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클롯호이드 곡선을 이용한 이동로보트의 경로제작

신 동 헌*

Path Generation of Mobile Robots Using Clothoid Curves

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ABSTRACT

이 논문에서는 이동로보트가 주행하기 쉬운 경로의 조건을 간단히 보인 후, 이러한 경로를 제작하는 방법을 제시하였다. 과거의 제작방법들은 대체적으로 주어진 점들을 직선으로 연결한 후 모서리를 원호, 클롯호이드 곡선 등으로 모서리 안쪽으로 곡선화하였다. 이 논문에서 제시된 방법은 두 단계로 되어있는데, 먼저 주어진 연속의 점들을 연속의 포우스쳐들로 바꾼 후, 이 포우스쳐들을 클롯호이드 곡선으로 연결한다. 클롯호이드 곡선의 특성상, 생성된 경로는 접선각, 곡률까지 연속적이며 곡률은 구분적으로 선형이며, 이 외에, 과거의 경로는 주어진 점들의 모서리 안쪽을 지나는 것에 비하여, 주어진 점들을 부드럽게 바깥쪽으로부터 통과하는데, 이것은 장애물들이 경로의 모서리의 안쪽에 있는 것을 생각하면 매우 유용한 점이다.

Key Words : Mobile Robot(이동 로보트), Path Generation(경로 제작), Clothoid Curve(클롯호이드 곡선), Linear Curvature(선형 곡률)

1. Introduction

Mobile robot navigation is typically composed of three phases-perception, planning, and control. A robot must perceive its state with respect to the world and use this information to plan its motion. Once a plan has been composed, it is necessary to enact the plan with requisite contol actions. Typically, path planning occurs at two levels. First, path planning uses the information about the world to obtain an ordered sequence of objective points that the robot must attain to avoid obstacles and to progress towards its goal. These objective points constitute a coarse plan and a second step is necessary to generate a path in finer detail for the purposes of robot contol.

This paper reports a method of path generation to develop a fine grained specification of a path from a sequence of objective points that immediately suggests a control law of the mobile robot at a sub second interval. This problem has been studied by other researchers

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[1,2,4]. This work is an extension of work reported by Kanayama[2] and accrues several advantages over the other reported schemes.

This paper suggests criteria for "goodness" of candidate path segments that may be used to join the objective points. As with any control system, the response of a mobile robot in tracking a path is partly dependent on the nature of the reference path. This paper propose a family of curves known as "clothoid" curves that satisfy the criteria of goodness, be used to join the objective points. Then, this paper presents a method how clothoid curves can be used to join two arbitray (within bounds) objective points that are each uniquely specified by position, orientation and instantaneous path curvature. And the results are shown to evaluate the performance of the presented method.

2. Robot and Path Modeling

For our analysis, a bicycle model is used as an archetype for modeling mobile robot with two degrees of freedom; conventional steering and propulsion. The advantage of such a model is that there exist simple geometric relationships between the curvature of the reference path and steering angle for the robot.[2,3]

The guide point is the point of the robot that is the point of the robot that is controlled to follow the given path. The choice of the guide point is an important decision-it affects the desired steering and propulsion functions required to follow the given path and speed. I have chosen the guide point to be at the midpoint of the rear axle (Fig. 1) resulting in the follwoing advantages:

- The steering angle at any point on the path is determined geometrically, independent of speed in the following manner:

$$\tan\phi = \frac{l}{r}; \quad \phi = \tan^{-1} cl \qquad (1)$$

- The robot is able to follow the minimum turning radius for the maximum steering angle. In other words, the peak steering angle is smaller than that for any other guide point
- The heading of the vehicle is aligned with the tangent direction of the path. This gives a more reasonable vantage point for sensors like a vision camera, and a smaller area is swept by a mobile robot.[4,6]

