Journal of the Korean Nuclear Society
Volume 27, Number 1, February 1995

Optimal Sampling Period of the Digital Control System for the Nuclear Power Plant Steam Generator Water Level Control

Woosung Hur and PoongHyun Seong
Korea Advanced Institute of Science and Technology
(Received May 2, 1994)

증기발생기 수위 제어를 위한 디지탈 제어기의 적정 샘플링 주기

허우성 · 성풍현
한국과학기술원
(1994. 5. 2 접수)

Abstract

A great effort has been made to improve the nuclear plant control system by use of digital technologies, and a long term schedule for the control system upgrade has been prepared with an aim to implementation in the next generation nuclear plants. In case of digital control system, it is important to decide the sampling period for analysis and design of the system, because the performance and the stability of a digital control system depend on the value of the sampling period of the digital control system. There is, however, currently no systematic method used universally for determining the sampling period of the digital control system. Generally, a traditional way to select the sampling frequency is to use 20 to 30 times the bandwidth of the analog control system which has the same system configuration and parameters as the digital one. In this paper, a new method to select the sampling period is suggested which takes into account of the performance as well as the stability of the digital control system. By use of the Irving's model of steam generator, the optimal sampling period of an assumptive digital control system for steam generator level control is estimated and is actually verified in the digital control simulation system for KORI-2 nuclear power plant steam generator level control. Consequently, we conclude the optimal sampling period of the digital control system for KORI-2 nuclear power plant steam generator level control is 1 second for all power ranges.

요 약

최근 디지탈 기술을 응용하여 원자력 발전소 제어 시스템의 성능을 향상시키려는 많은 노력이 있어 왔고, 차세대 원자로에 구현할 것을 목표로 디지탈 제어 시스템 개발에 관한 장기적인 계획이 수립되어 있다. 디지탈 제어 시스템을 구축하고자 할 때 적정한 샘플링 주기를 정하는 것은 중요한 과정이다. 제어기의 안정성과 성능은 샘플링 주기에 밀접한 관련이 있다. 현재 디지탈 제어기의 샘플링 주기를 체계적으로 정하는 전형적인 방법은 없다. 일반적으로 디지탈 제어기의 안정성을 고려해서 연속시간역 제어의 대

역폭의 20~30배의 역수 정도의 샘플링 주기를 흔히 쓴다. 이 논문에서는 안정성 뿐 아니라 시간역에서의 좋은 성능을 보장 받을 수 있는 적정한 샘플링 주기를 얻을 수 있는 방법을 제시하였다. 이 방법으로 Irving의 모델을 이용하여 디지탈제어기의 적정 샘플링 주기를 예측하고, 고리 2호기의 마이크로 시뮬레이터와 WDPF 디지탈 제어기를 이용한 증기 발생기 수위제어 모사시스템에서 검증해 본 결과, 고리 2호기 원자력발전소 증기 발생기 수위제어를 위한 디지탈 제어기의 원자로 전 출력영역에 대한 적정 샘플링 주기가 1초가 되는 것을 알게 되었다.

1. Introduction

Based on the significant progress in micro-electronics technology in recent years, microcomputer based digital controllers, which have high reliability, maintainability, and flexibility in modifying control functions and algorithms, are being applied to the control and safety systems in the nuclear power plants instead of conventional relays and analog controllers. In case of digital control systems, it is important to decide optimal sampling periods for the analysis and design of the systems. The performance of digital control systems depends on the value of the sampling period of digital control system. The selection of large sampling period usually gives rise to a high overshoot in the step response and may eventually cause instability. A sampling period that is too small, on the other hand, is wasteful in which a more complex and more costly digital processor may result.

The objective of this study is to determine the optimal sampling periods of the digital control system for the steam generator water level control. For the purpose of this study, the transfer function describing the dynamics of steam generator developed by Irving, et al[1], [2] is used and three-element control system commonly used in current nuclear plants is conceptually modeled. This model includes the steam generator as a process plant. By introducing samplers and holders, it is then digitalized. By use of two methods; one is the method currently used and the other is the one developed in this study, we determine the optimal sampling period of digital control system which can satisfy the control specifications and can yield the desirable system responses for all power ranges. In order to validate the optimal sampling period obtained from a linear steam generator model, the control performance is evaluated by use of the digital control simulation system for KORI-2 nuclear power plant steam generator level control with this sampling period.

