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Geometric Geoid Determination in South Korea using GPS/Levelling Data

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ABSTRACT

This paper describes the determination of geoid using height data measured by GPS and Spirit Levelling. The GPS data of the 88 stations were used to determine the geoid undulation (N) which can be easily obtained by subtracting the orthometric height(H) from the ellipsoidal height(h). From the geoid undulation (N) calculated at each station mentioned above, geoid plots with a contour interval of 0.25 m were drawn using two interpolation methods. The following interpolation methods were applied and compared with each other: Minimum Curvature Method and Least Squares Fitted Plane. Comparison between geometric geoid and gravimetric geoid undulation by FFT technique was carried out.

要 旨

본 논문은 GPS로부터 측정된 타원체고와 Spirit Levelling으로부터 측정한 정표고를 이용하여 기하학적인 방법에 의하여 지오이드를 결정하였다. 기하학적인 지오이드고의 계산을 위하여 88점의 GPS측정데이타를 사용하였으며, 지오이드고는 간단하게 타원체고와 정표고의 차이를 구하고 보간법에 의하여 지오이드면을 결정하였다. 또한 보간법의 정확도를 평가하기 위하여 Minimum Curvature 방법과 Least Square Plane Fit방법을 사용하여 보간한 결과와 실측치를 비교하였으며, FFT를 적용하여 중력학적인 방법으로 결정한 지오이드고와 비교, 분석하였다.

I. INTRODUCTION

The orthometric height is used extensively in the field of surveying, mapping and large civil engineering projects, e.g., for construction of railways, highways, bridges and long tunnels, as a basis for height systems throughout the world. The heights for the mentioned above have been acquired from the spirit levelling method. It is possible to obtain accurate ellipsoidal height differences by GPS technique.

Especially for the civil engineering work, the main purpose of GPS is to establish a control point, serving as a stable framework for checking and setting out relevant points of the objects under construction. Although GPS positioning yields 3 dimensional coordinates, heights derived from 3-D coordinates cannot be directly used as conventional heights derived from levelling.[4,5]

Here, ellipsoidal height can only be used if the separation (relative to an arbitrarily chosen reference point) between the ellipsoid and the geoid is known with sufficient accuracy. The determination of the spatial structure of this separation (height difference of the geoid relative to the ellipsoid) is important for solving the problem in question.

II. GPS/LEVELLING

The orthometric height of a point on the earth's surface can be determined by GPS observation if the geoid undulation is known at the same point. Let H be the orthometric height (referring to the

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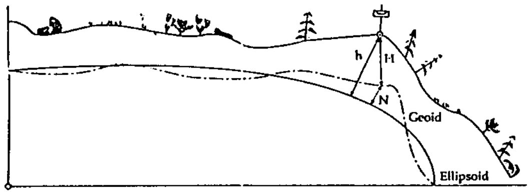

Fig. 1. Relationship between ellipsoidal height, MSL-height and geoid undulation[6]

geoid), h the ellipsoidal height (referring to the introduced reference ellipsoid) and N the geoid undulation, then the relation between these quantities for a certain point on the earth's surface is given by

$$H = h - N \qquad (1)$$

The approximation is due to the non-linearity of the curved plumb line along which the orthometric height(H) is measured. The ellipsoidal height (h) and the geoid undulation (N) are both measured normal to the ellipsoid. The difference in direction is the deflection of the vertical.[6]

Although it is possible to determine the ellipsoidal height and geoid undulation in an absolute sense, the evaluation of relative heights proves to be more accurate. Considering differences in the values of H, h and N between a reference point P of a conventional local height system on an arbitrary point P; equation (1) becomes

$$\Delta H = \Delta h - \Delta N \qquad (2)$$

GPS positioning yields h. If H is to be derived from h, the corresponding values of N must be provided. If a number of control stations exist in a level network that have GPS and orthometric levelling height, then geoid undulations can be obtained directly at the control station using equation (1).

Three or more control stations will define a linear or a non-linear surface of geoid undulation from which values of N can be determined for other stations. If the control stations surround other stations then the process is one of interpolation or if the stations are outside the control stations then the process is one of extrapolation. The surface of geoid undulations can be defined graphically or analytically.

IV. INTERPOLATION OF GEOID UNDULATION

3.1 Minimum Curvature Splines

Minimum curvature first calculates a set of initial estimates for all map locations based on the sampled data values. A surface that interpolates the data and has the least possible change in slope at all points is obtained by minimizing the total curvature.

Formally, minimum curvature surface has been derived as conforming to the variational criterion of smoothness obtained by minimizing the square of Laplacian, or the quadratic variation of the gradient, which leads to the biharmonic or bilaplacian equation[7].

The smoothing continues until successive changes at each map location are less than a specified "maximum absolute deviation", or a maximum number of iterations have been reached. This method can be applied to most data sets, but there is some difficulty if the range of heights is large, this appears as overshoot between data.

