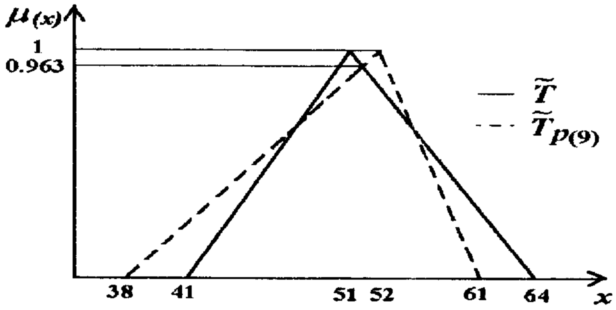
〈Table 2〉 Forward calculations using the comparison method

Activity(P)	$\widetilde{ES}_p(\tilde{t}_{i(p)}{}^E)$	$\widetilde{ES}_p(\tilde{t}_{i(p)}{}^E \oplus \tilde{d}_p)$
A	0	(4, 8, 13)
B	0	(3, 8, 8)
C	(4, 8, 13)	(11, 15, 20)
D	(4, 8, 13)	(5, 14, 21)
E	(4, 8, 13)	(14, 20, 26)
F	(3, 8, 8)	(7, 14, 15)
G	(3, 8, 8)	(11, 21, 23)
H	(11, 15, 20)	(22, 28, 34)
I	$max[\widetilde{EF}_D,\ \widetilde{EF}_F]=\widetilde{EF}_D=(5,\ 14,\ 21)$	(9, 25, 34)
J	$max[\widetilde{EF}_D,\ \widetilde{EF}_F]=\widetilde{EF}_D=(5,\ 14,\ 21)$	(10, 20, 28)
K	$max[\widetilde{EF}_E,\ \widetilde{EF}_G]=\widetilde{EF}_E=(14,\ 20,\ 26)$	(19, 27, 35)
L	$max[\widetilde{EF}_E,\ \widetilde{EF}_G]=\widetilde{EF}_E=(14,\ 20,\ 26)$	(24, 32, 43)
M	$max[\widetilde{EF}_H,\ \widetilde{EF}_I,\ \widetilde{EF}_K]=\widetilde{EF}_H=(22,\ 28,\ 34)$	(29, 37, 45)
N	$max[\widetilde{EF}_J\ \widetilde{EF}_L]=\widetilde{EF}_L=(24,32,43)$	(41, 51, 64)

The degree of criticality is then calculated for each path using $\tilde{T}$ and (12). The completion times and the degree of criticality for each path are summarized in Table 3.

〈Table 3〉 $Cp(i)$ using the comparison method.

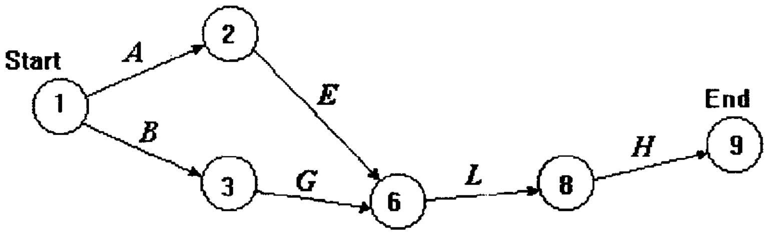
i	$P(i)$	$\tilde{T}_{P(i)}$	$C_{P(i)}$
1	$A - C - H - M$	(29, 37, 45)	0.22
2	$A - D - I - M$	(16, 34, 45)	0.19
3	$A - D - J - N$	(27, 39, 44)	0.40
4	$A - E - K - M$	(26, 36, 46)	0.25
5	$A - E - L - N$	(41, 51, 64)	1.00
6	$B - F - I - M$	(18, 34, 49)	0.00
7	$B - F - J - N$	(29, 39, 43)	0.14
8	$B - G - K - M$	(23, 37, 43)	0.13
9	$B - G - L - N$	(38, 52, 61)	0.96



[Figure 6] Illustration of the calculation $C_{p(9)}$

Figure 6. illustrates the calculation of the degree of criticality for path 9.

While path 5 is judged the most critical, path 9 also has a very high degree of criticality. Therefore, to determine the path with the highest degree of criticality and the fuzzy project completion time, the composite method is used on both paths. Table 4 lists the forward calculations through the network considered paths 5 and 9 in Figure 7.