2. Model of S/G Level Control System

2.1. Open Loop Transfer Function

Irving and Bihoreaux suggested a simple transfer function that successfully describes the shrink and swell effects based on the step response of the S/G water level for the step change of the feedwater flowrate and the steam flowrate[1] as follows :

$$\Delta L(s) = [\frac{G_1}{s} - \frac{G_2}{(\tau_2 s+1)}](\Delta W_f(s) - \Delta W_s(s)) + [\frac{G_3 s}{(s^2 + 2\tau^{-1}s + 4\pi^2 T^{-2} + \tau^{-2})}]\Delta W_f(s) \quad (1)$$

, where ΔL(s) represents the water level response of steam generator for feedwater or steam flowrate changes and $\Delta W_f(s)$ and $\Delta W_s(s)$ are the deviation of feedwater flowrate (W_f) and steam flowrate (W_s) from the reference values, which are given for the specific power levels in Ref.[2], respectively. In Eq.(1), G_1/s is the mass capacity term of the steam generator where G_1 is a measure of the steam generator's height to volume ratio. This mass capacity term represents changes in the steam generator level caused by mass influx or efflux from the volume of the particular steam generator. The $G_2/(\tau_2 s+1)$ term is of first order and represents the shrink and swell effects. G_2 is the variable which describes the magnitude of the shrink and swell effects. The τ_2 is the characteristic decay time for the shrink and swell effects caused by the feedwater and steam flowrate changes. The last term in the right hand side of Eq.(1) represents the effect of mechanical oscillation in the tube bundle due to the change of feedwater flowrate. The transfer functions of water level to feedwater flowrate and water level to steam flowrate are written as follows :

$$\Delta L(s) = G_f(s)\Delta W_f(s) + G_s(s)\Delta W_s(s) \qquad (2)$$

, where

$$G_f(s) = \frac{\Delta L(s)}{\Delta W_f(s)}$$

$$= \frac{G_1}{s} - \frac{G_2}{(\tau_2 s+1)} + \frac{G_3 s}{(s^2+2\tau^{-1}s+4\pi^2T^{-2}+\tau^{-2})}, \qquad (3)$$

$$G_s(s) = \frac{\Delta L(s)}{\Delta W_s(s)} = -\frac{G_1}{s} + \frac{G_2}{(\tau_2 s+1)} \qquad (4)$$

Table 1. Steam Generator Parameter Variations[2]

Power(%)	5	15	30	50	100
W(kg/s)	57.4	180.8	381.7	660.0	1435.0
G_1	0.058	0.058	0.058	0.058	0.058
G_2	9.63	4.46	1.33	1.05	0.47
τ_2(sec)	48.4	21.5	4.5	3.6	3.4
τ(sec)	41.9	26.3	43.4	34.8	28.6
T(sec)	119.6	60.5	17.7	14.2	11.7
G_3	0.181	0.226	0.310	0.215	0.105

The coefficients of the transfer functions depend on the operating range of reactor power. Values of steam generator parameter at specific powers are given in Table 1[2].

The third term in the transfer function of feedwater flowrate in Eq.(3) describes the mechanical oscillation that results from a direct addition of feedwater to the steam generator. This quantity only appears in response to a feedwater change and it decays rapidly

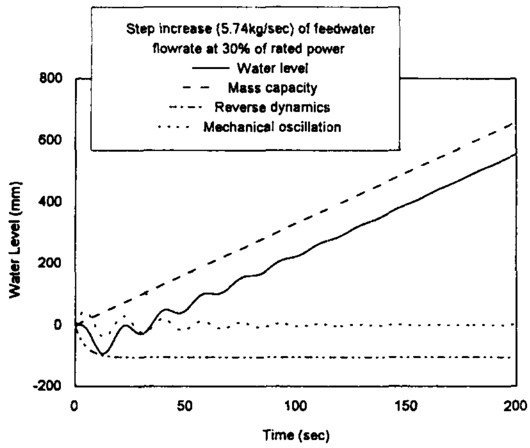


Fig. 1. Comparison of the Effects Caused by the Terms in Eq.(3) at the Step Increase of Feedwater Flowrate in Irving's S/G Model

