# Total Ordering by Fuzzy Inference

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ABSTRACT

Fuzzy inference method is introduced to order totally a partiallyordered system. When there are more than one order indices and fuzzy order rules, the proposed method provides one order index by mixing them.

## I. Introduction

The ordering is an important issue in the system analysis. A system can be structured by the total ordering or partial ordering. An order is given by ordering rules or criteria, and in general a system is partially ordered when there are several ordering rules[3][4].

If we want to order totally a system which is partially ordered, it is necessary to develop a method. Therefore, in this paper, a fuzzy approach is introduced which is used in the fuzzy inference. It is shown that this method is useful to tranfrom a partial ordered system into a total ordered system.

In the fuzzy inference, there are many methods for the inferencing and defuzzification. In this paper, we use the Mandani's method for the inferencing and the center of area method for the defuzzification because they are simple and popular[1][2].

## II. Fuzzy Inference

In the fuzzy inference, in general, the fuzzy rules ($R_1$, $R_2$, ..., $R_i$) are formulated as the forms of *If-then* rule as follows:

$R_1$: If $x_1$ is $A_1$ then $w$ is $C_1$

$R_2$: If $x_2$ is $A_2$ then $w$ is $C_2$

⋮

$R_i$: If $x_i$ is $A_i$ then $w$ is $C_i$

In the above rules, $x_i$ and $w$ are the input and output variable respectively. $A_i$ and $C_i$ are fuzzy sets re-

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spectively representing linguistic terms, and they are defined by membership functions $\mu_{A_i}(x_i)$ and $\mu_{C_i}(w)$ respectively. The numbers of rules and variables can be also extended.

When the input data is given as a scalar value such as $x_i = x_i^0$, the output value $C$ is given by the Mamdani's method as follows:

$$\mu_C(w) = \max_i \min[\mu_{A_i}(x_i^0), \mu_{C_i}(w)]$$

To defuzzify the fuzzy output $C$, we use the center of area method.

$$w^* = \frac{\sum_{j=1}^{n} w_j \cdot \mu_C(w_j)}{\sum_{j=1}^{n} \mu_C(w_j)} \quad \text{for the discrete value } w$$

$$= \frac{\int w \cdot \mu_C(w)\, dw}{\int \mu_C(w)\, dw} \quad \text{for the continuous value } w$$

Then, $w^*$ is the scalar output value.

## Ⅲ. Fuzzy Total Order

Suppose a system ordered by the relations $R_1$ and $R_2$. When two relations are applied to a system, it is possible that it is partially ordered. However, if we want to obtain a total order, we can use the fuzzy inference method.

Let $a_1, a_2, \ldots, a_i$ be the elements in the system and $r_1$ and $r_2$ be the functions applied by the relations $R_1$ and $R_2$ respectively. Then $r_1(a_i)$ and $r_2(a_i)$ are the function values of $a_i$ which can be used as ordering indices.

For example, let's construct 3 fuzzy rules to order the system. The rules represent our knowledge that if the ordering index is bigger the order is higher.

$R_1$ : If $r_1(a_i)$ is $Big_1$, then $w(a_i)$ is $High$
$R_2$ : If $r_2(a_i)$ is $Big_2$, then $w(a_i)$ is $High$
$R_3$ : If $r_2(a_i)$ is $Small$, then $w(a_i)$ is $Low$

The fuzzy terms $Big_1$, $Big_2$, $Small$, $High$ and $Low$ are defined as shown in Figure 1. The output $w^*(a_i)$ is the ordering index of $a_i$ which is obtained by mixing $r_1(a_i)$ and $r_2(a_i)$.

$$w^*(a_i) = \frac{\int w(a_i) \cdot \mu_C(w(a_i))\, dw(a_i)}{\int \mu_C(w(a_i))\, dw(a_i)}$$

To illustrate, consider a system $a_1, a_2, a_3, a_4$ which are ordered by ordering indices $r_1$ and $r_2$. The values of order function are as follows:

$r_1(a_1) = 30$ $r_2(a_1) = 4$
$r_1(a_2) = 20$ $r_2(a_2) = 6$
$r_1(a_3) = 50$ $r_2(a_3) = 5$
$r_1(a_4) = 60$ $r_2(a_4) = 8$

We see that the elements $a_2$ and $a_3$ are not comparable and thus this system is partially ordered as shown in Figure 2.