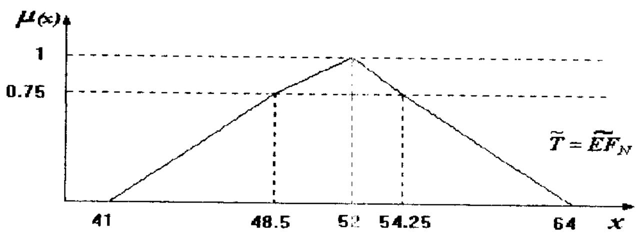


[Figure 7] Network considered path 5 and 9.

〈Table 4〉 Forward calculations using the composite method

Activity(P)	$\widetilde{ES}_p(\tilde{t}_{ip}^{E})$	$\widetilde{EF}_p(\tilde{t}_{ip}^{E} \oplus \tilde{d}_p)$
A	0	(4, 8, 13)
B	0	(3, 8, 8)
E	(4, 8, 13)	(14, 20, 26)
G	(3, 8, 8)	(11, 21, 23)
L	$\max[\widetilde{EF}_E, \ \widetilde{EF}_G]$ $= \begin{cases} \frac{x}{6}-\frac{14}{6}, & 14 \le x \le 18.5 \\ \frac{x}{10}-\frac{11}{10}, & 18.5 \le x \le 21 \\ -\frac{x}{2}+\frac{23}{2}, & 21 \le x \le 21.5 \\ -\frac{x}{6}+\frac{26}{6}, & 21.5 \le x \le 26 \\ 0, & elsewhere \end{cases}$	$\begin{cases} \frac{x}{8}-\frac{24}{8}, & 24 \le x \le 30 \\ \frac{x}{12}-\frac{21}{12}, & 30 \le x \le 33 \\ -\frac{x}{7}+\frac{40}{7}, & 33 \le x \le 34.75 \\ -\frac{x}{11}+\frac{43}{11}, & 34.75 \le x \le 43 \\ 0, & elsewhere \end{cases}$
N	$\begin{cases} \frac{x}{8}-\frac{24}{8}, & 24 \le x \le 30 \\ \frac{x}{12}-\frac{21}{12}, & 30 \le x \le 33 \\ -\frac{x}{7}+\frac{40}{7}, & 33 \le x \le 34.75 \\ -\frac{x}{11}+\frac{43}{11}, & 34.75 \le x \le 43 \\ 0, & elsewhere \end{cases}$	$\begin{cases} \frac{x}{10}-\frac{41}{10}, & 41 \le x \le 48.5 \\ \frac{x}{14}-\frac{38}{14}, & 48.5 \le x \le 52 \\ -\frac{x}{9}+\frac{61}{9}, & 52 \le x \le 54.25 \\ -\frac{x}{13}+\frac{64}{13}, & 54.25 \le x \le 64 \\ 0, & elsewhere \end{cases}$

The fuzzy project completion time $\tilde{T}$ is the $\widetilde{EF}_N$ and this is the project completion time for the pessimistic project planning in the example. The $\tilde{T}$ is illustrated in Figure 8.



[Figure 8] Illustration of the fuzzy project time $\tilde{T}$, for the pessimistic planning.

In the case of the pessimistic project planning, the path 9 is judged the most critical(the degree of criticality of the path 5 and 9 are 0.963 and 1 respectively). However, the path 5 also has a very high degree of criticality. Therefore, the activities on both paths should be managed more closely than the others.

In the optimistic project planning, minor TFNs in Table 1 are used. And we calculate the degree of criticality for all paths using the same way for pessimistic case. Finally, the path 5 is judged the most critical (the degree of criticality of the path 5 is 1. 0).

7. Conclusion

This paper shows possibilistic (fuzzy) project planning based on the membership function as an alternative to probabilistic project planning. If the uncertainty is due to a lack of information (or experience), then the project planning problem should be modeled with fuzzy components. The ultimate objective of the project planning is to find the critical path that represents the shortest duration needed to complete the project. But, if activity processing times are fuzzy, then it is impossible to identify a unique critical path. We can determine only the project completion time and the degree of criticality for each path in a project network.

The composite and comparison methods have been used to analyze the fuzzy project network. The comparison method is much easier to use than the composite method. However the composite method is a more realistic model.

In this paper, we proposed a combination of the two methods. The comparison method based on dominance property was used first to weed out activities not on highly critical paths and then the composite method was used to determine the path with the highest degree of criticality. In addition, we introduced a pessimistic project planning with major TFNs and an optimistic project planning with minor TFNs. In our example, the data were established by four experts as TFN, but in a practical example the number of experts depend on many factors : available experts, a kind of activity, resources, gravity of project and so on.

Following the pessimistic and optimistic calculations, the experts re-evaluate their TFN estimates and the process is repeated. The method described in this paper is useful especially when the opinions of many experts are required in a large scale project.

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