as shown in Fig. 1. Therefore, we simplify the transfer function of feedwater flowrate in Eq.(3) as follows

$$G_f(s) \approx \frac{G_1}{s} - \frac{G_2}{(\tau_2 s+1)} \qquad (5)$$

2.2. Schemes of S/G Level Control System

The block diagram of typical three element PI control system for S/G level control is shown in Fig. 2[3]. In this work, three elements are the incremental feedwater flowrate(ΔW_f), incremental steam flowrate (ΔW_s), and level set point (L_{sd}). The incremental values ΔW_f, ΔW_s are used instead of the values of W_f ,W_s. This comes from the characteristic of the input variables in Irving's linear steam generator model as shown in Eq.(1). The error signal between the water level and the set point and the difference signal between the incremental feedwater flowrate and steam flowrate are inputs to the control process, and the incremental feedwater flowrate is the control output for the steam generator level control. The control system is composed of two proportion-integral (PI) controllers, of which one is the feedback control of the level error signal and the other is the feedforward control of the flowrate difference signal. The reason of adopting the PI controllers is that these are widely used in S/G water level control and the derivative control is usually susceptible to the noisy input signal though it often well suited in the case where speedy reaction to input is required[4].

It is assumed in this study that the steam flowrate is constant for the purpose of simplicity. Hence, the incremental steam flowrate becomes therefore zero and the flowrate difference signal is equivalent to the incremental feedwater flowrate alone. With respect of control, the steam flowrate among three elements is regarded as a disturbance in the linear steam generator model and, then, the control process in Fig.2 becomes the combination of the feedback level error control and the feedforward feedwater flowrate control. The overall transfer function of two PI controllers, PI#1 and PI#2, in Fig.2 is as follows :

$$T(s) = K_1(1+\frac{1}{\tau_1 s}) \times (\frac{1}{1+K_2(1+\frac{1}{\tau_2 s})}) . \qquad (6)$$

where K_1 and K_2 are the proportional gains and τ_1, τ_2 are the integral time constants of PI#1 and PI#2 respectively. The steam generator in Fig.2 represents the simplified Irving's linear model and it is composed of two transfer functions of water level response to feedwater flowrate change and one to steam flowrate change as expressed in Eq.(1). It is assumed that the steam flowrate is constant, that is, $\Delta Ws(s) = 0$ and hence the steam generator model is reduced to $G_f(s)$. The closed loop transfer function of the control system is as follows :

$$C(s) = \frac{L(s)}{L_{set}(s)} = \frac{T(s) \times G_f(s)}{1+T(s) \times G_f(s)} . \qquad (7)$$

The continuous-time control system shown in Fig.2 is redesigned for discrete-time control with the input processed by a sample-and-hold device. The block diagram of redesigned digital control system for S/G level control is shown in Fig.3. This has the same system configuration and parameters as three-element continuous-time control system and the only difference to the continuous-time control system is that the sampler is added to the input port of control process and the zero-order holder (zoh) is added to the output port. The water level sensor in this case is analog one, such as the ball float. Before the water level error signal is processed by the digital processor, it must be sampled and quantitized to be the discrete-data signal. The feedwater flowrate feedback is not subject to sampling. This is the case that the feedwater flowrate sensor is a digital transducer and its output and input signals are both processed by a digital processor. In this control scheme, the one-step delay component is added to the feedwater flowrate feedback. This cannot accurately estimate the intermediate feedwater flowrate but has the merit to reduce the computational burden of the digital controller. Then, the overall transfer function T(s) of two controllers which considers the delay unit in the feedwater flowrate feedback loop is devised as follows :

$$T(s) = K_1(1+\frac{1}{\tau_1 s}) \times (\frac{1}{1+K_2(1+\frac{1}{\tau_2 s})e^{-Ts}}) . \qquad (8)$$

where K_1 and K_2 are also the proportional gains and τ_1, τ_2 are the integral time constants of two controllers, PI#1 and PI#2 respectively and T is the sampling period. The closed loop transfer function C(z) of the digital control system is obtained as follows:

$$C(z) = \mathcal{Z}\left[\frac{G_{oh}(s) \times C(s) \times G_{fw}(s)}{1+G_{oh}(s) \times C(s) \times G_{fw}(s)}\right] \qquad (9)$$

where   is the z-transform operator

$G_{oh}(s)$ is the Laplace transform of zoh,

$(1-e^{-Ts})/s$.

