Journal of the Korean Nuclear Society
Volume 27, Number 5, October 1995

Elastic Stiffness Analysis of Leaf Type Holddown Spring Assemblies

Hyun Tae Lim and Jae Won Kim
Korea Nuclear Fuel Company

Kee Nam Song
Korea Atomic Energy Research Institute
(Received April 29, 1995)

판형홀드다운 스프링 집합체의 탄성 강성도 해석

임현태 · 김재원
한국원전연료주식회사

송기남
한국원자력연구소
(1995. 4. 29 접수)

Abstract

A general method is proposed for elastic stiffness analysis of the leaf type holddown springs using only the geometric data and Young's modulus of the springs. In this method, an engineering beam theory and Castigliano's theory are applied to elastic stiff analysis of the leaf type holddown springs. To show reliability and effectiveness of this method, the elastic stiffness from the proposed method is compared with test results and from the comparison, the proposed method has been proven to be effective for estimating the elastic stiffness of the leaf springs.

요 약

Young's Modulus와 단지 기하학적 데이타를 이용하여 홀드다운스프링의 탄성강성도를 해석하는 방법을 제시 하였다. 제시된 이 방법은 엔지니어링 빔이론과 카스티릴아노 이론을 이용하여 판형홀드다운스프링의 탄성강성도 해석에 적용하였다. 이러한 방법의 효율성과 신뢰성을 보여주기 위하여 제안된 방법으로 부터의 탄성강성도를 여러가지 형태의 홀드다운스프링의 시험결과와 비교하였다. 이러한 결과비교에 의해 제안된 방법이 판형홀드다운스프링의 탄성강성도를 구하는데 있어서 효과적임을 입증하였다.

1. Introduction

There are two functions of the holddown spring which is stuck to the top of the fuel assembly. First, it prevents perpendicular drag upon the fuel assembly which is caused by coolant flow in reactor core during plant operation. Second, it can receive thermal expansion difference between reactor and fuel assembly structure and neutron irradiation growth of the fuel assembly. The holddown spring should be designed for possessing enough stiffness and force of restitution to maintain spring function during the lifecycle. It is of very important consideration in spring design to understand elastic stiffness of the leaf type holddown spring because elastic stiffness is one of the basic parameters affecting the characteristic curve of the holddown spring. However, it has not been easy to analytically obtain the elastic stiffness of leaf type holddown spring because its shape and structure for most fuel vendors is not geometrically arranged as shown in Fig. 1, and it is composed of 3 or 4 leaf springs. So Westinghouse company[1]) developed experimental correlations on the elastic stiffness of each leaf spring from much experience of designing and manufacturing, and they applied it to designing of their own leaf type spring. The KWU company in Germany[2]) each considered the leaf spring as a simple cantilever, obtained elastic stiffness of each leaf spring, and then they used this analysis in the primary design of leaf type holddown spring. Recently some researchers[3]) have studied stiffness analysis of leaf type holddown by using finite element method. But these methods are so simple and difficult for predicting the characteristics of holddown spring when spring design change is requested that a general method to enable prediction of the elastic stiffness of leaf type holddown springs is required. In this paper, considering the comparatively large slenderness ratio of leaf spring, the general method enabling the prediction of elastic stiffness for assembled leaf spring and each leaf spring was proposed. This method is based on the engineering beam theory and Castigliano's theorem.

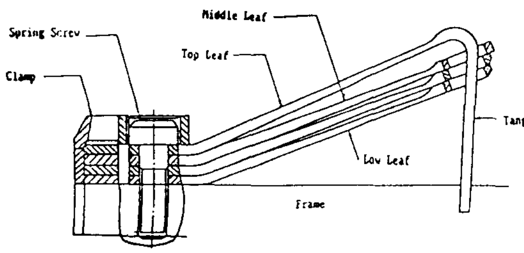

Fig. 1. Leaf Holddown Spring

2. Theory

2.1. Each Leaf Total Strain Energy by the Bending Moment

Assuming frictionless contact on each leaf, loads acting on each leaf are like Fig 2. and elastic strain energy(U) due to bending moment on each leaf can be obtained as follows.

$$U_{TOTAL} = U_{\mathrm{I}} + U_{\mathrm{II}} + U_{\mathrm{III}} + U_{\mathrm{IV}} = \sum_{i=I}^{IV} \int \frac{M_i^2}{2EI_i} ds \tag{1}$$

Here, Ⅰ, Ⅱ, Ⅲ, Ⅳ represent the region in Fig 2.