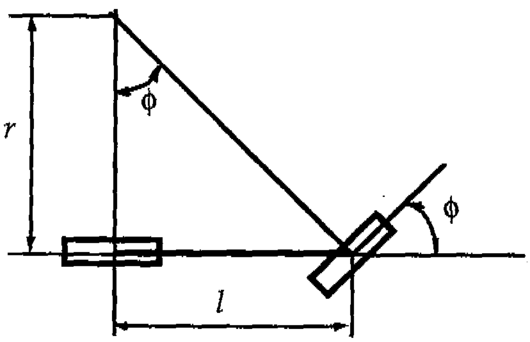


Fig. 1 Geometry of a Bicyclc Model

For the guide point at the center of the rear axle, paths with discontinuities of curvature will require infinite acceleration of the steering wheel. Consider the motion of the vehicle along a path which consists of two circular arcs. There is a discontinuity in curvature at the point where the two circular arcs meet. An infinite acceleration in steering is required for the vehicle to stay on the specified path provided that it does not come to a stop at the transition point simply because it takes a finite amount of time to switch to the new curvature. In reality, moving through a transition point with non-zero velocity results in an offset error along the desired path. Likewise, rotary body inertia requires continuity of heading, and steering inertia requires continuity of curvature.

A path can be parameterized in terms of path length s as $(x(s), y(s))$. Tangent direction $(\theta(s))$ and curvature $(c(s))$ can be derived along a path:

$$\theta(s) = \frac{dy(s)}{dx(s)} \tag{2}$$

$$c(s) = \frac{d\theta(s)}{ds} \tag{3}$$

The continuity of (x,y) is guaranteed if a path is continuous. But the effect of the continuity of vehicle heading and steering on the path depends on the guide point. Since the guide point is chosen at the center of the the rear axle, the heading of the vehicle is aligned with the tangent direction of the path θ , and the steering angle Φ is determined by the curvature of the path as in Eq (1). Thus, the continuity of the robot motion is tantamount to the continuity of (x,y,θ,c). If we define posture as the quadruple of parameters (x,y,θ,c), a posture describle the state of a conventionally steered vehicle, and a path is required to be posturecontinuous for easy tracking.[1]

Further, the rate of change of curvature (sharpness) of the path is important, too, since the linearity of curvature of the path dictates the linearity of steering motion along the path. If we assume that less control effort is required for an actuator to provide a linear velocity profile than an arbitrary nonlinear one, the extent to which steering motions are likely to keep a vehicle on a desired path can be correlated to the linearity of curvature of the path.

Certain spline curves[3,4] are good candidates for good paths, because they guarantee posture-continuity along paths. However, thse spline curves do not guarantee linear gradients of curvature. Clothoid curves, by contrast, do not have the same properties of curvature and vary linearly.

Clothoid curves[3,7] are a family of curves that are posture-continous, and are distinct in that their curvature varies linearly with the length of the curve:

$$c(s) = ks + c_i \tag{4}$$

where k is the rate of change of curvature(sharpness) of the curve and subscript i denotes the initial curvature and s is the curve length. Given an initial posture, sharpness of the clothoid segment, and the distance along that segment, postion, orientation, and curvature at any point are calculated as (4, 5, 6 and 7):

$$\theta(s) = \theta_i + \int_o^s c(\xi)d\xi = \frac{k}{2}s^2 + c_i s + \theta_i \tag{5}$$

$$x(s) = x_i + \int_o^s \cos\theta(\xi)d\xi$$

$$= \int_o^s \cos\{\frac{k}{2}\xi^2 + c_i\xi + \theta_i\}d\xi + x_i \tag{6}$$

$$y(s) = y_i + \int_o^s \sin\theta(\xi)d\xi$$

$$= \int_o^s \sin\{\frac{k}{2}\xi^2 + c_i\xi + \theta_i\}d\xi + y_i \tag{7}$$

1. Omni directional locomotion systems, such as an ilonator wheel system and legged systems, can follow paths with discontinuities of tangent direction and curvature, since they can subtend pure rotation or pure lateral motion. However, continuity of curvanture is still recommended because it provides smooth changes in the centrifugal force: $Fcg = \frac{v^2}{r} = cv^2$

3. Generating Posture-Continous Paths

The previous section discussed the condition for a path that is intrinsically easy to track. This section presents a method to generate intrinsically easy paths. The problem posed in generating paths is: how to produce a unique, easily trackable, continous path from a given sequence of points. This problem is similar to those addressed by previous path generations methods. [2,4]

Hongo[1] proposed a method to generate continuous paths composed of connected straight lines and circular arcs from a sequence of objective points. While paths composed solely of arcs and straight lines are easy to compute, such a scheme leaves curvature discontinuities at the transitions of the segments, as discussed previously.