The MATLAB[5] software package is used to simulate the steam generator control system shown in Fig.2 and Fig.3. Gains and integral time constants of tuned PI controllers for the simplified Irving's model at specific powers are listed in Table 2.

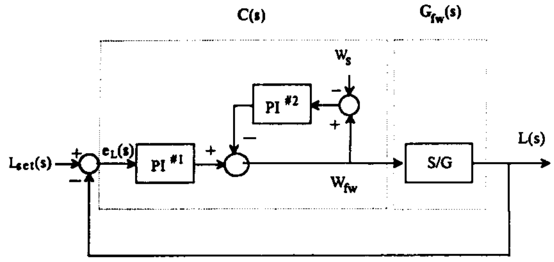


Fig. 2. Schematic Block Diagram of Three-Elements Analog Control System for S/G Water Level Control

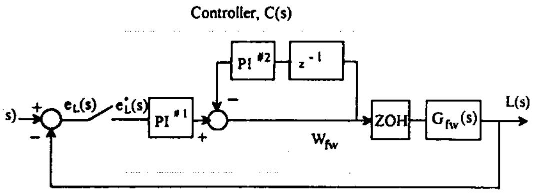


Fig. 3. Schematic Block Diagram of Digital Control System for S/G Water Level Control

Table 2. Gains and Integral Time Constants of the Tuned PI Controllers in Three Element Continuous-Time Control System at Specific Powers

Power(%)	K_1 (kg*s^{-1}/mm)	τ_1 (sec)	K_2	τ_2 (sec)
100	2.6	300	2.0	200
50	1.65	300	2.1	200
30	1.1	300	2.5	200
15	0.4	300	2.1	200
5	0.2	300	2.3	200

3. Optimal Sampling Period

Generally, a traditional way in control design to select the sampling frequency is to use one which is 20 to 30 times the bandwidth of the continuous-time control system emulated by the discrete-time control system[6]. Therefore, a rough calculation of the sampling period for the stability of digital control system is as follows :

$$T_s = \frac{1}{20 \times Bandwidth} . \tag{10}$$

The bandwidth *(BW)* is defined as the frequency ω at which the ratio of the magnitude of output signal to that of input signal drops to 70.7 percent of its zero-frequency value[7]. It is suggested that a general method[8], which reduces the high-order system to the low-order system, can be used in order to obtain easily the undamped natural frequency of the high-order system. The closed loop transfer function, C(s), of three element continuous-time control system in Eq.(7) can be rewritten as the following form ;

$$C(s) = \frac{\Delta L(s)}{L_{set}(s)} = k\frac{1+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2+b_3 s^3} . \tag{11}$$

It is desired that the transfer function with three-order characteristic equation in Eq.(11) can be reduced to one with second-order characteristic equation as

$$L(s) = k\frac{1+c_1 s}{1+d_1 s+d_2 s^2} . \tag{12}$$

in the sense that the frequency response of the system of Eq.(12) is similar to one of the system of Eq.(11). Moreover, the zero frequency gain, k, of the two transfer functions is the same. This will ensure that the steady-state behavior of the high-order system is preserved in the low-order system. The criterion of finding low-order *L(s)*, given *C(s)*, is that the following relation should be satisfied as closely as possible ;

$$\frac{|C(j\omega)|^2}{|L(j\omega)|^2} = 1 \qquad for\ 0 \le \omega \le \infty. \tag{13}$$

By using of Eq.(11) and Eq.(12), the ratio of C(s) to L(s) is

$$\begin{aligned}\frac{C(s)}{L(s)} &= \frac{1+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2+b_3 s^3} \times \frac{1+d_1 s+d_2 s^2}{1+c_1 s} \\ &= \frac{1+m_1 s+m_2 s^2+m_3 s^3+m_4 s^4}{1+l_1 s+l_2 s^2+l_3 s^3+l_4 s^4}\end{aligned} \tag{14}$$

