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Test for Trend Change in NBUE-ness Using Randomly Censored Data

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Abstract

Let F be a life distribution with finite mean μ. Then F is said to be in new better then worse than used in expectation (NBWUE(p)) class if $\varphi(u) \geq u$ for $0 \leq u \leq t_0$ and $\varphi(u) \leq u$ for $t_0 < u \leq 1$, where $\varphi(u)$ is the scaled total-time-on-test transform and $p = F(t_0)$. We propose a testing procedure for H_0 : F is exponential against H_1 : NBWUE(p), and is not exponential, (or H_1' : $F$is NWBUE (p), and is not exponential) using randomly censored data. Our procedure assumes knowledge of the proportion p of the population that fail at or before the change-point t_0. Knowledge of t_0 itself is not assumed. The asymptotic normality of the test statistic is established and a Monte Carlo experiment is performed to investigate the speed of convergence of the test statistic to normality. The power of our test is also studied.

1. Introduction

For many practical situations where it is more reasonable to assume a certain type of trend change for some parameters, the statistical inference regarding such parameters attracts a great deal of interests among reliability scientists, engineers, or other statisticians recently.

Because of its useful applicability, many authors have considered the testing procedures for non-monotone classes of life distributions such as bathtub-shaped failure rate (BTR), increasing then decreasing mean residual life (IDMRL) (for example, Matthews, Farewell and Pyke (1985), Guess, Hollander and Proschan (1986), Park (1988), etc.).

Klefsjö (1988) proposes a nonparametric procedure intended for testing exponentiality against the situation where the life distribution changes from the new better than used in expectation (NBUE) to new worse than used in expectation (NWUE), assuming knowledge of the proportion p of the population that fail at or before the change-point t_0. Such a trend

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change can be used in modelling for several maintenance and replacement policies. Mitra and Basu (1994) refer to Klefsjö's class as NBWUE (p) class.

In Section 2, we propose a testing procedure for H_0: F is exponential against H_1 : NBWUE(p), and is not exponential, (or H_1' : $F$is NWBUE (p), and is not exponential) using randomly censored data, assuming the same condition as in the Klefsjö (1988).

In Section 3, Monte Carlo experment is performed to investigate the power of our test procedure and the speed of convergence to normality of the proposed test statistic. Also we study the efficiency loss to the presence of censoring. Finally, Section 4 contains conclusion.

2. NBWUE(p) test with randomly censored data.

To derive our test statistic, we assume knowledge of the proportion p of population that fail at or before the change-point. Note that $t_0 = F^{-1}(p)$ is assumed to be unknown.

We are interested in testing

$$H_0 : F(x) = 1 - \exp(-x/\mu),\ x \geq 0,\ \mu \text{ is unspecified} \tag{2.1}$$

versus

$$H_1 : F \text{ is NBWUE}(p) \text{ (and is not exponential).} \tag{2.2}$$

A natural test statistic to consider is

$$\begin{aligned} T(F) &= \int_0^p (\varphi(u) - u)du + \int_p^1 (u - \varphi(u))du \\ &= \frac{1}{\mu}\left(\int_0^{t_0} \overline{F}^2(s)ds - \int_{t_0}^{\infty} \overline{F}^2(s)ds - 2\overline{F}(t_0)\int_0^{t_0} \overline{F}(s)ds\right) + 1/2 - p^2 \\ &= \frac{1}{\mu}\int_0^{\infty} s(J(F(s))dF(s), \end{aligned} \tag{2.3}$$

where $\varphi(u) = \dfrac{1}{\mu}\displaystyle\int_0^{F^{-1}(u)} \overline{F}(t)\,dt$, $0 \leq u \leq 1$ is the scaled total-time-on-test(TTT) transform and

$$J(u) = \begin{cases} -p^2 + 2p + 1/2 - 2u & \text{for } 0 \leq u \leq p, \\ -p^2 - 3/2 + 2u & \text{for } p < u \leq 1. \end{cases} \tag{2.4}$$

Let

$$B(t) \equiv \int_t^1 J(u)du$$

$$= \begin{cases} [p(2-p)-(1/2+t)](1-t) & \text{for } 0 \le t \le p, \\ [-p^2-1/2+t](1-t) & \text{for } p < t \le 1. \end{cases} \quad (2.5)$$