2.1.1. Bending Moment and Inertia Moment at Each Region

Bending moment and inertia moment at each re-

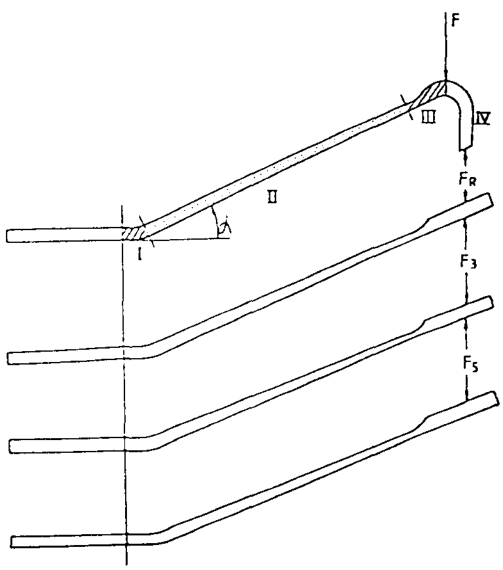

Fig. 2. Reaction Force Acting on the Each Spring

gion of leaf spring can be obtained from equilibrium condition in the free-body diagrams. (refer Fig. 3−Fig. 8).

(a) Region I

$$M_I = \{(L+a)\cos\alpha + R\sin\alpha + R\sin(\theta-\alpha)\}F - \{(L+c)\cos\alpha + R\sin\alpha + R\sin(\theta-\alpha)\}F_R \quad (0 \leq \theta \leq \alpha) \qquad (2)$$

$$I_I = \frac{1}{12}bh^3$$

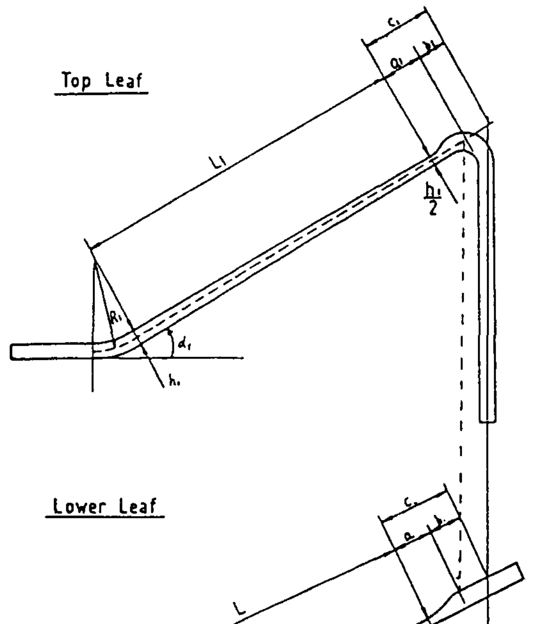

Fig. 3. Variables Used in the Analysis model

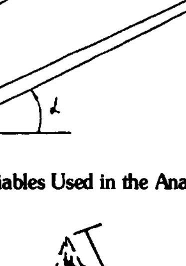

Fig. 4. Bending Moment M_{I} for Region I

(b) Region Ⅱ

Bending moment and inertia moment in tapered region are as follows. (Refer Fig. 5)

$$M_{\mathrm{II}} = (a+x)F\cos\alpha - (c+x)F_R\cos\alpha$$
$$I_{\mathrm{II}} = \frac{1}{96}bh^3\left(1+\frac{x}{L}\right)^3 \qquad (3)$$