Then, Eq.(13) can be written as

$$\frac{|C(j\omega)|^2}{|L(j\omega)|^2}=\left.\frac{C(s)C(-s)}{L(s)L(-s)}\right|_{s=jw}$$

$$=\frac{1+m_1s+m_2s^2+m_3s^3+m_4s^4}{1+l_1s+l_2s^2+l_3s^3+l_4s^4}$$

$$\times\left.\frac{1-m_1s+m_2s^2-m_3s^3+m_4s^4}{1-l_1s+l_2s^2-l_3s^3+l_4s^4}\right|_{s=jw}$$

$$=\left.\frac{1+e_2s^2+e_4s^4+e_6s^6+e_8s^8}{1+f_2s^2+f_4s^4+f_6s^6+f_8s^8}\right|_{s=jw}$$

$$=1+\left.\frac{(e_2-f_2)s^2+(e_4-f_4)s^4+(e_6-f_6)s^6+(e_8-f_8)s^8}{1+f_2s^2+f_4s^4+f_6s^6+f_8s^8}\right|_{s=j\omega}. \tag{15}$$

From the relation between the second equation and the third one in Eq.(15), the following constitutive equations are obtained :

$$\begin{aligned} e_2 &= 2m_2-m_1^2 \\ e_4 &= 2m_4-2m_1m_3+m_2^2, \\ e_6 &= 2m_2m_4-m_3^2 \\ e_8 &= m_4^2 \end{aligned} \tag{16}$$

and

$$\begin{aligned} f_2 &= 2l_2-l_1^2 \\ f_4 &= 2l_4-2l_1l_3+l_2^2 \\ f_6 &= 2l_2l_4-l_3^2 \\ f_8 &= l_4^2 \end{aligned} \tag{17}$$

In order to satisfy the condition of Eq.(13), the second term of the right-hand side in Eq.(15) is to be zero, and it is done by

$$\begin{aligned} e_2 &= f_2 \\ e_4 &= f_4 \\ e_6 &= f_6 \\ e_8 &= f_8 \end{aligned} \tag{18}$$

Solving Eq.(16), Eq.(17) and Eq.(18) constitutively, the values of c_1, d_1, and d_2 in Eq.(12) can be obtained. By comparing Eq.(12) with the prototype second-order system, the undamped natural frequency ω_n and the damping ratio ξ can be obtained as follows ;

$$\omega_n=\sqrt{\frac{1}{d_2}}, \qquad \zeta=\frac{1}{2}\frac{d_1}{\sqrt{d_2}}. \tag{19}$$

The bandwidth of the approximated second-order system can be obtained from the relation as

$$BW=\omega_n\left[(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}\right]^{1/2}. \tag{20}$$

The bandwidths of the control system are obtained through the procedure derived above at specific reactor powers, and the corresponding sampling periods are obtained from the relation of Eq.(10) and listed in Table 3.

Table 3. Bandwidths of Three-Element Control System and Corresponding Sampling Periods at Specific Powers

Power(%)	Bandwidth ($sec^{-1}\times10^{-2}$)	T_s(sec)
100	14.9	0.34
50	7.0	0.71
30	5.1	0.98
15	3.0	1.67
5	1.5	3.33

Although these sampling periods make the operation of the control system stable, more information is needed in order to select the optimal sampling period for the desirable performance within the stable bounds. New method is introduced as follows. Performing the z-transform of Eq.(9) by partial fraction method, the characteristic equation of closed loop transfer function is written of the form

$$F(z)=z^3+a_2(K,T_s)z^2+a_1(K,T_s)z+a_0(K,T_s)=0, \tag{21}$$

where a_0, a_1, and a_2 are the functions of the sampling time T_s and the gain K. It is noted that the feed back PI control for the level error is more significant to the control performance than the feedforward PI control for the flow error in this digital control system. Therefore, the feedback gain K_2 and integral time constant τ_2 are fixed to constants for the simplicity of analysis. Then, the range of the feedforward gain K_1 and the sampling period Ts in which the system may be asymptotically stable is determined by use of Routh's stability criterion, which states that the number of poles of the closed loop transfer function with positive real parts is equal to the number of changes in sign of the coefficients of the first column of Routh's table. For stability, all the coefficients in the first column of the Routh's tabulation must be of the same sign. By use of this fact, the inequality condition is obtained, including the gain K and the sampling period Ts as parameters to satisfy the stability condition. In practice, the stability boundary and the selection region under the line of the maximum feedback gain K_m versus the sampling period Ts of the digital control system at specific powers are shown in Fig.4.

