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UNIVERSAL DISPERSION EQUATION FOR MAGNETOSTATIC WAVES(MSW)

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Abstract — A universal dispersion equation for magnetostatic waves(MSW) propagating in the film with arbitrary-multiple magnetic layers magnetized in an arbitrary direction was derived with a matching boundary condition method. The computing result curves of delay time were shown.

I. INTRODUCTION

Magnetostatic waves(MSW) are the spinwaves with a longer wavelength, a slower propagating velocity and a high operating frequency and can be controlled by magnetic field, MSW propagating within a ferrimagnetic plane magnetized in the direction of tangential and vertical to MSW propagating was first treated theoretically by Damon and Eshback[1], and was called as magnetostatic surface waves(MSSW), Later, MSW propagating within the plate magnetized in the direction of vertical to the surface of the plate, magnetostatic forward volume waves(MSFVW), was treated by M. R. Daniel et al[2], and MSW propagating within the plate magnetized in the direction of parallel to MSW propagating, magnetostatic backward volume waves(MSBVW), was treated by N. S. Chang et al[3], Many researchers[4~6] tried their best to work at MSW and issued a lot of papers on MSW propagating in the film with a single magnetic layer or with multiple magnetic layers. However, there is still not a universal dispersion equation for MSW, Based on the outhor's work, the universal dispersive equation for MSW propagating in the film with arbitrary multiple magnetic layers magnetized in an arbitrary direction was derived theoretically. The computing results curves of delay time were shown.

II. THEORY

The schematic of the multilayered magnetic YIG film structure and the coordinate system are shown in Fig. 1. The magnetic layers and the dielectric layers separate each other. The uppest and the lowest dielectric layers make contact with ground plates which is supposed as perfect conducting. θ and φ are the direction angles of internal magnetic field, H. Consider the MSW propagating in y direction and assume the film width along z-axis to be infinite, Assumed D. C. magnetization of the film in an arbitrary direction(θ, φ). The permeability tensor[7] $\overline{\mu i}$ of the ith magnetic layer can be derived.

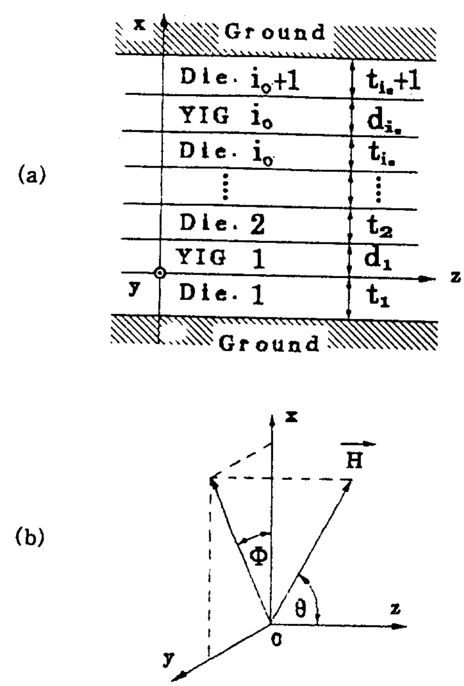

Fig. 1. Schematic of the layered structure.
(a) Layered structure;
(b) Coordinate system.

$$\overleftrightarrow{\mu_i} = \begin{pmatrix} \mu_{11}^{(i)} & \mu_{12}^{(i)} & \mu_{13}^{(i)} \\ \mu_{12}^{(i)*} & \mu_{22}^{(i)} & \mu_{23}^{(i)} \\ \mu_{13}^{(i)*} & \mu_{23}^{(i)*} & \mu_{33}^{(i)} \end{pmatrix} \qquad (1)$$

where $\mu_{ab}^{(i)*}$ is the conjugate complex number of $\mu_{ab}^{(i)}$, and

$$\mu_{11}^{(i)} = \mu_i + (1-\mu_i)\sin^2\theta\cos^2\varphi$$
$$\mu_{12}^{(i)} = 0.5(1-\mu_i)\sin 2\varphi\sin^2\theta + jK_i\cos\theta$$
$$\mu_{13}^{(i)} = 0.5(1-\mu_i)\sin 2\theta\cos\varphi - jK_i\sin\theta\sin\varphi$$
$$\mu_{22}^{(i)} = \mu_i + (1-\mu_i)\sin^2\theta\sin^2\varphi$$
$$\mu_{23}^{(i)} = 0.5(1-\mu_i)\sin 2\theta\sin\varphi + jK_i\sin\theta\cos\varphi$$
$$\mu_{33}^{(i)} = 1 - (1-\mu_i)\sin^2\theta$$