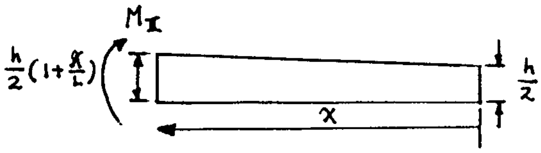

Fig. 5. Bending Moment M_{II} for Region Ⅱ

(c) Region Ⅲ

Bending moment and inertia moment in region Ⅲ are as follows. (Refer Fig. 6)

$$M_{\mathrm{III}} = F(a-x)\cos\alpha - F_R(c-x)\cos\alpha$$
$$I_{\mathrm{III}} = \frac{1}{12}b\left\{\gamma(1-\cos\xi) + \frac{h}{2}\right\}^3 \qquad (4)$$
$$x = \gamma\sin\xi \quad (0 \leq \xi \leq \xi_0)$$

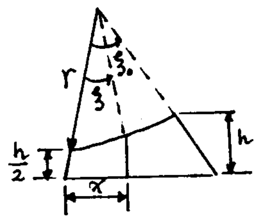

Fig. 6. Bending Moment M_{III} for Region Ⅲ

(d) Region Ⅳ

First leaf: Bending moment and inertia moment are as follows. (Refer Fig. 7)

$$M_{\mathrm{IV},a} = F_R R_{1b}\{1+\sin(\theta_0-\theta)\} - FR_{1b}\sin(\theta_0-\theta)$$

$$M_{\mathrm{IV},b} = F_R R_{1b}(1-\cos\beta)$$
$$I_{\mathrm{IV}} = \frac{1}{12}bh^3 \qquad (5)$$

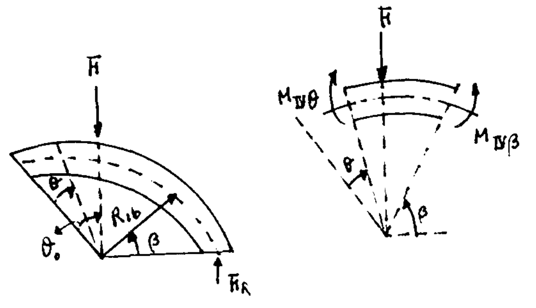

Fig. 7. Bending Moment M_{IV} for Region IV

Second leaf : Bending moment and inertia moment are as follows. (Refer Fig. 8)

$$M_{IV} = F_R \cdot \cos\alpha\,(c-x)$$

$$I_{IV} = \frac{1}{12} b^* h^3 \qquad (6)$$

$$b^* \begin{cases} b, & \gamma\sin\xi_0 < x < (\gamma\sin\xi_0 + d), \\ b-b_0, & (\gamma\sin\xi_0 + d_0) < x \le c \end{cases}$$

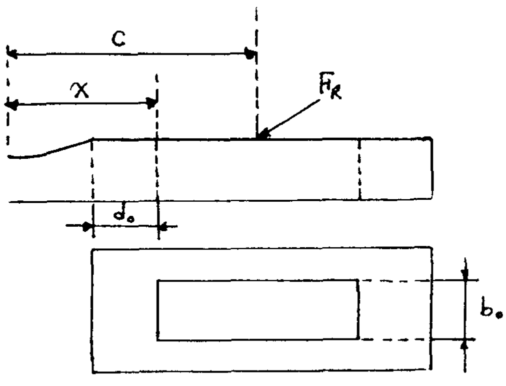

Fig. 8. Bending Moment M_{IV} for Region IV

2.1.2. Total Strain Energy in First Leaf

Total strain energy in first leaf is as follows, from equation (1), (2), (3), (4), (5)

$$U_{1.TE} =$$

$$\frac{6R\alpha}{Ebh^3}(C_1F - C_2F_R)^2$$

$$-\frac{12R^2}{Ebh^3}(1-\cos\alpha)(F-F_R)(C_1F-C_2F_R)$$

$$+\frac{6R^3}{Ebh^3}C_3(F-F_R)^2$$

$$+\frac{48L^3}{Ebh^3}\cos^2\alpha\{C_4(F^2-2FF_R)+C_5F_R^2-C_6F\cdot F_R\}$$

$$+\frac{6\gamma\cos^2\alpha}{Eb}\{F^2\cdot\psi_3 - 2F\cdot F_R\cdot\psi_1 + F_R^2\cdot\psi_2\} \qquad (7)$$

