▩ 연구논문

Comparison of the Kaplan-Meier and Nelson Estimators using Bootstrap Confidence Intervals

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Abstract

The bootstrap confidence intervals are a computer-based method for assigning measures of accuracy to statistical estimators. In this paper we examine the small sample behavior of the Kaplan-Meier and Nelson-type estimators for the survival function using the bootstrap and asymptotic normal-theory confidence intervals. The Nelson-type estimator is nearly always better than the Kaplan-Meier estimator in the sense of achieved error rates. From the point of confidence length, the reverse is true. Also, we show that the bootstrap confidence intervals are better than the asymptotic normal-theory confidence intervals in terms of achieved error rates and confidence length.

1. Introduction

In recent years, the nonparametric estimation of the survival(reliability) function has become an important topic in industrial life testing and medical research. The two popular estimators were proposed by Kaplan & Meier(1958) and Nelson(1972) and are applicable when dealing with censored data. Kaplan & Meier derived the product limit estimator or Kaplan & Meier estimator(KME) of the survival function and showed that the KME reduced to the usual empirical survival function in the absense of censoring. Large sample properties of the KME have been studied by Breslow & Crowley(1974) and Peterson(1977). Nelson introduced the estimator for the cumulative hazard function under random censoring model. Aalen(1978) and Anderson & Borgan(1985) investigated the asymptotic properties of the estimator of the cumulative hazard function using the counting process approach.

Whereas much is known about the asymptotic properties of the KME and the Nelson-type estimator(NE) of the survival function, the small sample performance, particularly interest to the applied statistician, is neglected. Asymptotic properties suffer from the drawback of being valid only for large samples to be correct. The computer-intensive interval estimation procedure known as the bootstrap(Efron, 1979 & 1981) is designed to avoid this drawback.

In this paper we investigate small sample performances of the KME and NE for the survival function using the asymptotic normal-theory and bootstrap methods in terms of achieved error rates and confidence length. In Section 2, we construct approximate confidence intervals based on the bootstrap and asymptotic normal-theory using the KME and NE. In Section 3, the actual coverage probabilities, length, and shape of these intervals are examined by computer simulations and recommendations are made for their use.

2. Bootstrap and Normal-Theory Confidence Intervals

Let $X_1, X_2, \cdots, X_n$ be a random sample from a continuous distribution function $F(x)$ on $[0, \infty)$. Let $S(x)=1-F(x)$ denote the survival function. Let $Y_1, Y_2, \cdots, Y_n$ be a random sample from a continuous distribution function $G(y)$. Suppose that X_i is assumed to be independent of Y_i for each i. In random censoring model, the true survival times X_i's are censored on the right by the censoring times Y_i's, so that we can only observe (Z_i, δ_i), where

$$Z_i = \min(X_i, Y_i)$$

and
$$\delta_i = \begin{cases} 1, & \text{if } X_i \le Y_i \\ 0, & \text{if } X_i > Y_i, \end{cases} \qquad (1)$$

for $i=1, \cdots, n$.

Let $Z_{(1)} < \cdots < Z_{(n)}$ denote the ordered observed survival times, and let $\delta_{(1)}, \cdots, \delta_{(n)}$ be their corresponding unordered indicator values.

In reliability and survival analysis, the KME plays an important role and has wide range of applications. The KME of the survival function $S(t)$ is defined by

$$\hat{S}_{KM}(t) = \prod_{i:Z_{(i)} \le t} \left(\frac{n-i}{n-i+1}\right)^{\delta_{(i)}} \qquad (2)$$

