

직사각형 어레이를 위한 공간체감 방법

A Space-Tapering Approach for a Rectangular Array

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요 약

균일한 소자 (안테나 또는 감지기) 간격으로 계수치를 체감하는 것보다 균일한 계수치로 소자의 간격을 체감하는 것이 실용적이며, 직사각형 어레이에서는, 삼각형 격자 구조가 직사각형 격자구조 보다 소자 수를 줄이는데 더 경제적이다. 접지판 위에 설치된 삼각형 격자 구조를 가진 직사각형 위상어레이의 성능을 향상시키기 위하여 소자간격 체감 방법을 제안하였다. 소자간격 체감이 주빔(main beam)의 폭과 측면로브(sidelobe)의 높이에 미치는 영향을 논의하였다. 제안된 방법을 사용한 결과 측면로브의 성능이 향상되었으나 주빔폭은 약간 넓어지는 것이 밝혀졌다.

ABSTRACT

It is practical to taper the element (e.g., antenna or sensor) spacing with uniform weight rather than to taper the weights with uniform spacing. In a rectangular array, a triangular grid geometry of elements is more economical than a rectangular grid geometry in terms of reducing the number of elements.

A space-tapering approach is proposed to improve the performance of a rectangular phased array with a triangular grid geometry of elements above a ground plane. The effects of space tapering on the main beam width and sidelobe level are discussed. It is shown that the proposed approach improves the sidelobe performance while the main beam width becomes a little broader.

I. Introduction

To improve the transmitting and receiving performance of an antenna/sensor array, the array needs to be designed in such a way that the main beam width is narrow and the sidelobe level is low enough to satisfy prespecified performance criteria. Some factors which affect the performance of the main beam width and sidelobe level are the number of elements, array geometry,

element configuration in a given geometry, type of individual elements, amplitude and phase distribution of the currents feeding the elements, and mutual coupling effects [1].

If the array is large and the cost of each element is expensive, one major concern in the array design is to achieve a desired array performance with fewer number of elements. A practical way of synthesizing such an array is to taper the element spacing with uniform element currents. Another advantage of a space-tapered array is that the mutual coupling effects are not significant due to larger distance between neighboring elements compared to a normal uniformly spaced

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array. The application areas of the array include sonar [2], radar [3], and seismology [4].

In this paper, a space-tapering approach is used to improve the performance of a rectangular phased array with a triangular grid geometry of elements above a ground plane. The effects of space-tapering on the array performance in terms of main beam width and sidelobe level are discussed. It is assumed that the array is large and mutual coupling effects are negligible, and the array is above a ground plane which is sufficiently large such that there are no edge effects due to the ground plane.

II. Performance of Main Beam Width

A. Uniformly Spaced Array

Consider a rectangular array with a triangular grid geometry of isotropic elements as shown in Fig.1. Assuming uniform current of unit magnitude at each element, the array factor steered to (θ_o, ϕ_o) is given by

$$H(\theta, \phi) = \sum_{n=-n_x/4}^{n_x/4} \sum_{m=-n_y/2}^{n_y/2} e^{-j\beta[(2b_n+0.5)d_x(u-u_o) + (b_m+0.25)d_y(v-v_o)]}$$

n and $m \neq 0$

(1)

$$\sum_{n=-n_x/4}^{n_x/4} \sum_{m=-n_y/2}^{n_y/2} e^{-j\beta[(2b_n-0.5)d_x(u-u_o) + (b_m-0.25)d_y(v-v_o)]}$$

n and $m \neq 0$

$$u-u_o = \cos \theta \cos \phi - \cos \theta_o \cos \phi_o \tag{2}$$

$$v-v_o = \cos \theta \sin \phi - \cos \theta_o \sin \phi_o \tag{3}$$

$$b_k = s(k)(|k| - 0.5) \tag{4}$$

$$\beta = 2\pi/\lambda_c \tag{5}$$

θ and ϕ are the elevation and azimuth angles respectively, d_x and d_y are the spacings between neighboring elements in the x -axis, $s(k)$ denotes the sign of k , and λ_c is the wave length for the