Kanayama[2] proposed pairs of clothoids segments as a method to interpolate between given points. The problem with clothoids is that though they guarantee smoothness and linearity, it is non-trivial to generate clothoid segments for arbitrary starting and ending postures; the expressions for these curves are underconstrained, and further, no closed form expression is available. (the expression must be evaluated using numerical methods.) Kanayama's scheme is tenable under the simplification made by the requirement that the starting and the ending curvature at the via points are always zero. This model is adequate for simple paths for which symmetric pairs of clothoid curves cna be found or for paths with mild curvatures that can be easily broken down in separate clothoid segments.

Certain polynomial spline curves are candidates for path segments, because they guarantee continuity of posture. Nelson[4] proposed quintic spline and polar spline curves obtained from two-point boundary conditions. However, these spline curves do not guarantee linear gradients of curvature. Clothoid curves, by contrast, do vary linearly with the distance along the curve.

The following two-step method generates a unique posture-continuous path from a sequence of points. The first step is to derive a sequence of unique postures from the objective points; the second, is to interpolate between those postures with clothoid segments. Heading and curvature at the starting and ending positions are presumed from the configuration of the vehicle, using equation(1).

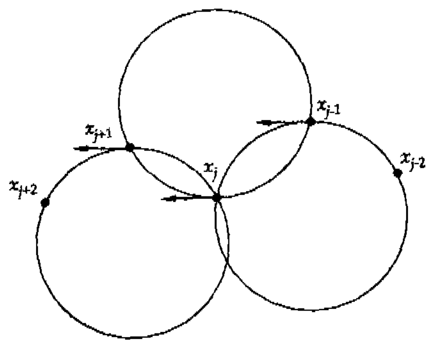


Fig. 2 Obtaining Postures and Associated Circles from Objective Points

Let $[X=(X_0,\ldots,X_n)=\{(x_0,y_0),\ldots,(x_n,y_n)\}]$ be sequence of objective points. The associated circle at X_j is defined to be the circle which passes through points X_{j-1}, X_j, X_{j+1} as in Fig. 2. Then the heading of the vehicle at X_j is taken as the direction of the tangent to the associated circle at X_j, and the curvature is the reciprocal of the osculating radius of the associated circle, denoting the posture thus obtained as the associated postures, The next step is to connect neighboring postures with clothoid segments.

It is not always possible to connect two neighboring postures with one clothoid curve segment, because four governing euqations

(4), (5), (6), and (7) cannot be satisfied simultaneously with only the two parameters (sharpness k and length s) that a clothoid curve provides. To satisfy these four equations, at least two clothoid segments are needed. However, the general problem cannot be solved with only two clothoid segments, Fig. 2 shows two pairs of associated postures and thier associated circles. Let P_i, P_j denote the starting and the ending a postures, respectively. and C_i, C_f denote the associated circles corresponding to the curvatures at P_i, P_j. They are drawn by solid lines and dotted lines, respectively.

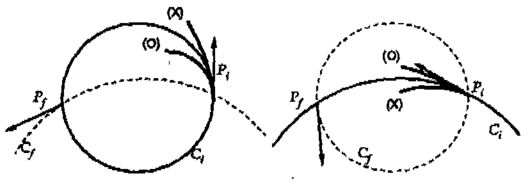


Fig. 3 Determination of the Sign of the First Clothoid Segment

As in (A) of Fig. 3, if the orientation of P_f is outward from C_i, the ending part of a solution curve should be inside C_i. Then, it is plausible that the starting part of a solution curve also lies inside C_i. similarly, if the direction of the P_f is inward into C_i, as in (B) of Fig. 3, it is plausible that the starting part of a solution curve lies outside C_i. Note that the sign of the sharpness determines the side of the associated circle on which the clothoid segment lies: (1) If the sharpness is zero, then the clothoid curve remains on the associated circle in question. (2) If the sharpness is positive, then the clothoid curve will be in the left side of the associated circle. (3) Otherwise, it will be in the right side of the first and last segments of the solution curve to the circles C_i and C_f (The convention used is that negavtive curvature equates to a right turn and a positive curvature equates to left turn). Hence, a solution curve should satisfy the following proposition:

If the direction of P_f is outward from C_i, the sign of k and the first clothoid segment is chosen so that the curve lies inside C_i.

If the direction of P_f is inward into C_i. the sign of k a of the first clothoid segment is chosen so that the curve lies outside C_i.

Otherwise, k of the first clothoid segments is chosen so that the curve remains on C_i.[2]

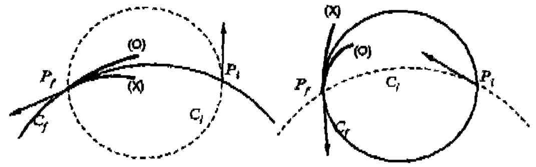


Fig. 4 Determination of the Sign of the Last Clothoid Segment

As a corollary to the above proposition, the sign of k of the last clothoid segment can be determined (Fig. 4):

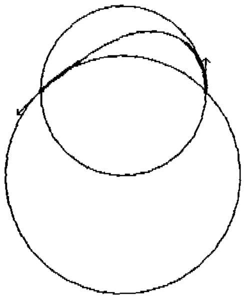
If the direction of P_i is outward from C_f, the sign of k of the last clothoid segments is chosen so that the curve lies outside C_f.

If the direction of P_i is inward into C_f, the sign of k of the last clothoid segment is chosen so that the curve lies inside C_f.

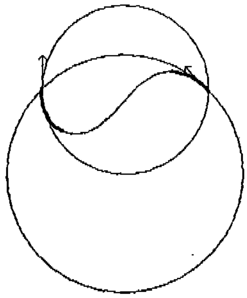
Otherwise, k of the last clothoid segment is chosen so that the curve lies on C_f.

Fig. 5 shows all possible cases of curvature variations between a pair of neighboring postures. Notice that the signs of k for the first

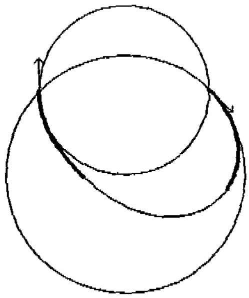
2. In this case, P_i and P_f share the same associated cirvcle, two postures are connected with the part of their associated circle, which is a clothoid curve of zero sharpness.



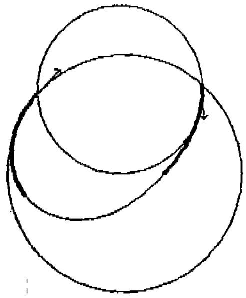
(A) when $c_i \geq c_f \geq 0$, $k_i, k_f \geq 0$



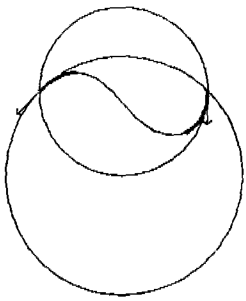
(B) when $c_i \geq 0 \geq c_f$, $k_i, k_f \geq 0$



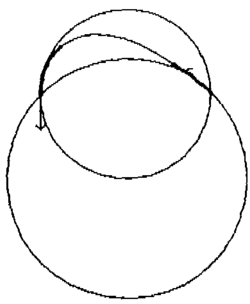
(C) when $0 \geq c_i \geq c_f$, $k_i, k_f \leq 0$



(D) when $0 \geq c_f \geq c_i$, $k_i, k_f \geq 0$



(E) when $c_f \geq 0 \geq c_i$, $k_i, k_f \leq 0$



(F) when $c_f \geq c_i \geq 0$, $k_i, k_f \leq 0$

Fig. 5 Cases by Curvature Variation of P_i and P_f

and the last segments are the same for each case. However, the sign is the opposite of that required for the curvature variation between the postures for all the cases except (C) and (D) of Fig. 5. Thus, the general problem to connect a pair of neighboring associated postures cannot be solved with two clothoid curve segments.