On the other hand, there are some comparable estimators of the KME. Nelson proposed the estimator $\hat{\Lambda}(t)$ for the cumulative hazard function $\Lambda(t)$ as follows

$$\hat{\Lambda}(t) = \sum_{i:Z_{(i)}\le t} \frac{\delta_{(i)}}{n-i+1}. \tag{3}$$

From the relationships $\Lambda(t) = -\log S(t)$, the Nelson-type survival estimator by substituting $\hat{\Lambda}(t)$ for $\Lambda(t)$ can be considered as

$$\hat{S}_N(t) = e^{-\hat{\Lambda}(t)}. \tag{4}$$

It is known that the estimators $\hat{S}_{KM}$ and $\hat{S}_N$ are asymptotically unbiased and uniformly strongly consistent, and when properly normalized that they converge weakly to the Gaussian process(Breslow & Crowley). That is, the asymptotic distribution of the estimators $\hat{S}_{KM}$ and $\hat{S}_N$ is

$$N(S(t), \frac{S(t)^2}{n}\int_0^t \frac{dF(u)}{(1-F(u))^2(1-G(u))}). \tag{5}$$

Thus we have approximate confidence interval with coverage $1-2\alpha$ based on the large-sample normal theory of the KME as follows:

$$(\hat{S}_{KM}(t) - z^{(1-\alpha)}\cdot\sqrt{\widehat{Var}(\hat{S}_{KM}(t))},\ \hat{S}_{KM}(t) + z^{(1-\alpha)}\cdot\sqrt{\widehat{Var}(\hat{S}_{KM}(t))}) \tag{6}$$

where $z^{(\alpha)}$ is the 100 αth percentile point of a standard normal distribution, and

$$\widehat{Var}(\hat{S}_{KM}(t)) = \hat{S}^2_{KM}(t)\sum_{i:Z_{(i)}\le t}\frac{\delta_{(i)}}{(n-i)(n-i+1)}.$$

In the similar manner, we can obtain an approximate normal-theory interval with coverage $1-2\alpha$ based on the NE.

Although large-sample normal theory confidence intervals are easier to compute, more accurate approximate confidence intervals can often be obtained, especially for relatively small-sample situations. To achieve this purpose, we use approximate confidence intervals based on bootstrap method such as pecentile method, bias-corrected percentile (BC) method, and biased correct accerlation (BC_a) method. This is a simulation-based resampling procedure of the available data that uses a Monte Carlo estimator of the distribution of an appropriate estimating statistic to set confidence limits for an unknown quantity of interest.

In order to obtain bootstrap confidence intervals, it is necessary to employ the following algorithm. (a) We draw a bootstrap sample $(Z_1^*, \delta_1^*), \cdots, (Z_n^*, \delta_n^*)$ by independent sampling n times with replacement from the distribution putting mass $1/n$ at each point (z_i, δ_i); (b) Evaluate the bootstrap replications of $\hat{S}_{KM}(t)$ and $\hat{S}_N(t)$ corresponding to each bootstrap sample, say $\hat{S}_{KM}^*(b)$ and $\hat{S}_N^*(b)$ $(b=1, 2, \cdots, B)$, respectively.

Let $\hat{S}_{KM}^{*(\alpha)}$ indicate the 100αth percentile of B bootstrap replications $\hat{S}_{KM}^*(1), \cdots, \hat{S}_{KM}^*(B)$. The percentile interval $(\hat{S}_L, \hat{S}_U)$ of coverage $1-2\alpha$, is obtained directly from these percentiles,

$$\text{percentile method} : (\hat{S}_L, \hat{S}_U) = (\hat{S}_{KM}^{*(\alpha)}, \hat{S}_{KM}^{*(1-\alpha)}). \quad (7)$$

For example, if $B=1000$ and $\alpha=0.05$, then the percentile interval $(\hat{S}_{KM}^{*(0.05)}, \hat{S}_{KM}^{*(0.95)})$ is the interval extending the 50th to the 950th ordered values of the 1000 numbers $\hat{S}_{KM}^*(b)$, $b=1, 2, \cdots, 1000$.