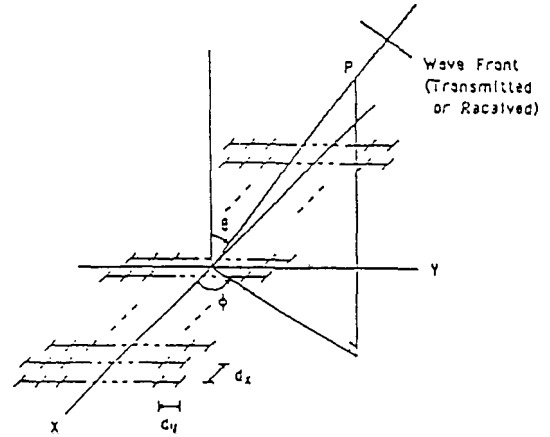


Fig. 1 A rectangular array of a triangular grid geometry.

array center frequency. When the array is steered to (θ_o, ϕ_o) , the amplitudes of the array factor at two half-power angles are related to its maximum value as

$$H(\theta_1, \phi_1) = H(\theta_2, \phi_2) = 0.7071 H(\theta_o, \phi_o) \tag{6}$$

where $\theta_1 = \theta_o - \Delta\theta_1$ and $\theta_2 = \theta_o + \Delta\theta_2$. Note that $F(\theta_o, \phi_o) = n_x n_y$ for unit current at each element. Assuming an array consisting of a large number of elements, which has a narrow main beam, the directional factors can be approximated as

$$u-u_o = \Delta\theta \cos \theta_o \cos \phi_o \tag{7}$$

and

$$v-v_o = \Delta\theta \cos \theta_o \sin \phi_o \tag{8}$$

where $\Delta\theta_1$ and $\Delta\theta_2$ are assumed to be approximately equal to $\Delta\theta$. Expanding $H(\theta_1, \phi_1)$ in a Taylor series assuming a large array with $\Delta\theta$ small [5], we have the half-power beam width θ_h (i.e., $2\Delta\theta$) in the elevation angle plane as

$$\theta_h^2 = \frac{2.3432}{\cos^2 \theta_o [2.3432 \cos^2 \phi_o / \theta_{hxo}^2 + 2.3432 \sin^2 \phi_o / \theta_{hyo}^2] + 0.25\beta^2 d_x d_y \sin \phi_o \cos \phi_o} \quad (9)$$

where θ_{hxo} and θ_{hyo} are the half-power beam widths at broadside in the x and y -axis and are given by

$$\theta_{hxo} = \frac{0.886\lambda_c}{L_x} \quad (10)$$

and

$$\theta_{hyo} = \frac{0.886\lambda_c}{L_y} \quad (11)$$

where L_x and L_y are the array dimensions in wavelength along the x and y -axis respectively. Similarly, it can be shown that the half-power beam width in the plane orthogonal to the elevation angle plane is given by

$$\theta_{ah}^2 = \frac{2.3432}{[2.3432 \sin^2 \phi_o / \theta_{hxo}^2 + 2.3432 \cos^2 \phi_o / \theta_{hyo}^2] - 0.25\beta^2 d_x d_y \sin \phi_o \cos \phi_o} \quad (12)$$

It is observed that the half-power beam width along the elevation angle is inversely proportional to the cosine of the elevation angle from the broadside while the half-power beam width in the plane orthogonal to the elevation angle plane does not depend on the elevation angle. Also, it is shown that the variations of θ_h and θ_{ah} along the azimuth angle depend on the array geometry such that the beam width decreases toward the azimuth angle where the array dimension is large and increases toward the azimuth angle where the array dimension is small.