$$\mu_i = 1 - \frac{r^2 H M_{si}}{F^2 - r^2H^2} \qquad K_i = \frac{r M_{si} F}{F^2 - r^2 H^2}$$

where F, r, $\vec{H}$ and Msi are the operation frequency, the gyromagnetic ratio(0.03518MHz · m/A), the internal magnetic field and the saturation magnetization of the ith layer YIG, respectively. i = 1, 2, 3, ⋯, i_o is the ordinal number of the magnetic or dielectric layers.

Do not consider the magnetic anisotropy, then

$$\vec{H} = \vec{He} + \vec{Hd}. \qquad (2)$$

Where $\vec{H}$, $\vec{He}$ and $\vec{Hd}$ is a internal magnetic field, a external magnetic field and a demagnetic field. The constitutive relation between the rf magnetic field $\vec{h}$ and the magnetic flux density $\vec{b}$ are given in the form:

$$\vec{b_i} = \mu_o \vec{h_i} \text{ (in the ith dielectric)} \qquad (3)$$

$$\vec{b_i} = \mu_o \overleftrightarrow{\mu_i}\, \vec{h_i} \text{ (in the ith YIG layer).} \qquad (4)$$

Where the μ_o is the permeability of vacuum. Under the magnetostatic approximation[8], Maxwell equations in the ith YIG layer are

$$\nabla \times \vec{h_i} = 0 \qquad (5)$$

$$\nabla \times \vec{b_i} = 0. \qquad (6)$$

The $\vec{h_i}$ can be represented by the gradient of a scalar potential as

$$\vec{h_i} = \nabla \Psi_i \qquad (7)$$

$$\nabla \cdot \overleftrightarrow{\mu_i}\, \nabla \Psi_i = 0 \quad \text{(in the ith YIG layer)} \qquad (8)$$

$$\nabla \cdot \overleftrightarrow{b_i}\, \nabla \Psi_{oi} = 0 \quad \text{(in the ith dielectric).} \qquad (9)$$

Do not consider the effect of the sampple width and consider waves propagating along y direction, then Ψ_i in the ith YIG layer satisfies,

$$\mu_{11}^{(i)} \frac{\partial^2 \Psi_i}{\partial X^2} + \mu_{22}^{(i)} \frac{\partial^2 \Psi_i}{\partial Y^2} + (\mu_{12}^{(i)} + \mu_{11}^{(i)*}) \frac{\partial^2 \Psi_i}{\partial X \cdot \partial Y} = 0 \qquad (10)$$

$$\Psi_{oi} = [A_i \exp(kx) + B_i \exp(-kx)] \exp[j(ky - \omega t)] \quad \text{(in ith dieletric)} \qquad (11)$$

$$\Psi_i = [C_i \sin(\alpha_i x) + D_i \cos(\alpha_i x)] \times \exp[j(-\beta_i x + ky - \omega t)] \quad \text{(in ith YIG)} \qquad (12)$$

Substituting Eq. (12) into (10),

$$\alpha_i = \frac{k\{-[\mu_i^2 + \mu_i(1-\mu_i)\sin^2\theta]\}^{1/2}}{\mu + (1-\mu)\sin^2\theta\cos^2\varphi} \qquad (13)$$

$$\beta_i = \frac{k(1-\mu_i)\sin^2\theta\sin\varphi\cos\varphi}{\mu_i + (1-\mu_i)\sin^2\theta\cos^2\varphi} \qquad (14)$$

are obtained. Substituting Eq. (12) into Eq. (7) and (4), respectitvely, expressions of $\vec{h_i}$ and $\vec{b_i}$ in the ith YIG layer are obtained. Substituting Eq. (11) into (7), the expressions of $\vec{h_{oi}}$ and $\vec{b_{oi}}$ in the ith dielectric are obtained. According to the boundary condition relations and considering assumed of two ground plates as perfect conducting, the constants of Ai, Bi, Ci and Di in the above equations are eliminated with the substituting and eliminating method[10], Then a universal dispersion equation for MSW is obtained:

$$\exp(2kx_{o\,i_o+1}) = P_{i_o} \tag{15}$$

where P_{i_o} is the P_i at $i = i_o$.