Now, BC_a interval of coverage $1-2\alpha$, is given by

$$BC_a \text{ method} : (\hat{S}_L, \hat{S}_U) = (\hat{S}_{KM}^{*(\alpha_1)}, \hat{S}_{KM}^{*(1-\alpha_2)}), \quad (8)$$

where

$$\alpha_1 = \Phi\left(z_0 + \frac{z_0 + z^{(\alpha)}}{1 - a \cdot (z_0 + z^{(\alpha)})} \right)$$

and

$$\alpha_2 = \Phi\left(z_0 + \frac{z_0 + z^{(1-\alpha)}}{1 - a \cdot (z_0 + z^{(1-\alpha)})} \right)$$

Here $\Phi(\cdot)$ is the standard normal cumulative distribution function. The value of the biased-correction z_0 is obtained directly from the proportion of bootstrap replications less than the original estimate $\hat{S}_{KM}$.

That is,

$$z_0 = \Phi^{-1}\left(\frac{\#\{\hat{S}_{KM}^*(b) < \hat{S}_{KM}\}}{B} \right), \quad (9)$$

where $\Phi^{-1}(\cdot)$ indicate the inverse function of $\Phi(\cdot)$.

There are various ways to compute the acceleration a. Let $(\boldsymbol{z}, \boldsymbol{\delta})_{(i)}$ be the original sample with the ith point (z_i, δ_i) deleted, let $\hat{S}_{KM_{(i)}}$ be the KME using sample $(\boldsymbol{z}, \boldsymbol{\delta})_{(i)}$, and define $\hat{S}_{KM_{(\cdot)}} = \sum_{i=1}^{n} \hat{S}_{KM_{(i)}}/n$.
Thus a simple acceleration constant is

$$a = \frac{\sum_{i=1}^{n} (\hat{S}_{KM_{(\cdot)}} - S_{KM_{(i)}})^3}{6\{\sum_{i=1}^{n} (\hat{S}_{KM_{(\cdot)}} - S_{KM_{(i)}})^2\}^{3/2}}. \tag{10}$$

Particularly, when acceleration a equals zero, we call equation (8) BC interval of coverage $1-2\alpha$.
That is,

$$BC \text{ method} : (\hat{S}_L, \hat{S}_U) = (\hat{S}_{KM}^{*(\alpha_1)}, S_{KM}^{*(1-\alpha_2)}), \tag{11}$$

where

$$\alpha_1 = \Phi(2z_0 + z^{(\alpha)})$$

and $$\alpha_2 = \Phi(2z_0 + z^{(1-\alpha)}).$$

Note that if a and z_0 in equation (8) equal zero, then

$$\alpha_1 = \Phi(z^{(\alpha)}) = \alpha$$
$$\alpha_2 = \Phi(z^{(1-\alpha)}) = 1-\alpha,$$

so that the BC_a interval is reduced to the percentile interval.
Similarly, we can obtain the bootstrap confidence intervals for the NE.

3. Numerical Comparison and Conclusion

Monte Carlo simulation is performed for the following three lifetime distributions: (i) Weibull with scale parameter $\alpha=1$ and shape parameter $\beta=1$ (denote it by Exp(1)) (ii) Weibull with scale parameter $\alpha=1$ and shape parameter $\beta=2$(Weib(1, 2)), and (iii) Weib(1, 0.5). There were chosen to represent hazard rates that are constant, increasing, and decreasing, respectively. Furthermore, we investigate the effects of varing the censoring rates(10%, 30%) and sample sizes

($n=10$, 30, 50, 100). Censoring distributions are used an uniform distribution on $[0, \lambda]$ (Unif(0, λ)) and Exp(λ). The parameter λ of the censoring distribution is calculated by using the numerical integration routine DECADRE in IMSL library so that the portion of censoring, $P(X>Y)$, is equal to 10% or 30% approximately Since simulation results of 10% and 30% censoring rates are similar, we don't report in the tables for 10%.