The Ziegler-Nichols tuning method suggests that the proper control gain yielding the desirable time-responses for a PI type controller is the 0.45 times of the maximum gain for the sake of stability. The tuned gains of the PI controllers in the three element control system are selected from the Ziegler-Nichols' suggestion, and the corresponding sampling periods can be determined in the constraints in Fig.4 and are listed in Table 4.

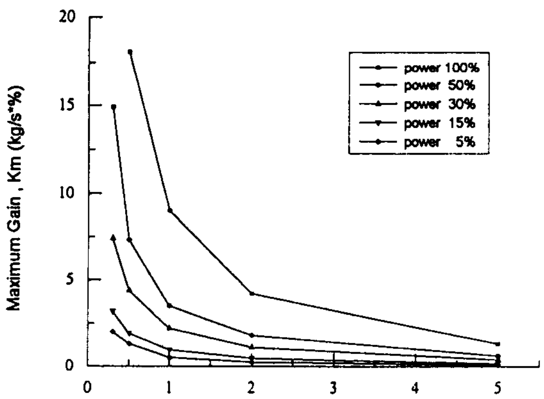


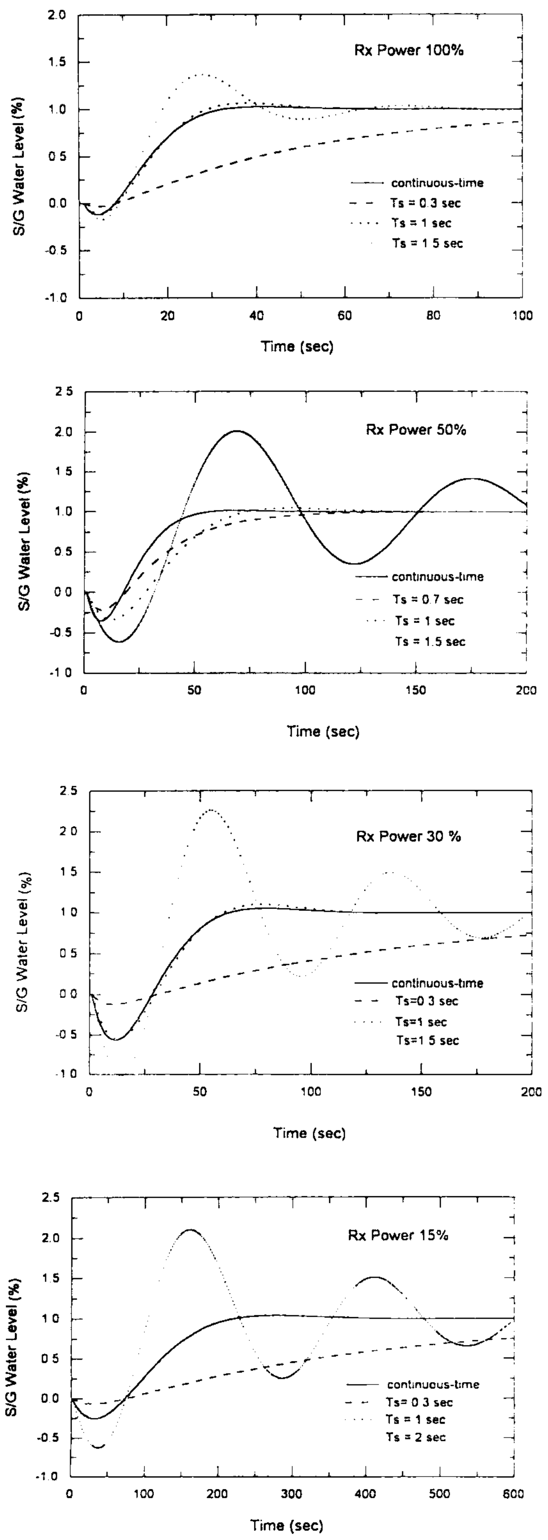
Fig. 4. Maximum Gains vs. Sampling Periods

Table 4. Sampling Periods Recommended by the Method Developed in This Paper at Specific Powers.