If F has a finite mean μ, then

$$\int_0^\infty s\ J(F(s))dF(s) = -\int_0^\infty s\ dB(F(s)) = \int_0^\infty B(F(s))ds \quad (2.6)$$

and thus we obtain another expression of $T(F)$ as

$$T(F) = \frac{1}{\mu} \int_0^\infty B(F(s))ds. \quad (2.7)$$

Klefsjö (1988) obtains the test statistic for testing H_0 versus H_1 by replacing F by F_n in the expression of (2.7), where F_n is an empirical distribution function of F. $T(F_n)$ can be expressed as

$$T(F_n) = \frac{1}{\mu_n} [\sum_{i=0}^{n-1} B(\frac{i}{n})(X_{(i+1)} - X_{(i)})]$$

$$= \frac{1}{\mu_n} \sum_{i=1}^{n} X_{(i)} [B(\frac{i-1}{n}) - B(\frac{i}{n})].$$

In this paper, we extend Klefsjö (1988) results to the case when the data is incomplete. In many medical setting and industry the data are incomplete due to a number of reasons. Let $X_1, X_2, \cdots, X_n$ be i.i.d. F. F is the life distribution of the person (or item). Let $Y_1, Y_2, \cdots, Y_n$ be i.i.d. G. G is the censoring distribution. We assume F and G are continuous. In this setting we observe (Z_i, δ_i), where

$$Z_i = \min(X_i, Y_i),$$

and

$$\delta_i = I\ [X_i \le Y_i] = \begin{cases} 1 & \text{for } X_i \le Y_i, \\ 0 & \text{for } X_i > Y_i, \end{cases} \quad i = 1, 2, \cdots, n.$$

Note that if $\delta_i = 0$ the i-th observation is censored. If $\delta_i = 1$ we observe the actual time of failure (end-point event). We also assume that X_i 's are independent of Y_i 's. Thus $Z_1, Z_2, \cdots, Z_n$ are i.i.d according to the distribution K, where $1-K = \overline{K} = \overline{F}\ \overline{G} = (1-F)(1-G)$.

To derive our test statistic the NBWUE(p) procedure based on censored data, we use the

Kaplan-Meier estimator (1958), $\hat{F}_n$. The Kaplan-Meier estimator of $\overline{F}(x)=1-F(x)$ is defined as

$$1-\hat{F}_n(x)=\hat{\overline{F}}_n(x)=\prod_{(i\,|\,Z_{(i)}\leq x)}\left(\frac{n-i}{n-i+1}\right)^{\delta_{(i)}} \quad \text{for } 0\leq x < Z_{(n)}, \tag{2.8}$$

where $Z_{(1)}\leq Z_{(2)}\leq\cdots\leq Z_{(n)}$ are the order statistics formed from $Z_1,Z_2,\cdots,Z_n$ and $\delta_{(i)}$ indicates whether $Z_{(i)}$ is uncensored ($\delta_{(i)}=1$) or censored ($\delta_{(i)}=0$), $i=1,2,\cdots,n$. When censored observations are tied with uncensored observations, the convention is to treat uncensored observations of the tie as preceding the censored observations of the tie. Also, we treat $Z_{(n)}$ as an uncensored observation whether or not it is uncensored by convention. It is also assumed that $\hat{\overline{F}}_n=0$ for $x\geq Z_{(n)}$. $\hat{\overline{F}}_n$ reduces to empirical distribution function when all observations are uncensored. Our test statistic is obtained by replacing F of (2.7) by $\hat{F}_n$, Kaplan-Meier estimation of F and we have

$$T(\hat{F}_n) = \frac{1}{\mu_n}\int_0^\infty B(\hat{F}_n(s))ds, \tag{2.9}$$

where $\mu_n = \int_0^\infty \hat{\overline{F}}_n(s)\,ds.$

To obtain the asymptotic distribution of our test statistics, we assume the following conditions on F and G.

$$\text{(i)} \quad \int_0^{F^{-1}(1)} [\overline{G}(x)]^{-1}dF(x) < \infty \tag{2.10}$$

and

$$\text{(ii)} \quad \int_0^\infty [\overline{F}^2(x)\int_0^x [\overline{F}^2\overline{G}]^{-1}dF]^{1/2} < \infty. \tag{2.11}$$

Let

$$T(\hat{F}_n)=\frac{1}{\mu_n}\int_0^\infty B(\hat{F}_n(t))dt \quad \text{and} \quad T(F)=\frac{1}{\mu}\int_0^\infty B(F(x))dx.$$

The derivation of asymptotic normality of $T(\hat{F}_n)$ is similar to that of Guess (1984), using the techniques of Joe and Proschan (1982) and Gill (1983).