B. Space-Tapered Array

The array is space-tapered such that the

inter-element spacings in the x and y -axis increase linearly and symmetrically toward the four array sides. The spacings between the x and y directions are given by

$$d_{xm} = (1 + n \alpha_x) d_{xc} \quad (13)$$

and

$$d_{ym} = (1 + n \alpha_y) d_{yc} \quad (14)$$

respectively. The ratios α_x and α_y are obtained from a given array size and center spacing d_{xc} and d_{yc} for each axis and expressed as

$$\alpha_x = 4(n_x - 1)(d_x / d_{xc} - 1) / [n_x(n_x - 2)] \quad (15)$$

and

$$\alpha_y = 4(n_y - 1)(d_y / d_{yc} - 1) / [n_y(n_y - 2)] \quad (16)$$

where n_x and n_y are the number of elements in the x and y -axis respectively and assumed to be even and greater than two. The half-power beam width in the elevation angle plane can be obtained using the same procedure as for the uniform array and is given by

$$\theta_h^2 = \frac{2.3432}{\cos^2 \theta_o [2.3432 \cos^2 \phi_o / \theta_{hxo}^2 + 2.3432 \sin^2 \phi_o / \theta_{hyo}^2] + n_{xs} d_x \beta^2 d_{xc} d_{yc} \sin \phi_o \cos \phi_o / n_x} \quad (17)$$

where

$$n_{xs} = \sum_{n=1}^{n_x/2} (-1)^n [n - 0.5 + n(n-1)\alpha_x / 2] \quad (18)$$

and

$$d\gamma_y = 1 + (n_y / 2 - 1)\alpha_y \quad (19)$$

Also, the half-power beam width in the plane

orthogonal to the elevation angle plane is given by

$$\theta_{ah}^2 = \frac{2.3432}{[2.3432 \sin^2 \phi_o / \theta_{hxo}^2 + 2.3432 \cos^2 \phi_o / \theta_{hyo}^2 - n_{xs} dr_y \beta^2 d_{xc} d_{yc} \sin \phi_o \cos \phi_o / n_x]} \quad (20)$$

It can be shown that if the array is large, the θ_{hxo} and θ_{hyo} in the space-tapered array are approximated as

$$\theta_{hxo}^2 = \frac{2.3432 n_x}{\beta^2 d_{xc}^2 \sum_{n=1}^{n_x/2} 2[n-0.5+0.5n(n-1)\alpha_x]^2} \quad (21)$$

and

$$\theta_{hyo}^2 = \frac{2.3432 n_y}{\beta^2 d_{yc}^2 \sum_{n=1}^{n_y/2} \{2[n-0.5+0.5n(n-1)\alpha_y]^2 + 0.125 dr_x^2\}} \quad (22)$$

The half-power beam widths for a 208×32 uniformly spaced and space-tapered arrays are evaluated to find the performance of the main

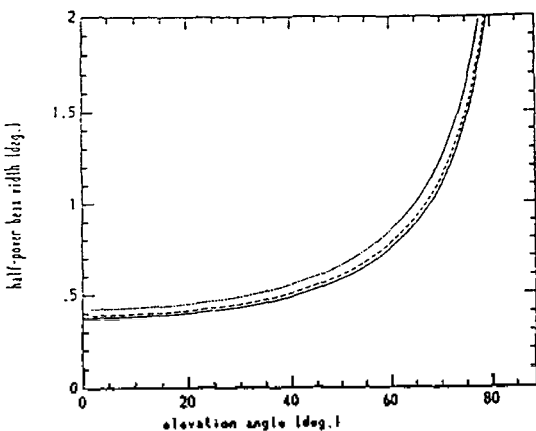


Fig. 2 Half-power beamwidths in terms of steering angle in the x-axis ($\phi_o = 0^\circ$).

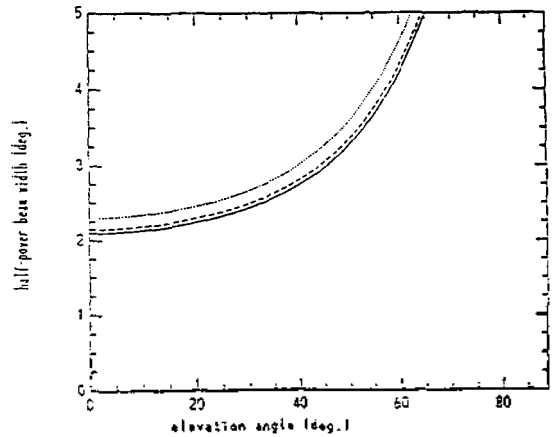


Fig. 3 Half-power beamwidths in terms of steering angle in the y-axis ($\phi_o = 90^\circ$).