Here

$$\psi_1 = \int_0^{\xi_0}\frac{(a-\gamma\sin\xi)(c-\gamma\sin\xi)^2\cdot\cos\xi}{\{\gamma(1-\cos\xi)+h/2\}^3}d\xi \qquad (8)$$

$$\psi_2 = \int_0^{\xi_0}\frac{(c-\gamma\sin\xi)^2\cdot\cos\xi}{\{\gamma(1-\cos\xi)+h/2\}^3}d\xi \qquad (9)$$

$$\psi_3 = \int_0^{\xi_0}\frac{(a-\gamma\sin\xi)^2\cdot\cos\xi}{\{\gamma(1-\cos\xi)+h/2\}^3}d\xi \qquad (10)$$

$$C_1 = (L+a)\cos\alpha + R\sin\alpha \qquad (11)$$

$$C_2 = (L+c)\cos\alpha + R\sin\alpha \qquad (12)$$

$$C_3 = \frac{\alpha}{2} - \frac{\sin 2\alpha}{4} \qquad (13)$$

$$C_4 = \frac{3a^2}{8L^2} + \frac{a}{4L} + (\ln 2 - \frac{5}{8}) \qquad (14)$$

$$C_5 = \frac{3c^2}{8L^2} + \frac{c}{4L} + (\ln 2 - \frac{5}{8}) \qquad (15)$$

$$C_6 = \frac{b}{4L}(\frac{3a}{L} + 1) \qquad (16)$$

2.1.3. Total Strain Energy in the n+1 th Leaf

$$U_{n+1.TE} = \frac{6R\alpha}{Ebh^3}(C_1F - C_2F_R)^2 - \frac{12R^2}{Ebh^3}$$

$$(1-\cos\alpha)(F-F_R)(C_1F - C_2F_R)$$

$$+\frac{6R^3}{Ebh^3}C_3(F - F_R)^2$$

$$+\frac{48L^3}{Ebh^3}\cos^2\alpha[\,(C_4(F^2$$

$$-2F\cdot F_R) + C_5F_R^2 - C_6\cdot F\cdot F_R]$$

$$+\left(\frac{6\gamma\cos^2\alpha}{Eb}\right)F_R^2\cdot\psi_2 + \frac{2\cos^2\alpha}{Ebh^3}F_R\cdot$$

$$[\,(c-\gamma\sin\xi_0)^3 - (c-\gamma\sin\xi_0 - d_0)^3]$$

$$+\frac{2\cos^2\alpha F_R}{E(b-b_0)h^3}(c-\gamma\sin\xi_0 - d_0)^3 \qquad (17)$$

2.2. Vertical Displacement for Each Leaf Spring

2.2.1. Vertical Displacement of First Leaf Spring by Reaction Force.

$$\delta_{1R} = \frac{\partial U_{1.TE}}{\partial F_R}$$

$$= \frac{1}{Ebh^3}(AA_1F + BB_1F_R) \qquad (18)$$

Here,

$$AA_1 = -12RaC_1C_2 + 12R^2\cdot(1-\cos\alpha)(C_1 + C_2) - 12R^3\cdot C_3 - 48L^3\cos^2\alpha(2C_4 + C_6) - 12\gamma h^3\cos^2\alpha\cdot\psi_1 - 12R^3_{1b}\cdot\left(1 + \frac{\theta_0}{2} - \cos\theta_0 - \frac{\sin 2\theta_0}{4}\right) \qquad (19)$$

$$BB_1 = 96L^3\cos^2\alpha\cdot C_5 + R[\,12\alpha C_2^2 - 24R(1-\cos\alpha)\cdot C_2 + 12R^2\cdot C_3] + 12\gamma h^3\cos^2\alpha\cdot\psi_2 + 12R^3_{1b}\cdot\left[\frac{3}{4}(\pi+\theta_0) - 2\cos\theta_0 - \frac{\sin 2\theta_0}{4}\right] \qquad (20)$$