Under the assumptions (2.10) and (2.11), using Theorem 1 of Joe and Proschan (1982) it can be shown that

$$n^{1/2}[T(\hat{F}_n)-T(F)] \to^D N(0, \sigma^2_{J}(F,G)) \quad \text{as } n\to\infty, \tag{2.12}$$

where

$$J^{*}(u) = J(u) - T(F)$$

and $$\sigma^{2}_{J^{*}}(F,G)=\left(\int_{0}^{\infty}\int_{0}^{\infty} J^{*}(F(x))J^{*}(F(y))\,\overline{F}(x)\,\overline{F}(y)\int_{0}^{\min(x,y)}[\overline{F}^{2}\,\overline{G}]^{-1}dF\,dx\,dy\right)/\mu^{2}.$$

Note that under H_0, $\int_{0}^{\infty} B(F(x)) = 0$ and $\sigma^{2}_{J^{*}}(F,G) = \sigma^{2}_{J}(F,G)$.

Straightforward calculations show that under H_0, we have

$$n^{1/2}\,T(\hat{F}_n) \rightarrow^{D} N(0,\ \sigma^{2}_{0}(F,\ G)) \text{ as } n \rightarrow \infty, \qquad (2.13)$$

where

$$\sigma^{2}_{0}(F,G)=\int_{0}^{1} B^{2}(t)\,(1-t)^{-1}[\,\overline{K}(\,\overline{F}^{-1}(t))]^{-1}dt. \qquad (2.14)$$

When there is no censoring, (2.12) reduced to an asymptotic result in Klefsjö (1988).

The null asymptotic variance $\sigma^{2}_{0}(F,\ G)$ depends on the nuisance parameter G. To define the test statistic, we must obtain an estimator of the null asymptotic variance which is consistent under $H_0 \cup H_1$. The following proposition can be proved using the similar techniques as in Jeong (1992).

Proposition 2.1. Suppose that F and G are continuous distributions and $\overline{G}^{-1}(1) \geq \overline{F}^{-1}(1)$. Define $h(t) = \int_{0}^{t} B^{2}(u)\,(1-u)^{-1}\,du$, $0 \leq t \leq 1$. Let $0 < \eta < 1$.

Then

$$\int_{0}^{F_n^{-1}(\eta)} [\,\overline{K}_n(x^{-})]^{-1}dh(\,\hat{F}_n(x)) \rightarrow^{a.s.} \int_{0}^{F^{-1}(\eta)} [\,\overline{K}(x)]^{-1}dh(F(x)) \text{ as } n\rightarrow\infty, \qquad (2.15)$$

where $\overline{K}_n$ is the empirical distribution function based on the observations $Z_1,\cdots,Z_n$ and $\overline{K}_n \equiv 1 - K_n$.

Note that $\sigma^{2}_{0}(F,\ G) = \int_{0}^{F^{-1}(1)} [\overline{K}(x)]^{-1}\,dh(F(x))$. We have been unable to show that $\int_{0}^{Z_{(n)}} [\,\overline{K}_n(x^{-})]^{-1}dh(\,\hat{F}_n(x))$ converges in probability to $\sigma^{2}_{0}(F,\ G)$ as $n\rightarrow\infty$. Thus we use $\hat{\sigma}_0^{2} = \int_{0}^{F_n^{-1}(\eta)} [\,\overline{K}_n(x^{-})]dh(\,\hat{F}_n(x))$ as an estimator of $\sigma^{2}_{0}(F,\ G)$, where

$\eta \in (0, 1)$ is chosen so that the limit $\int_0^{F^{-1}(\eta)} [\overline{K}(x)]^{-1} dh(F(x))$ is approximately $\sigma^2{}_0(F, G)$.

As our test statistic for testing H_0 versus H_1 for randomly censored data, we propose the following scale invariant statistic

$$T_n^c \equiv n^{1/2}[T(\hat{F}_n)]/\hat{\sigma}_0. \tag{2.16}$$

Consider the test which rejects the null hypothesis of exponentiality in favor of the alternative H_1 if $T_n^c \geq z_\alpha$, where z_α is the upper α-percentile of the standard normal distribution. Under some conditions given in Theorem 2.1 this test has approximate α -level for n sufficiently large under H_0.