Minimum curvature can be extrapolated and tend to a horizontal plane at some distance from the data set.

This tendency can cause some distortion at the edge of the data set. However, a systematic trend in the data can be simultaneously determined and expressed by a low order curvature surface; and adding it to the polynomial surface, the distortion is reduced. For N data, N+3 simultaneous equations are set up as follows (here illustrated for four data)[7]:

$$\begin{bmatrix} 1 & x_1 & y_1 & 0 & c(p_1-p_2) & c(p_1-p_3) & c(p_1-p_4) \\ 1 & x_2 & y_2 & c(p_2-p_1) & 0 & c(p_2-p_3) & c(p_2-p_4) \\ 1 & x_3 & y_3 & c(p_3-p_1) & c(p_3-p_2) & 0 & c(p_3-p_4) \\ 1 & x_4 & y_4 & c(p_4-p_1) & c(p_4-p_2) & c(p_4-p_3) & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 \\ 0 & 0 & 0 & y_1 & y_2 & y_3 & y_4 \end{bmatrix}$$

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (3)$$

This system establishes any linear trend in the data, and builds the minimum curvature surface as departure from that trend which then is added to the surfaces. Although the matrix shown is for only four data, any number of data can be used. For any interpolation point X=(x,y), the interpolated value is[7]

$$F(x,y) = b_o + b_1x + b_2y + a_1c(p_1 - x) + a_2c(p_2 - x) + a_3c(p_3 - x) + a_4c(p_4 - x) \qquad (4)$$

In practice, the process is done on a coarse map grid and repeated from finer grid spacing until the desired grid spacing and smoothness is reached. As with Kriging, the estimated values often exceed the range of the original data values and things can go berserk in areas without sample values.

3.2 Least Squares Plane Fit Method

The simplest surface to generate is a first-order plane surface. This often gives the wrong impression that the plane surface is flat. Actually, it represents the linear relationship between the plane and curved geoid. The analytical function of a plane surface will be of the form.[1]

$$N = Ae + Bn + C \qquad (5)$$

where, A, B and C are coefficients defining the plane. e and n are the easting and northing in a plane coordinate system, N is the geoid undulation at point (e, n)

If more than three points exist then the plane is overdetermined and the coefficients A, B and C can be determined from the least squares method and takes the form,

$$\hat{x} = (A^TA)^{-1}(A^Tl) \qquad (6)$$

where

$$\hat{x} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \quad A = \begin{bmatrix} e_1 & \cdots & n_1 & 1 & 0 \\ e_2 & \cdots & n_2 & 1 & 0 \\ e_3 & \cdots & n_3 & 1 & 0 \\ . & \cdots & . & . & . \\ . & \cdots & . & . & . \\ e_1 & \cdots & n_1 & 1 & 0 \end{bmatrix} \quad l = \begin{bmatrix} N_1 \\ N_1 \\ N_1 \\ . \\ . \\ N_1 \end{bmatrix}$$

The size of the residual at each point and the standard deviation of the residuals gives an indication of how well each point fits the plane.

V. PRACTICAL COMPUTATIONS AND RESULTS

The 88 GPS/Levelling stations measured using Trimble 4000SST receiver were used to determine the geoidal undulation (N) which can be easily obtained by subtracting the orthometric height (H) from the ellipsoidal height (h). The interpolated geoid undulations by the mentioned above two interpolation methods are also compared.

Using the geoid undulation (N) calculated at each station, geoid plots with a contour interval of 0.25m were drawn by minimum curvature method that fits best than others from the test of interpolation methods.