The simulation procedure is repeated 1000 times in order to get approximate 90% confidence intervals based on asymptotic normal-theory and bootstrap methods of the KME and NE evaluated at $t: F(t)=0.25$, 0.50, 0.75. Bootstrap confidence intervals are based on $B=1000$ bootstrap replications. To observe the properties of each confidence interval, we define the following measures described by its error rate, length, and shape,

$$\textit{error rate} = \frac{\text{number of interval } (\hat{S}_L, \hat{S}_U) \text{ fails to cover the true survival}}{\text{number of simulation}}$$

$$\textit{length} = \hat{S}_U - \hat{S}_L \text{ , and}$$
$$\textit{shape} = (\hat{S}_U - \hat{S})/(\hat{S} - \hat{S}_L).$$

Shape measures the asymmetry of the interval about the point estimate $\hat{S}$.

Shape > 1.00 indicates greater distance from $\hat{S}_U$ to $\hat{S}$ than from $\hat{S}$ to $\hat{S}_L$.

Tables 1-3 illustrate the following findings:

(i) Under the approximate confidence intervals based on the large-sample normal theory, the NE is nearly always better than the KME in the sense of achieved error rates regardless of the censoring rates.

(ii) From the point of confidence length, the KME is better than the NE.

(iii) For the NE, the interval lengths based on bootstrap methods are shorter than those of normal-theory approximation.

(iv) The approximation to the nominal confidence level 90% of bootstrap methods is often close to or better than that of normal-theory method for both estimators.

(v) The shape's of the all approximate intervals by the bootstrap methods tend to decrease as survival increase.

Bootstrap methods can require even more computing than normal-theory method, and up to hundreds to thousands of times more computing time than using the normal-theory method. However, with high speed computers, even this may not be a severe problem, and the improvement may often be worth the extra cost

〈 Table 1 〉 Achieved error rates, length, and shape for the bootstrap and normal-theory intervals under lifetime distribution Exp(1)

			KME				NE			
sample size	reference time		A	B	C	D	A	B	C	D
10	25th percentile	error rate	0.0920	0.0960	0.1530	0.1350	0.0760	0.0940	0.1480	0.1310
		length	0.5096	0.5000	0.5000	0.5000	0.5470	0.4750	0.4741	0.4741
		shape	1.0000	1.5000	0.6667	0.6667	1.0000	1.5035	0.6695	0.6695
	50th percentile	error rate	0.1660	0.1690	0.0840	0.0930	0.1210	0.1680	0.0920	0.0880
		length	0.5096	0.6000	0.6000	0.6000	0.5470	0.4750	0.4741	0.4741
		shape	1.0000	1.0000	0.5000	0.5000	1.0000	1.5035	0.6695	0.6695
	75th percentile	error rate	0.2680	0.2440	0.2140	0.2140	0.0600	0.1640	0.1710	0.1720
		length	0.4937	0.5000	0.5000	0.5000	0.5694	0.4362	0.4557	0.4362
		shape	1.0000	0.8750	0.8750	0.8750	1.0000	0.9875	0.9070	0.9875
30	25th percentile	error rate	0.1410	0.1300	0.1070	0.0850	0.1400	0.1360	0.1030	0.0850
		length	0.2169	0.2199	0.2297	0.2483	0.2175	0.2159	0.2260	0.2439
		shape	1.0000	0.9366	0.8216	0.7491	1.0000	0.9379	0.8203	0.7494
	50th percentile	error rate	0.1270	0.1200	0.1080	0.1040	0.1230	0.1200	0.1120	0.1060
		length	0.3052	0.3039	0.2996	0.3142	0.3079	0.2977	0.2959	0.3077
		shape	1.0000	0.9947	0.9429	0.9490	1.0000	0.9959	0.9203	0.9479
	75th percentile	error rate	0.1440	0.1240	0.1020	0.1280	0.0810	0.1160	0.1050	0.1200
		length	0.3674	0.3685	0.3682	0.3644	0.3809	0.3582	0.3575	0.3544
		shape	1.0000	1.0150	1.0093	1.0202	1.0000	1.0226	1.0069	1.0009
50	25th percentile	error rate	0.1160	0.1130	0.1000	0.0760	0.1130	0.1160	0.1030	0.0830
		length	0.2326	0.2271	0.2262	0.2277	0.2342	0.2250	0.2237	0.2253
		shape	1.0000	1.0040	0.8581	0.8582	1.0000	1.0044	0.8593	0.8579
	50th percentile	error rate	0.1100	0.1080	0.1020	0.0970	0.1010	0.1100	0.1000	0.0970
		length	0.2358	0.2313	0.2316	0.2264	0.2399	0.2291	0.2289	0.2234
		shape	1.0000	1.0208	0.9323	0.9517	1.0000	1.0152	0.9318	0.9493
	75th percentile	error rate	0.1250	0.1230	0.1110	0.1310	0.0920	0.1160	0.1090	0.1270
		length	0.2287	0.2314	0.2316	0.2248	0.2395	0.2246	0.2258	0.2190
		shape	1.0000	1.0903	1.1433	1.1492	1.0000	1.1003	1.1599	1.1441
100	25th percentile	error rate	0.1130	0.1110	0.1040	0.0950	0.1120	0.1110	0.1040	0.0930
		length	0.1557	0.1578	0.1578	0.1612	0.1561	0.1570	0.1570	0.1603
		shape	1.0000	0.9147	0.9920	0.9841	1.0000	0.9145	0.9912	0.9849
	50th percentile	error rate	0.1080	0.1000	0.1030	0.1030	0.1000	0.1020	0.1010	0.1040
		length	0.1732	0.1768	0.1773	0.1770	0.1744	0.1759	0.1761	0.1761
		shape	1.0000	0.9873	1.0002	0.9974	1.0000	0.9861	0.9984	0.9978
	75th percentile	error rate	0.1020	0.1010	0.0980	0.1120	0.0990	0.1020	0.1060	0.1130
		length	0.1683	0.1707	0.1707	0.1637	0.1721	0.1678	0.1678	0.1622
		shape	1.0000	1.0237	1.0237	1.0262	1.0000	1.0295	1.0295	1.0337