Theorem 2.1. Let $\varepsilon > 0$ be a fixed constant which is "very small" and let $0 < \eta < 1$ be such that $\int_\eta^1 (1-t)^{-2} dh(t) < \varepsilon$. Let F be the exponential distribution with mean μ.

Suppose that the censoring distribution G is continuous and that $\overline{G}(x) \geq [\overline{F}(x)]^\theta$ for all $x \geq F^{-1}(\eta)$, where $\theta \in [0, 1)$. Also suppose that ε is "much smaller" than $\sigma^2(F, G)$. Then for $z > 0$, $P(T_n^c > z) \fallingdotseq 1 - \Phi(z)$ for all n sufficiently large, where Φ is the standard normal distribution function.

Proof.

By Proposition 2.1, $\hat{\sigma}_0^2 \rightarrow \int_0^{F^{-1}(\eta)} [\overline{K}(x)]^{-1} dh(F(x))$ a.s. Assumptions (2.10) and (2.11) are satisfied. By the definition of η and ε,

$$\int_{F^{-1}(\eta)}^{F^{-1}(1)} [\overline{K}(x)]^{-1} dh(F(x)) = \int_\eta^1 [\overline{K}(F^{-1}(t))] dh(t)$$
$$\leq \int_\eta^1 (1-t)^{-1-\theta} dh(t) < \varepsilon.$$

By Slutsky's Theorem, the conclusion of the theorem follows.

Because of the difficulties in proving the consistency of the estimator of asymptotic variance σ_0^2, we are more insterested in the speed of convergence of T_n^c to normality by simulation study. These simulation results are given in Table 1.

The NBWUE(p) test procedure rejects the null hypothesis of exponentiality in favor of the alternative H_1: F is NBWUE(p) (and is not exponential) at the approximation level α if

$$T_n^c \geq z_\alpha. \qquad (2.17)$$

Analogously, the approximate α level test of H_0 versus H_1': F is NWBUE(p) (and is not exponential) rejects H_0 if

$$T_n^c \leq - z_\alpha. \qquad (2.18)$$

Using (2.12) and Proposition 2.1, it can be shown that the NBWUE(p) test or NWBUE(p) test is consistent against H_1.

Computational formulae for the given statistics are as follows. Let $\xi_1 < \cdots < \xi_m$ be the ordered distinct uncensored values among $Z_1, \cdots, Z_n$ and let $\xi_0 = 0$. Since we treat $Z_{(n)}$ as an uncensored observation whether or not it is uncensored, $\xi_m = Z_{(n)}$. Computational formulae for $T(\hat{F}_n)$ and $\widehat{\sigma}_0$ are given below.

$$T(\hat{F}_n) = \sum_{j=1}^{m} \xi_j [B(\hat{F}_n(\xi_{j-1})) - B(\hat{F}_n(\xi_j))],$$

$$\widehat{\sigma_0}^2 = \sum_j [\overline{K}_n(\xi_j^-)]^{-1} [h(\hat{F}_n(\xi_j) - h(\hat{F}_n(\xi_{j-1})))],$$

where the sum is over $\{ j: \xi_j \leq \hat{F}_n^{-1}(\eta) \}$. We assume that $\hat{F}_n^{-1}(\eta) = Z_{(n)} = \xi_m$. Direct, but tedious calculations yield

$$h(t) = \begin{cases} -\frac{1}{4}(1-t)^4 - \frac{2}{3}(-p^2+2p-\frac{3}{2})(1-t)^3 - \frac{1}{2}(-p^2+2p-\frac{3}{2})^2(1-t)^2 \\ \quad + \frac{1}{6}(-p^2+2p-\frac{3}{2})(-3p^2+6p-\frac{1}{2}) + \frac{1}{4} & \text{if } 0 \leq t \leq p, \\ -\frac{1}{4}(1-t)^4 + \frac{2}{3}(-p^2+\frac{1}{2})(1-t)^3 - \frac{1}{2}(-p^2+\frac{1}{2})^2(1-t)^2 \\ \quad + \frac{2}{3}(2p^2-2p+1)(1-p)^3 + (2p^3-4p^2+3p-1)(1-p)^2 \\ \quad + \frac{1}{6}(-p^2+2p-\frac{3}{2}(-3p^2+6p-\frac{1}{2}) + \frac{1}{4} & \text{if } p < t \leq 1. \end{cases}$$