beam width. Figs.2 and 3 show the half-power beam width in terms of elevation angle for $\phi_o = 0^\circ$ for the uniformly spaced ($d_x = 0.656\lambda_c$, $d_y = 7.572\lambda_c$) and two space-tapered arrays ($d_{xc} = 0.6\lambda_c$, $d_{yc} = 0.7\lambda_c$; $d_{xc} = 0.55\lambda_c$, $d_{yc} = 0.65\lambda_c$) in the x and y axis. It is shown that the half-power beam width is a little broader in the space-tapered array than that in the uniformly spaced array. The half-power beam width in the elevation angle plane in terms of azimuth angle plane with $\theta_o = 0^\circ$ is shown in Fig.4. It is observed that the half-power beam

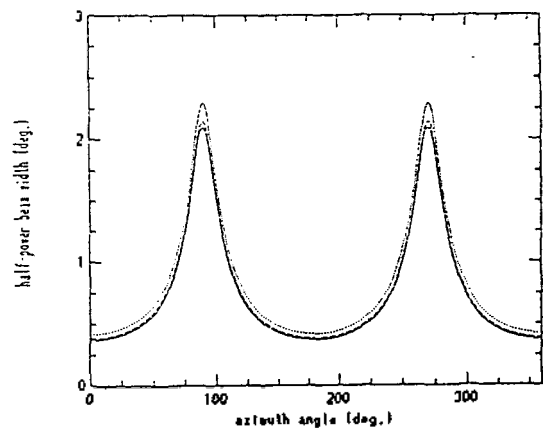


Fig. 4 Half-power beamwidths in terms of azimuth angle with $\theta_o = 0^\circ$.

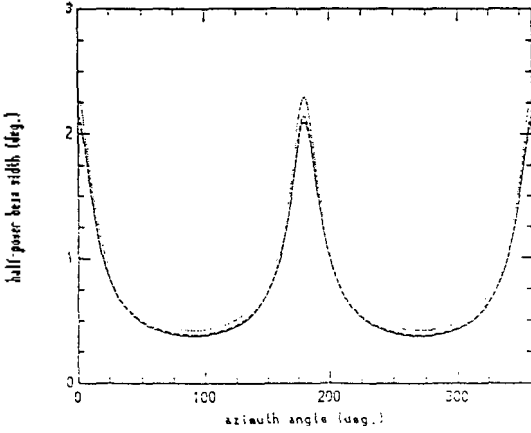


Fig. 5 Half-power beamwidths in the plane orthogonal to the elevation angle plane in terms of azimuth angle.

Note for the figures :

uniformly spaced :

$$d_x = 0.656\lambda_c ; d_y = 0.7572\lambda_c \text{ (solid curve)}$$

space-tapered :

$$d_{xc} = 0.6\lambda_c ; d_{yc} = 0.65\lambda_c \text{ (dotted curve)}$$

$$d_{cx} = 0.55\lambda_c ; d_{cy} = 0.65\lambda_c \text{ (dotted curve)}$$

width in the large dimension (i.e., x -axis) varies less than that in the small dimension (i.e., y -axis) in the space-tapered array. It is shown that the variation in the x -axis is 0.06° and that in the y -axis is 0.2° . Fig.5 shows the half-power beamwidth in the plane orthogonal to the elevation angle plane for the uniformly spaced and the two space-tapered arrays. The similar phenomenon was observed as in the elevation angle plane, except that the maximum and minimum beam widths are interchanged compared to Fig. 4.

III. Sidelobe Performance

The location and level of sidelobes in the x and y -axis are examined in the uniformly spaced and space-tapered rectangular array of a triangular grid geometry of elements. An iterative method is used to find the sidelobe locations and the corresponding sidelobe levels are calculated via the array factor.