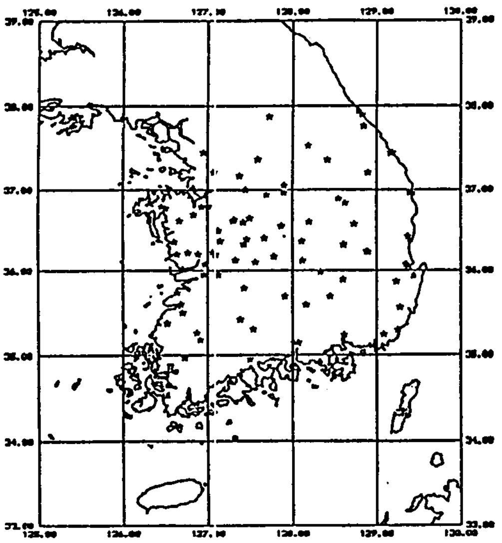

Fig. 2. Distribution of GPS stations

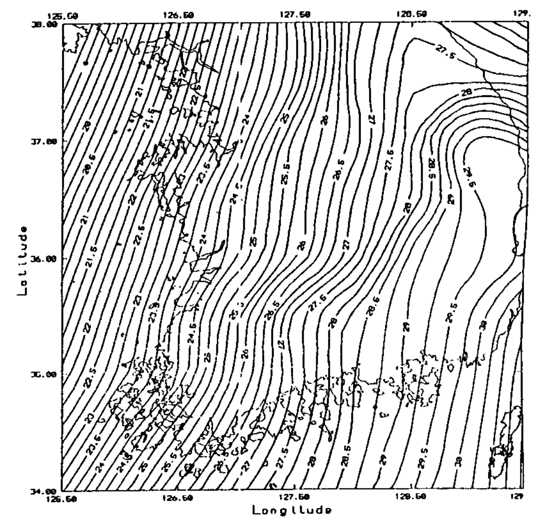

Fig. 3. GPS/Leveling derived geoid undulations. Contour Interval: 0.25[m]

Figure 2 shows the distribution of 88 GPS/Levelling stations in South Korea. Since 1991, Korea Astronomy Observatory (KAO) has measured these GPS stations using Trimble 4000 SST. Figure 3. shows the results of geoid surface computed by minimum curvature method.

Program LESQPL is also used to calculate a least square determined plane surface[1]. The coefficients A, B and C of the plane were computed to be,

Table 2. Statistics of the absolute geoid undulations derived from GPS/Leveling and gravimetric solution using FFT methods. Unit:[m][8]

	Planar FFT Window		Spherical FFT 4 bands
	NO	YES	
Max	1.600	1.400	1.402
Min.	-1.800	-0.703	-0.819
Mean	0.270	0.263	0.240
S.D	0.490	0.437	0.430

Table 1. The statistics of the differences between interpolation methods. Unit :[m]

Interpolation Method	Min.(m)	Max.(m)	Max.-Min.	Mean of Residuals	S.D. of Residuals
Minimum Curvature	22.56	30.21	7.65	-0.035	0.254
Least Square Plane Fit	2261	30.21	7.60	0.000	0.342

A = -.000005597103
B = 0.000024961932
C = 20.8956

with the standard deviation of the residuals being 0. 342 m. The range of residuals was -0.6~+0.7 m, indicating that the "fit" of the plane to the control is not so good fit. The quantitative results of each interpolation method are shown in Table 1. The results are slightly different between each method. To evaluate the quality of geometric geoid, GPS/Levelling was compared with the corresponding geoid undulations derived from both planar and spherical FFT solutions. Using 145×145 gridded residual free-air anomalies with spacing 9.30 km and 7.29 km in x and y sirection respectively, the geoid was determined by planar and spherical FFT[8].

Table 2 shows the comparison of GPS/Levelling and different fast Fourier Transform (FFT) results. Less accurate results were obtained from this numerical test, where as the S.D. of the differences between GPS/Levelling derived heights and corresponding heights derived from spherical FFT with 4 bands reached a value of about 40 cm. Figure 3 shows the geoid rising from the west to the

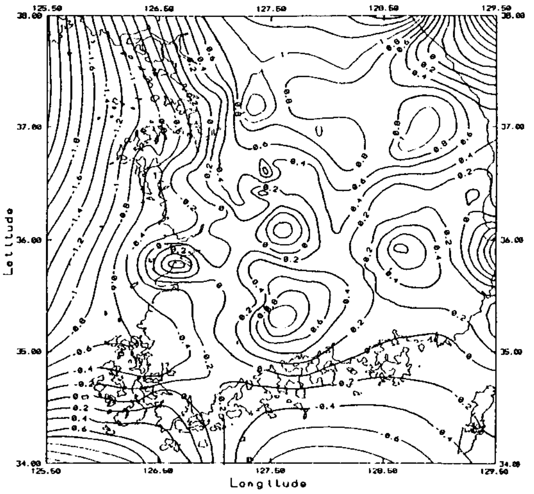

Fig. 4. Distribution of differences between GPS/Leveling and Spherical FFT with 4 band. Contour Interval: 0.2[m]

east, from 22 m to 30 m. Figure 4 shows the error distribution obtained.

IV. CONCLUSION

This study shows that the geometric geoid determination using GPS/Levelling data can be used to obtain geoidal heights and orthometric heights with appropriate interpolation method. The main points are written here.

(1) The interpolating surface can be defined analytically using interpolation method. An indication of how well the plane "fits" to the control station is given by the size of the residuals at each station. (2) From earlier test, minimum curvature method is better fit than others. (3) The difference of geoid undulations from west to east is about 7.65 m.

(4) The standard deviation of residuals obtained by subtracting the measured and interpolated values is about 0.25m. (5) The standard deviation of residuals between the gravimetric solution and each interpolation method is about 40 cm. It is imagined that there are some errors in Spirit Levelling data because Korea is very hilly mountainous area.

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