3. Monte Carlo Experiment

To investigate the speed of convergence of the test statistic T_n^c to $N(0, 1)$ under H_0 and the power of the proposed test procedure, a Monte Carlo experiment is performed. The life distribution F that we use is exponential(1) i.e., $F(x) = 1 - \exp(-x)$, $x \geq 0$. The censoring distribution G is exponential(θ) for θ = 1/4, 1/9, i.e., $G(t)=1-\exp(-t/4)$, $G(t)=1-\exp(-t/9)$. This results in censoring pattern of about 20%, 10%. The sample sizes are $n=10,20(20)60,100$. Table 1 presents the fraction of times that H_0 is rejected in favor of H_1 : F is NBWUE (p) (and is not exponential), with 1000 replications for selected values of sample size n and p for the given censoring distributions. Also, Table 2 shows the simulated power of the NBWUE(p) test at 5% level of significance against the lognormal alternatives for the same censoring distributions, where the lognormal random numbers are generated for $\mu=0$ and various choices of σ by the Statistical Analysis System (SAS) program. It is well known that the lognormal distribution is upside-down bathtub-shaped failure rate (UBR). If a continuous and strictly increasing life distribution function F is UBR with mean μ, then F is NBWUE (Mitra and Basu(1994)). Using the results by Park (1988) it can be shown that the failure rate of the lognormal distribution with parameters μ and σ^2 changes from increasing to decreasing at $t_0(\mu, \sigma^2)$, where t_0 satisfies

$$gauf((\log t_0-\mu)/\sigma)$$

$$= 1-(1/\sqrt{2\pi})(\sigma/(\sigma^2+\log t_0-\mu))\exp(-(\log t_0-\mu)^2/2\sigma^2).$$

Thus, the lognormal distribution has the NBWUE distribution with change-point at $p = F(t_0) = gauf((\log t_0 - \mu)/\sigma)$, *where* $F(\cdot)$ and gauf($\cdot$) are the lognomal and standard normal cdf's respectively.

In addition, we study the efficiency loss due to the presence of censoring. Since the statistic introduced in Section 2 is a generalization of the T_n statistic of Klefsjö (1988), we find it interesting to compare the power of T_n test based on n observations in the uncensored case with the power of T_n^c test based on n' observations in the randomly censored model. Since T_n and T_n^c have the same asymptotic means, the ARE of T_n^c with respect to T_n can be computed as

$$k = e_G(T_n^c, T_n)$$

$$= [1/12-p^2(1-p)^2] / \int_0^1 B^2(t)(1-t)^{-1}[\overline{K}(F^{-1}(t))]^{-1}dt.$$

Note that the efficiency loss due to censoring is measured by $1-k$. We consider the case where the censoring distribution is exponential, that is $\overline{G}(x) = \exp(-\theta x),\ x \geq 0$. To satisfy the condition $\sup\{[\overline{F}(x)]^{1-\epsilon}[\overline{G}(x)]^{-1}, x \in [0, \infty)\} < \infty$, for some $0 < \epsilon < 1$, we must impose the restriction $\theta < 1$. Then we obtain

$$e_G(T_n^c; T_n) = [1/12 - p^2(1-p)^2] / [(2p-p^2-1.5)^2 + (4p^3-8p^2+6p-2)(1-p)^{1-\theta}](1-\theta)^{-1} + [(4p-2p^2-3)-(4p-4p^2-2)(1-p)^{2-\theta}](2-\theta)^{-1} + (3-\theta)^{-1}. \quad (2.19)$$

In Table 3, the values of $e_G(T_n^c, T_n)$ are given for several choice of $\theta < 1$.

4. Conclusions

In Table 1, we may conclude that convergence of the test statistic T_n^c to $N(0,1)$ under H_0 is somewhat slow in general and not always regular. In view of Table 2, when p is further away from 0 or 1, the test does not perform very well. However, if p is close to 0 or 1, our test performs reasonably well. In fact, the most interesting situation would be when p is relatively small, which describes the phenomenon known as "infant mortality". Such situations also arise "Burn-in" model. Thus, it is encourging to know that our test procedure performs well when p is close to 0.

Table 3 shows that as θ tends to 0 (corresponding to the case of no censoring), the efficiency loss $1-k$ tends to 0, which is as expected.

TABLE 1

Empirical test size of NBWUE (p) test from 1000 replications when the censoring distributions are $\overline{G}(x) = e^{-\frac{1}{4}x}$ and $\overline{G}(x) = e^{-\frac{1}{9}x}$.