A. Uniformly Spaced Array

The array factor of a uniformly spaced array can be expressed as

$$H(\theta, \phi) = \sum_{n=0}^{n_x/2-1} \sum_{m=0}^{n_y-1} \{1 + e^{j\beta[d_x(u-u_0) + 0.5d_y(v-v_0)]}\} e^{j\beta[(2nd_x(u-u_0) + md_y(v-v_0))]} \quad (23)$$

Changing the phase center with normalization, we have the array factor in the x -axis(e.g., $\phi = \phi_0 = 0^\circ$) as

$$H(\theta, 0^\circ) = \sin x / \sin(x/n_x) ; x = n_x \alpha / 4 \quad (24)$$

where

$$\alpha = 2\beta d_x (\sin \theta - \sin \theta_0). \quad (25)$$

The solution of (24) can be obtained iteratively using the Newton's method [6] with its derivative. It can be shown that the iterative equation is given by

$$x_n = x_{n-1} - \frac{x_n \tan(x_{n-1}/n_x) - \tan x_{n-1}}{1/\cos^2(x_{n-1}/n_x) - 1/\cos^2 x_{n-1}}, \quad x_n \neq 0 \quad (26)$$

and

$$x_0 = \pm (2m+1)\pi/2 + \Delta x, \quad 1 \leq m < (n_x-1)/2 \quad (27)$$

where Δx is a small real number which is determined by convergence considerations. Note that $x_n = 0$ corresponds to the location of the main beam. A root x_r can be found by applying (26) iteratively with an initial value x_0 . Then the angular location of a sidelobe is given by

$$\theta_x = \sin^{-1}[\lambda x_r d_x] + \sin \theta_0, \quad (28)$$

$$\theta_x = \sin^{-1}[\lambda x_r / (\pi n_x d_y)] + \sin \theta_0.$$

In (28), it is observed that the angular distance of the sidelobe from the steering direction is inversely proportional to element spacing and the number of elements.

It can be shown that the iterative equation for the locations of sidelobes in the y -axis (i.e., $\phi = \phi_0 = 90^\circ$) is given by

$$x_n = x_{n-1} - \frac{2n_y \tan(x_{n-1}/(2n_y)) - \tan x_{n-1}}{1/\cos^2(x_{n-1}/(2n_y)) - 1/\cos^2 x_{n-1}} \quad (29)$$

and the location of the sidelobes is given by

$$\theta_y = \sin^{-1} \left[\frac{\lambda x_y}{\pi n_y d_y} + \sin \theta_0 \right] \quad (30)$$

B. Space-Tapered Array

If the array is space-tapered as in Section 2. B, the array factor in the x -axis can be expressed as

$$H_x(\theta, 0^\circ) = \sum_{n=1}^{n_x/2} \cos b_{xn} u \quad (31)$$

where

$$b_{xn} = [n - 0.5 + n(n-1)\alpha_x/2] \beta d_{xc} \quad (32)$$

and

$$u = \sin \theta - \sin \theta_0 \quad (33)$$

It can be shown that the iterative equation and the location of the sidelobes are given by

$$u_n = u_{n-1} - \frac{\sum_{n=1}^{n_x/2} b_{xn} \sin(b_{xn} u_{n-1}) / \sum_{n=1}^{n_x/2} b_{xn}^2 \cos(b_{xn} u_{n-1})}{\quad} \quad (34)$$

and

$$\theta_x = \sin^{-1}(u_x + \sin \theta_0) \quad (35)$$

Similarly, the iterative equation and sidelobe locations in the y -axis are given by

and

$$\theta_y = \sin^{-1}(v_y + \sin \theta_0) \quad (37)$$

where

$$a = [1 + (n_y - 2)\alpha_y/2] \beta d_{yc}/4 \quad (38)$$

$$b_{yn} = [n - 0.5 + n(n-1)\alpha_y/2] \beta d_{yc} \quad (39)$$

$$v = \sin \theta - \sin \theta_0 \quad (40)$$

The initial value of u (or v) can be roughly determined from x_0 in (27).