One adequate solution set of the clothoids is the set of three clothoid segment (k, s_1), $(-k, s_2)$, (k, s_3). The subscripts denote the order of the clothoid segments from P_i. This combination is plausible for the following reasons:

1. The signs of sharpness for the first and last clothoid segments are the same.
2. The sharpness for the second clothoid segment is equal in magnitude and opposite in sign to the first and last segments. This enables the curve of three clothoid segments to satisfy the curvature variation between the starting and the ending postures by varying s_1, s_2, s_3, even though the sign of the first and the last clothoid segments satisfies the curve location requirement.
3. There are four variables in the combination: k, s_1, s_2, s_3. It is possible to find a unique solution satisfying the following four equations which descirbe the mathematical relationship between the staring and ending postures:

$$c_f = c_i k(s_1 - s_2 + s_3) \quad (8)$$

$$\theta_f = \theta_i + c_i(s_1 + s_2 + s_3) + k(s_1 s_2 - s_2 s_3 + s_3 s_1) + \frac{k}{2}(s_1^2 - s_2^2 + s_3^2) \quad (9)$$

$$x_f = x_i + \int_0^{s_1} \cos\theta_1(\xi) d\xi + \int_0^{s_2} \cos\theta_2(\xi) d\xi + \int_0^{s_3} \cos\theta_3(\xi) d\xi \quad (10)$$

$$y_f = y_i + \int_0^{s_1} \sin\theta_1(\xi) d\xi + \int_0^{s_2} \sin\theta_2(\xi) d\xi + \int_0^{s_3} \sin\theta_3(\xi) d\xi \quad (11)$$

where

$$\theta_1(\xi) = \theta_i + c_i\xi + \frac{k}{2}\xi^2$$

$$\theta_2(\xi) = \theta_i + c_i s_1 + \frac{k}{2}s_1^2 + (c_i + ks_1)\xi - \frac{k}{2}\xi^2$$

$$\theta_3(\xi) = \theta_i + c_i(s_1 + s_2) + k(s_1 s_2) + \frac{k}{2}(s_1^2 - s_2^2) + \{c_i + k(s_1 - s_2)\}\xi + \frac{k}{2}\xi^2$$

Since Equations (10) and (11) contain Fresnel integrals, for which there is no closed form solution, the values of k, s_1, s_2, s_3 are computed using the numerical method oulined in Fig. 6.

Initial values for s_1 and s_2 are chosen to be $\frac{1}{3}$ of the average of the lengths of two of the arcs that connect P_i and P_f. Equations (8) and (9) are used to compute k and s. Then, x_c, y_c can be computed for the quadruple (k, s_1, s_2, s_3) using Simpson's approximation. Ideally x_c,

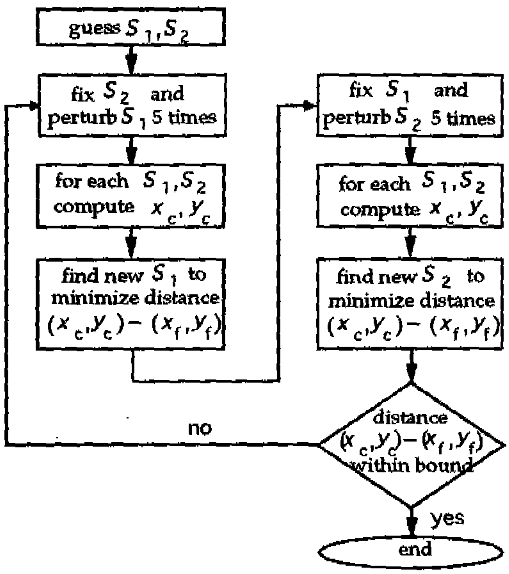


Fig. 6 Numerical Method to Compute the Fresnel Integrals

$y_c = x_f$, y_f and in fact values of s_1 and s_2 are adjusted until the difference is within a threshold.