In order to order totally the system, let's use $r_1(a_i)$ and $r_2(a_i)$ as the input data of the above fuzzy rules. In Figure 1, the inferencing mechanism for the element $a_1$ is shown, and the output of each rule is indicated by the grey area. Figure 3 shows the union of outputs and its defuzzified value $w^*(a_1) = 4.14$. In the same way we obtain the ordering indices $w^*(a_2) = 6.48$, $w^*(a_3) = 6.07$ and $w^*(a_4) = 6.89$. By the indices, we can order the system such as

$a_1 < a_3 < a_2 < a_4$

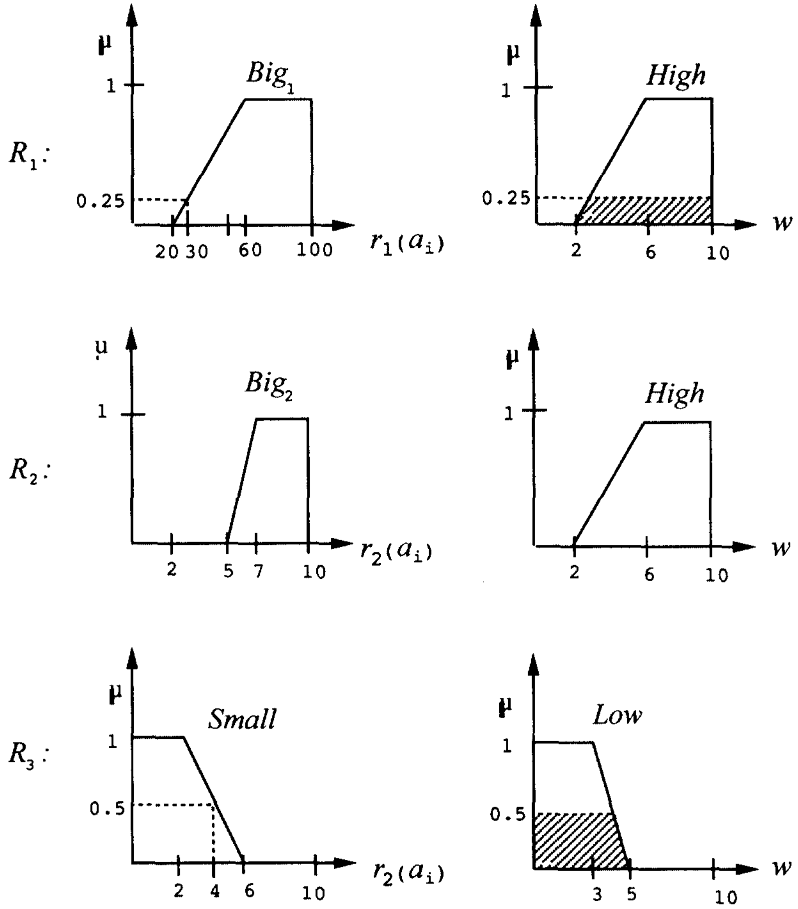


Figure 1 : Fuzzy sets and Fuzzy inferencing

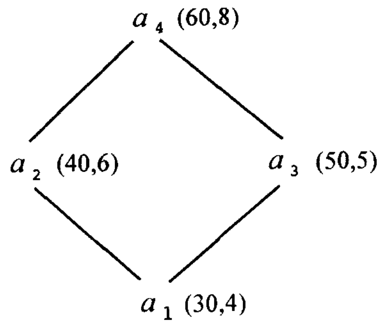


Figure 2 : Partial ordered system

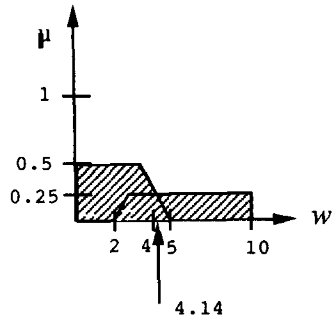


Figure 3 : Defuzzification

## Ⅳ. Conclusion

The fuzzy inference method has been introduced to order totally a system when there are one or more ordering rules. It was shown that the proposed method can efficiently mix several rules and provide one ordering index when there are fuzzy ordering knowledge.

## References

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