2.2.2. Vertical Displacement of n+1 th Leaf Spring Due to Reaction Force.

$$\delta_{n+1.R} = \frac{\alpha U_{n+1.TE}}{\partial F_{Ri}}$$

$$= \frac{1}{Ebh^3}BB_{n+1}F_{Ri} \qquad (21)$$

Here,

$$BB_{n+1} = 96L^3\cos^2\alpha\cdot C_5 + R[\,12C_2^2\cdot\alpha - 24C_2\cdot R\cdot(1-\cos\alpha) + 12R^2\cdot C_3] + 12rh^3\cos^2\alpha\cdot\psi_2 + 4\cos^2\alpha\,[\,(c-\gamma\sin\xi_0)^3 - (c-\gamma\sin\xi_0 - d_0)^3 + \frac{b}{b-b_0}(c - \gamma\sin\xi_0 - d_0)^3] \qquad (22)$$

2.2.3. Vertical Displacement of First Leaf Spring Due to External Force, F

$$\delta_{ass} = \frac{\partial U_{1.TE}}{\partial F}$$

$$= \frac{1}{Ebh^3}[\,CC_1F + AA_1F_R] \qquad (23)$$

Here,

$$CC_1 = 12R\alpha C_1^2 - 24R^2\cdot(1-\cos\alpha)C_2 + 12R^3C_3 + 96L^3\cos\alpha\cdot C_4 + 12rh^3\cos^2\alpha\cdot\psi_3 + 12R^3_{1b}\cdot\left(\frac{\theta_0}{2} - \frac{\sin 2\theta_0}{4}\right) \qquad (24)$$

2.3. Constraint Conditions on the Vertical Displacements

When the assembled leaf spring is deformed by load, vertical displacements must be equal on the contact point between leaves, so constraint conditions on the vertical displacements are as follows.

$$-\delta_{1R} = \delta_{2R}$$
$$\delta_{2R} = \delta_{3R} \qquad (25)$$
$$\delta_{3R} = \delta_{4R}$$

2.4. Spring Constant of Assembled Leaf Spring

The stiffness, Kass of assembled leaf spring is as follows, from equation (18), (21), (23), (25).

$$K_{ass} = \frac{F}{\delta_{ass}} = \frac{Ebh^3}{CC_1 - \dfrac{AA_1^2}{BB_1 + \dfrac{1}{\sum_{i=2}^{n}\dfrac{1}{BB_i}}}} \qquad (26)$$

3. Experimental Method

Two kinds of leaf type holddown spring have been used in domestic nuclear power plants. One is called a 14×14 type holddown spring which is composed of one top leaf and two lower leaves, the other is called 17×17, a type of holddown spring which consists of one top leaf and three lower leaves. These two types of holddown spring have different leaf numbers and geometric dimensions, so that the characteristics differ from each other. On the whole the characteristic curve has been determined by characteristic test on the five sets of assembled holddown spring sampled from commercial products.[6,7] Experimental equipment for characteristic testing are schematically illustrated in Fig. 9.

4. Results and Discussions

Using geometric dimensions of two kinds of leaf type holddown spring, elastic stiffness of leaf type holddown spring could be obtained from Eq.(26).