Power(%)	K_1	K_m	T_s(sec)
100	2.6	5.94	1.2
50	1.65	3.660	1.1
30	1.1	2.444	0.9
15	0.4	0.888	1.0
5	0.2	0.444	1.1

4. Results and Conclusions

There are two experiments to investigate the performance of control system for the steam generator water level control. One is the water level tracking experiment by use of linear steam generator model in which the water level set point is varied and accordingly whether the control system can track the changed setpoint or not is observed. Of course, it is well known that the steam generator water level should be kept constant for all power ranges. However, the simulations in this study is to show the effect of the sampling periods on the performance of the control system. The other is the water level regulating experiment by use of KORI-2 nuclear power plant simulator in which the set point is fixed and under the variation of environment such as turbine load change, whether the control system can regulating the water level to setpoint or not is observed. Fig.5 shows the level responses of Irving's steam generator model in time-domain by 1% step change of level set point with various sampling periods at specific powers. For example, amongst the time responses of digital control system for various sampling periods at power 100%, the time response of the digital control system with the sampling period 1 second is almost the same as that of analog control system emulated by the digital control system, and it has no overshoot and the short settling time of about 30 seconds. And the time response of the digital control system with the sampling period 1.5 second is not satisfactory



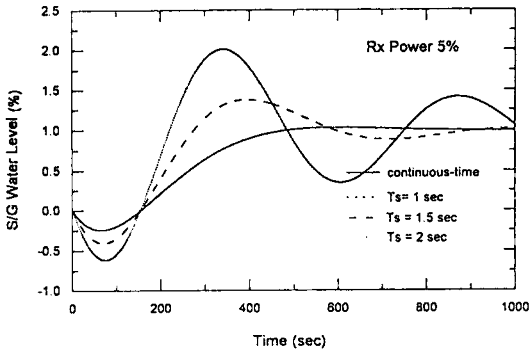
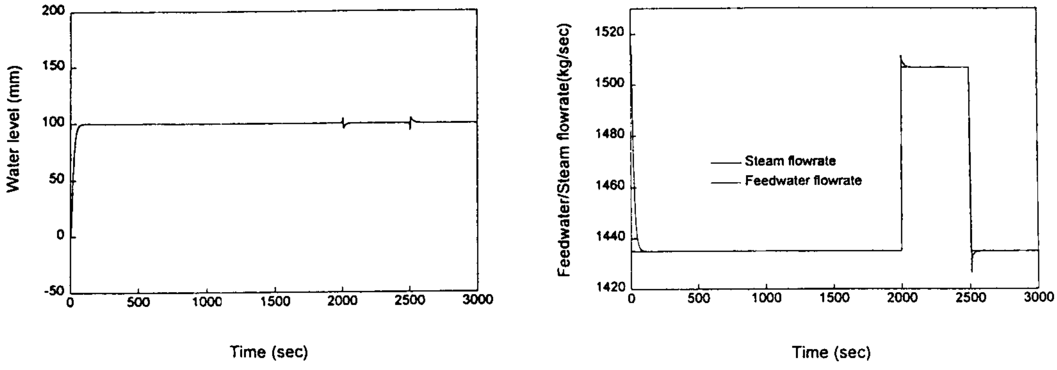


Fig. 5. Comparison of the Level Tracking Performance of the Digital Control System With Various Sampling Periods at Specific Powers

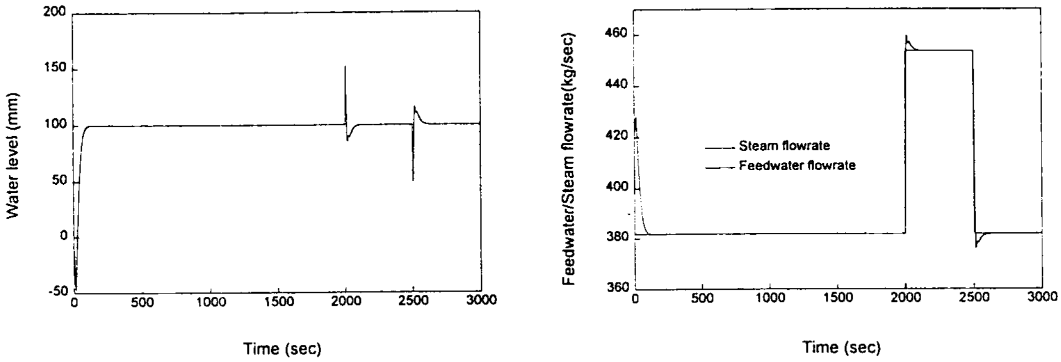
because it has maximum overshoot 1.35 and the long settling time of about 70 seconds. On the other hand, the time response of the digital control system with the sampling period 0.3 second is also not satisfactory because it has such a large rise time of about 100 seconds. The situations at the other specific powers are almost similar to that of 100% power.