The location and level of the first and second sidelobes for the beam pattern of the 208×32 array are evaluated with respect to three element conditions : isotropic element in free space, isotropic element $\lambda_c/4$ above ground plane, and dipole $\lambda_c/4$ above ground plane. It is observed that for a center spacing of $d_{xc} = 0.6\lambda_c$ and $d_{yc} = 0.7\lambda_c$, the first and second sidelobes decrease by about 1.3 and 0.5 decibels respectively in the x -axis and 1.2 and 0.4 decibels respectively in the y -axis compared with those of the uniformly spaced array and the amount of decrease is insensitive to the steering angle. For the case of $d_{xc} = 0.55\lambda_c$ and $d_{yc} = 0.65\lambda_c$, the first and second sidelobes decrease by about 2.7 and 0.7 decibels respectively in the x -axis and 2.3 and 0.7 decibels respectively in the y -axis. It is shown that the first sidelobe decreases about three times more than the second sidelobe for all cases. From these results, it is found that the space-tapered array is more efficient in counteracting the interferences incident at the angular region of the first sidelobe than the uniformly spaced array. It is observed that the effect of

$$v_n = v_{n-1} - \frac{a \tan(av_{n-1}) \sum_{n=1}^{n_y/2} \cos(b_{yn} v_{n-1}) + \sum_{n=1}^{n_y/2} b_{yn} \sin(b_{yn} v_{n-1})}{a \sum_{n=1}^{n_y/2} b_{yn} \sin(b_{yn} v_{n-1}) - 2a \tan(av_{n-1}) \sum_{n=1}^{n_y/2} b_{yn} \sin(b_{yn} v_{n-1}) + \sum_{n=1}^{n_y/2} b_{yn}^2 \cos(b_{yn} v_{n-1})} \quad (36)$$

Table 1. First sidelobe level : uniformly spaced ; x -axis :

$$d_x = 0.656\lambda_c, d_y = 0.7572\lambda_c ; (\theta_o, \phi_o) = (0^\circ, 0^\circ)$$

	left	right
location	0.6006°	-0.6006°
isotropic element in free space (decibel)	-13.2608	-13.2608
isotropic element $\lambda_c/4$ above ground plane (decibel)	-13.2608	-13.2608
dipole $\lambda_c/4$ above ground plane (decibel)	-13.2615	-13.2615

Table 2. First sidelobe level : space-tapered ; x -axis :

$$d_x = 0.6\lambda_c, d_y ; (\theta_o, \phi_o) = (0^\circ, 0^\circ)$$

	left	right
location	0.6082°	-0.6082°
isotropic element in free space (decibel)	-14.5349	-14.5349
isotropic element $\lambda_c/4$ above ground plane (decibel)	-14.5349	-14.5349
dipole $\lambda_c/4$ above ground plane (decibel)	-14.5356	-14.5356

Table 3. First sidelobe level : space-tapered ; x -axis :

$$d_{xc} = 0.55\lambda_c, d_{yc} = 0.65\lambda_c ; (\theta_o, \phi_o) = (0^\circ, 0^\circ)$$

	left	right
location	0.6146°	-0.6146°
isotropic element in free space (decibel)	-15.9148	-15.9148
isotropic element $\lambda_c/4$ above ground plane (decibel)	-15.9148	-15.9148
dipole $\lambda_c/4$ above ground plane (decibel)	-15.9155	-15.9155

Table 4. Second sidelobe level : space-tapered ; x -axis :

$$d_{xc} = 0.55\lambda_c, d_{yc} = 0.65\lambda_c ; (\theta_o, \phi_o) = (10^\circ, 0^\circ)$$

	left	right
location	11.9511°	-8.9522°
isotropic element in free space (decibel)	-18.5196	-18.5196
isotropic element λ_c above ground plane (decibel)	-18.5208	-18.5187
dipole λ_c above ground plane (decibel)	-18.5638	-18.4801

dipole pattern on the sidelobes is negligible. Some results for relevant sidelobe locations and levels are shown in Tables 1-4.

IV. Conclusions

The effects of space-tapering on the main beam width and the sidelobe levels are discussed with

respect to uniformly spaced and space-tapered rectangular phased arrays of a triangular grid geometry. It is shown that the first sidelobe of the space-tapered array is noticeably reduced compared with the uniformly spaced array while yielding a little broader main beam. It is found that the space-tapering approach is more effective in a large array in terms of achieving a better

performance of the sidelobe with a less increase of the main beam width. It is to be noted that since the mutual coupling effects get reduced as element spacing increases, the space-tapered array will yield a better sidelobe performance than the uniformly spaced array in the presence of mutual coupling effects.

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