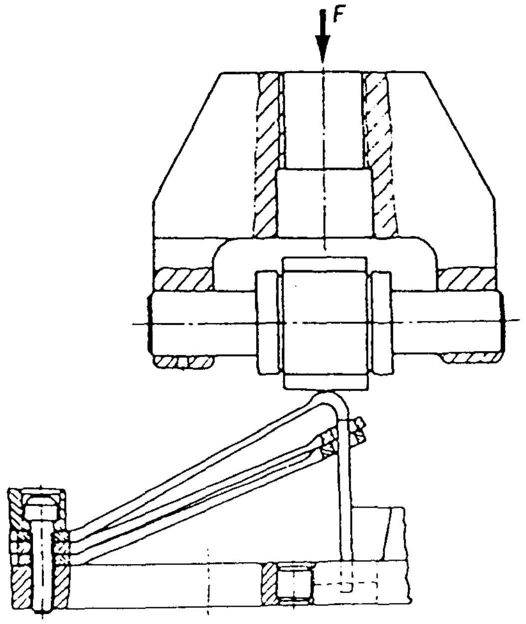

Fig. 9. Experimental Equipments

These values were compared with elastic stiffness from characteristic testing carried out in KNFC. Table 1 shows that elastic stiffnesses of 14×14 leaf type holddown spring obtained from Eq.(26) are about 7.8~12% larger than those from the characteristic test. And Table 2 shows that in the case of 17×17 leaf type holddown spring, elastic stiffnesses from (26) is about 6.6~9.0% larger than that from the characteristic test. Fig. 10 shows that all data are located a little above the line of $K_{estimate} = K_{experiment}$. And there are some discrepancies of elastic stiffnesses from Eq.(26) and characteristic test. This discrepancy may be attributed to the use the geometric dimensions which do not represent the actual dimensions of the specimens used in the characteristic test.

Table 1. Comparison of Calculated Values With Experimental Values for 14×14type Leaf Springs (Domestic Products)

Specimen #	Stiffness K (N/mm)		
	Calculated value	Experimental value	*ERROR(%)
K1	174.67	162	7.8
K2	174.56	161	8.4
K3	175.31	160	9.6
K4	174.72	156	12
K5	175.16	157	12

$* \frac{\text{Cal.} - \text{Exp.}}{\text{Exp.}} \times 100$

Table 2. Comparison of Calculated Values With Experimental Values for 17×17type Leaf Springs (Domestic Products)

Specimen #	Stiffness K (N/mm)		
	Calculated value	Experimental value	*ERROR(%)
K1	143.96	135	6.6
K2	144.38	135	7.0
K3	145.05	135	7.4
K4	143.84	132	9.0
K5	143.86	132	9.0

$* \frac{\text{Cal.} - \text{Exp.}}{\text{Exp.}} \times 100$

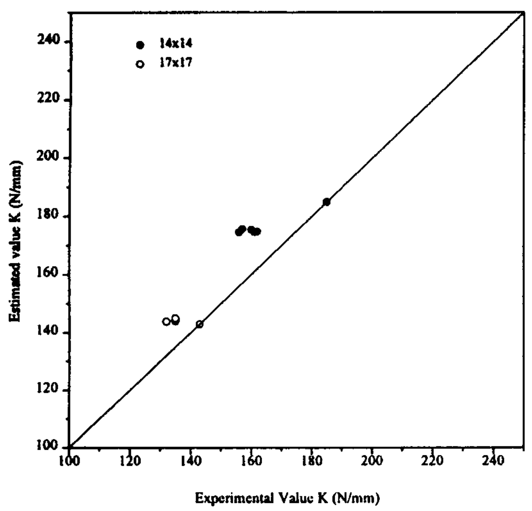

Fig. 10. Comparison of Spring Constants with Calculated Values and Experimental Values for Domestic Leaf Springs

5. Conclusions

A general method for estimating the elastic stiffness of the leaf type holddown spring within the confidence level was developed by an engineering beam theory and Castigliano's theorem. The comparison of elastic stiffnesses from characteristic testing with those estimated by the method in this study shows that the elastic stiffness estimated by the method in this study agrees well with test results. Therefore, it is hoped that the method in this study can be used effectively in preliminary design of leaf spring, and it is estimated that it can reduce cost and time for the final design of leaf spring.

References

1. Gesinski, L., "Mechanical Test and Evaluation of the 17×17 Fuel Assembly," WCAP-8286 (1974)
2. Garter and Gerscha, "determination of Spring characteristics and Fatigue Test on FA Holddown Leaf Spring," KWU Work Report U9/213/87/103 (1987)
3. Carnahan B., Luther H.A. and Wiles J.O, "Appled Numerical Methods," p. 101−112, John Wiley & Sons, New York (1969)
4. Dym, C.L. and Shames, I, H, "Solid Mechanics; A Variational Approach," p. 145−150, McGraw-Hill, New York (1973)
5. Paul J. Sipush, "PWR Fuel Rod Design and Fuel Assembly Design," Lecture Notes, KAERI, Dae Jeon (1984)
6. "Fuel Design Report for 14×14 Fuel Assembly," KAERI, September (1987)
7. "Fuel Design Report for 17×17 Fuel Assembly," KAERI, September (1987)