As a fact, within the operating range of reactor power, the sampling periods which give the best performance in time and frequency domains are a little smaller or larger than 1 second. The sampling period furthermore cannot be changed during operation only for the sake of the control performance of digital control system. Therefore, the sampling period of the digital control system which is able to give good performance over all operating power ranges must be chosen. Taking into account of this fact, the sampling period 0.3 second is possibly chosen based on the traditional method in Table 3 and on the other hand, the sampling period 1 second is possibly chosen based on the suggested method in Table 4.

Fig.6 shows the performance of level tracking of the digital control system with the 1 second sampling period in Irving's steam generator model with the disturbances of the step increase of steam flowrate from 1435(kg/s) to 1505(kg/s) at reactor power 100% at the time 2000 seconds and the step decrease from



(a) Reactor Power 100%



(b) Reactor Power 30%

Fig. 6. Level Responses and Feedwater/Steam Flowrate of Irving's S/G Model With Ts=1sec Under the Disturbance of Steam Flowrate

1505(kg/s) to 1435(kg/s) at the time 2500 seconds. The water level set point regulating performances of the digital control system with each sampling period suggested by two methods are compared at the digital control simulation system for KORI-2 nuclear power plant steam generator level control at the step increase of reactor power from 80% to 90% and the results are shown in Fig.7. In Fig.7, the performances of two types of controllers, a PI controller and a fuzzy controller, are both displayed and the result of PI control performance is only meaningful to this paper. The performance of the level control system with the sampling period 1 second is better than that of one with the sampling period 0.3 second. Consequently we conclude that the optimal sampling period of the digital control system for KORI-2 nuclear power plant steam generator level control is 1 second.

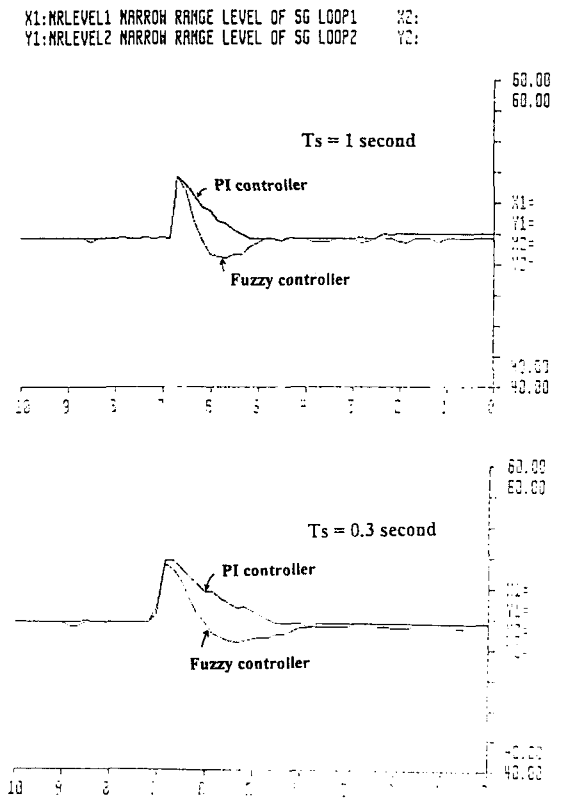


Fig. 7. Comparison of the Level Set Point Regulating Performance of the Digital Control System With the Sampling Periods Recommended by two Methods in KORI-2 Nuclear Power Plant Steam Generator Simulator, at the Step Increase of Rx Power 80% → 90%

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