$$P_i = \exp(2kx_i)\,(k - R_i)/(k + R_i) \tag{16}$$

$$R_i = \frac{M_i[G_i - \tan(\alpha_i x_i)] + jN_i[G_i \tan(\alpha_i x_i) + 1]}{G_i \tan(\alpha_i x_i) + 1} \tag{17}$$

$$G_i = \frac{-kT_i - M_i \tan(\alpha_i x_{oi}) + jN_i}{kT_i \tan(\alpha_i x_{oi}) - M_i - jN_i \tan(\alpha_i x_{oi})} \tag{18}$$

$$G_1 = \frac{k\,\mathrm{th}(-kx_{oi}) - jN_1}{M_i} \tag{19}$$

$$T_i = \frac{\exp(2kx_{oi}) - P_{i-1}}{\exp(2kx_{oi}) + P_{i-1}} \tag{20}$$

$$x_{oi} = \sum_{I=2}^{i} t_r + \sum_{I=1}^{i-1} d_r \tag{21}$$

$$x_{oi} = t_1$$

$$x_i = \sum_{I=2}^{i} t_r + \sum_{I=1}^{i} d_r \tag{22}$$

$$M_i = \mu_{11}^{(i)}\,\alpha_i$$

$$N_i = k\mu_{12}^{(i)} - \beta_i\,\mu_{11}^{(i)}$$

Ⅲ. COMPUTING RESULT CURVES

Let $\varphi = 0$, $i_o = 1$, changed θ, computing results is the same as the results from the Bajpai's equation [9]. The relation of H and He is

$$H \sin(90° - \theta) = He \sin\eta, \tag{23}$$

as Fig. 2.

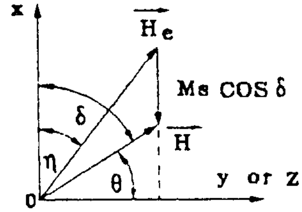

Fig. 2. The relation of H and He.

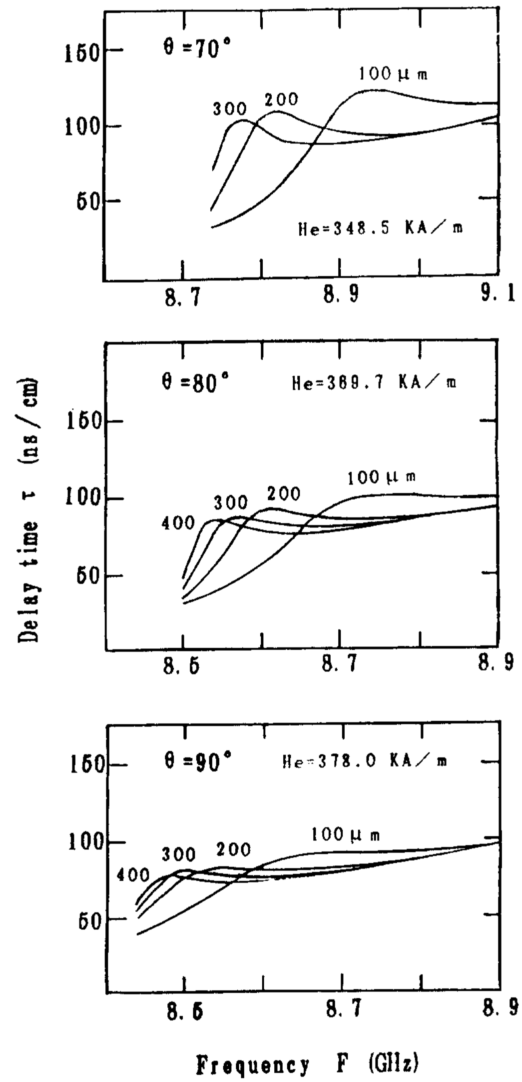

Fig. 3. Computing results curves of delay time.
H = 238.7KA/m.

Delay time τ :

$$\tau = L / Vg \quad (24)$$

$$Vg = d\omega / dk \quad (25)$$

where L, Vg and ω is propagating distance, group velocity and angular frequency of MSW, respectively. $d_1 = 20\ \mu m$, $H = 238.7 KA/m$, $Ms_1 = 139.4 KA/m$, $t_1 = 100 \mu m$, $200 \mu m$, $300 \mu m$ and $400\ \mu m$, respectively. The computing result curves of delay time are shown Fig. 3. Let $\varphi = 0$, $\theta = 0$, the Eq. (15) can be reduced into the Eq. (9) in reference [6].

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