1% 5% 10%	P=0.1	P=0.3	P=0.5	P=0.7	P=0.9
n = 10	0.004(0.004) 0.112(0.012) 0.022(0.025)	0.002(0.009) 0.014(0.023) 0.026(0.046)	0.019(0.014) 0.055(0.038) 0.083(0.086)	0.027(0.039) 0.095(0.106) 0.159(0.161)	0.086(0.076) 0.196(0.192) 0.289(0.266)
n = 20	0.006(0.007) 0.022(0.022) 0.033(0.051)	0.012(0.015) 0.034(0.036) 0.056(0.061)	0.006(0.016) 0.033(0.058) 0.076(0.099)	0.025(0.021) 0.070(0.066) 0.111(0.137)	0.042(0.030) 0.125(0.095) 0.202(0.185)
n = 40	0.008(0.005) 0.025(0.031) 0.047(0.055)	0.011(0.010) 0.028(0.030) 0.053(0.064)	0.009(0.014) 0.037(0.042) 0.069(0.087)	0.014(0.018) 0.068(0.085) 0.129(0.151)	0.020(0.016) 0.079(0.074) 0.164(0.139)
n = 60	0.009(0.011) 0.035(0.033) 0.067(0.065)	0.015(0.015) 0.043(0.041) 0.067(0.066)	0.009(0.010) 0.034(0.033) 0.071(0.073)	0.018(0.013) 0.058(0.067) 0.103(0.132)	0.009(0.018) 0.054(0.069) 0.136(0.124)
n = 100	0.014(0.012) 0.039(0.035) 0.068(0.067)	0.008(0.008) 0.027(0.038) 0.052(0.084)	0.012(0.014) 0.035(0.054) 0.067(0.098)	0.011(0.012) 0.058(0.058) 0.114(0.115)	0.008(0.012) 0.061(0.070) 0.127(0.153)

※ Numbers in the brackets represent empirical test size when censoring distribution is $\overline{G}(x) = e^{-\frac{1}{9}x}$.

TABLE 2

Empirical power of the NBWUE(p) test against lognormal distribution alternatives with parameters $\mu = 0$ and $\sigma^2 > 0$ when the censoring distributions are $\overline{G}(x) = e^{-\frac{1}{4}x}$ and $\overline{G}(x) = e^{-\frac{1}{9}x}$.

σ [p]	α	n=10	n=20	n=40	n=60	n=100
0.70 [0.9595]	0.05	0.672(0.576)	0.684(0.621)	0.690(0.673)	0.766(0.725)	0.882(0.800)
0.90 [0.5734]	0.05	0.269(0.306)	0.360(0.469)	0.559(0.683)	0.680(0.775)	0.848(0.915)
1.20 [0.1018]	0.05	0.039(0.117)	0.110(0.220)	0.209(0.342)	0.256(0.442)	0.387(0.601)
1.30 [0.0493]	0.05	0.056(0.116)	0.131(0.236)	0.223(0.404)	0.289(0.533)	0.424(0.623)
1.50 [0.0097]	0.05	0.070(0.168)	0.158(0.333)	0.316(0.534)	0.409(0.631)	0.577(0.825)
2.50 [0.0000]	0.05	0.088(0.215)	0.231(0.430)	0.434(0.680)	0.587(0.828)	0.768(0.939)

※ Numbers in the brackets represent empirical power when censoring distribution is $\overline{G}(x) = e^{-\frac{1}{9}x}$.

TABLE 3

Efficiency of T_n^c with respect to T_n when the censoring distribution is exponential with mean $\frac{1}{\theta}$.

P \ θ	1/100	1/20	1/10	1/4	1/3	1/2	2/3	3/4
0.10	0.985913	0.929475	0.858948	0.651376	0.541810	0.344279	0.185546	0.122844
0.20	0.984289	0.921828	0.844846	0.625017	0.512957	0.317767	0.167467	0.109728
0.25	0.983367	0.917516	0.836963	0.610636	0.497398	0.303738	0.157934	0.102886
0.50	0.988234	0.940740	0.880460	0.694957	0.590719	0.388714	0.212248	0.139811
0.70	0.994120	0.970530	0.941102	0.852794	0.804087	0.708310	0.615857	0.571180
0.75	0.993037	0.965094	0.929986	0.823894	0.764776	0.646853	0.528847	0.467908
0.90	0.988686	0.943217	0.886020	0.714065	0.619971	0.439828	0.275994	0.201305

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