4. Results

Connecting a pair of neighboring associated postures was accomplished successfully, as in Fig. 5, using three clothoid curves. Fig. 7 shows the graphic result of a posture-continuous path generated by clothoid curves through the given seven points. As the first step of the proposed method, a sequence of seven postures were generated. (Seven arrows in Fig. 7). Then, three clothoid segments are used to interpolate between neighboring postures.

Fig. 8 shows a comparison of the performance of Kanayama's method, the method using arcs, and the proposed method. Parameters of curvature and sharpness were constrained euqally for all methods. The maximum sharpness of Kanayama's method and the maximum curvature used in the arc method are set at the same levels as in the proposed method. Paths, curvatures and sharpness along the paths are compared. Paths resulting from the proposed method have the following advantages over other methods:

- The method proceeds from an arbitrary sequence of points. Generation of postures is essential to exploratory planning where goals are commonly posed as an evolving string of points. Paths generated by the proposed method pass through all the objective points, as in Fig. 7; whereas paths from Kanayama's method and the arc mekthod are only proximate to many of the points because these methods start from a sequence of postures.
- The method guarantees continuity of position, headning, and curvature along the path. Further, sharpness is piecewise constant.
- Paths generated by the method always

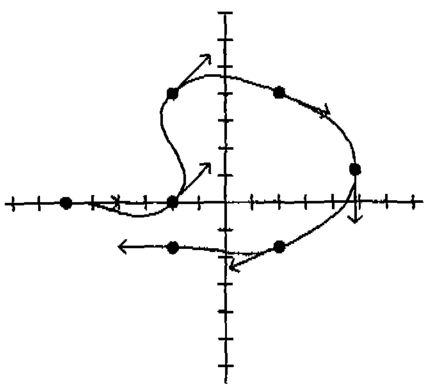


Fig. 7 Posture-continuous Path From a Sequence of Seven Points

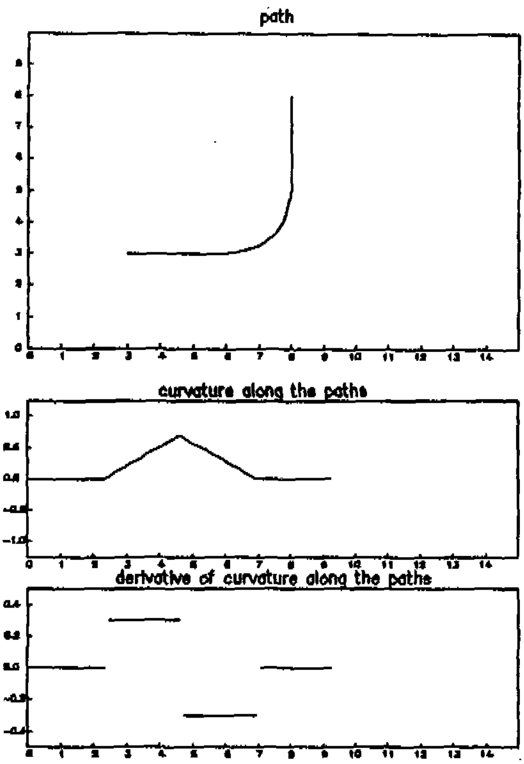


Fig. 8(a) Clothoid path with zero curvature transition

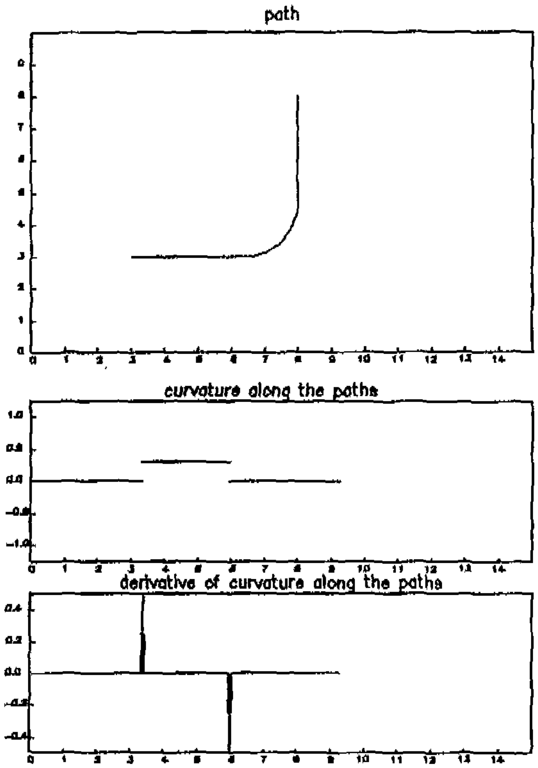


Fig. 8(b) Path with circular arc and straight lines

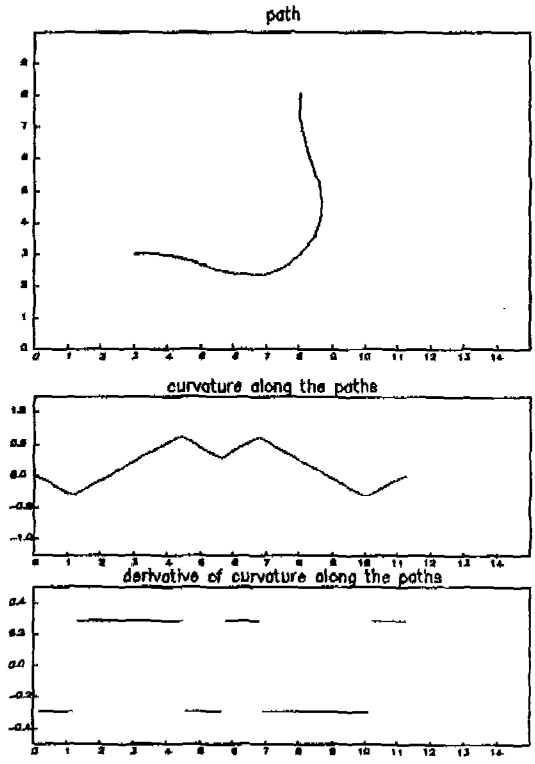


Fig. 8(c) Clothoid path with the proposed mothod

sweep outside the acute angles formed by straight line connection of the way points, as in Fig. 7. The resulting paths are especially useful for interpolating around obstacles that are commonly on the inside of angles. In contrast, Kanayama's paths are always inside path angles.

5. Conclusion

Posture-continuity is presented as the condition for a path that is intrinsically easy to track. In addition, the linearity of curvature of the path is correlated to linear steering motion, facilitating path tracking. Clothoid curves are good path candidates to satisfy the condition. Simulation shows better performance of a vehicle tracking paths of clothoids versus paths of arcs.

A method for generating a continous path was developed. This method uses clothoid segments and consists of two steps: First, a sequence of the postures is obtained using the objective points. Then, each pair of neighboring postures is connected with three clothoid curve segments.

The method provides additional advantages in that preprocessing of the objective points is not necessary, as with arc and zero curvature clothoids. Further, the geometry of the paths generated alwasy sweeps outside the acute angles formed by a straight-line connection of the way points. These are especially useful for interpollating around obstacles that are commonly on the inside of angles.

The method of obtaining postures, as in

Fig. 2, requires that the circles formed by the radii of curvature of two postures intersect. Relaxing such a constraint would require heuristics to determine intermediate postures. Assuming intermediate postures could be found, such that the associated circles intersect, the method could then be used on the new set of postures. Thus far, the search for a completely general method that would generate a path between two completely arbitrary postures has not been fruitful.

Directions for future research include the following: (1) optimizing the lengths of more than three clothoid segments on the premise that some cost funcition can be used to find a better solution than the results here; (2) improving the numerical method to connect postures through the Fresnel Integral can improve speed and accuracy.

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