A : asymptotic normal-theory　B : percentile method
C : *BC* method　D : BC_a method

〈 Table 2 〉 Achieved error rates, length, and shape for the bootstrap and normal theory intervals under lifetime distribution Weib(1, 0.5)

sample size	reference time		KME A	KME B	KME C	KME D	NE A	NE B	NE C	NE D
10	25th percentile	error rate	0.1230	0.1160	0.1420	0.1320	0.0980	0.1150	0.1420	0.1320
		length	0.5296	0.5667	0.5000	0.5000	0.5811	0.5401	0.4741	0.4741
		shape	1.0000	1.1795	0.9231	0.9231	1.0000	1.1633	0.8991	0.8991
	50th percentile	error rate	0.1580	0.1500	0.1060	0.1110	0.1180	0.1570	0.0920	0.0740
		length	0.5296	0.5667	0.5000	0.5000	0.5811	0.5401	0.4741	0.4741
		shape	1.0000	1.1795	0.9231	0.9231	1.0000	1.1633	0.8991	0.8991
	75th percentile	error rate	0.1290	0.1060	0.1870	0.1870	0.0570	0.1670	0.1760	0.1740
		length	0.4781	0.5000	0.5000	0.4800	0.5637	0.4566	0.4557	0.4282
		shape	1.0000	1.0833	1.0833	1.0000	1.0000	1.1208	1.1258	1.1497
30	25th percentile	error rate	0.1410	0.1280	0.1110	0.0830	0.1410	0.1310	0.1130	0.0850
		length	0.2162	0.2043	0.2243	0.2443	0.2167	0.2002	0.2192	0.2394
		shape	1.0000	0.7149	0.5808	0.5352	1.0000	0.7138	0.5828	0.5344
	50th percentile	error rate	0.1220	0.1160	0.1060	0.1040	0.1170	0.1200	0.1120	0.1050
		length	0.3028	0.3042	0.3049	0.3393	0.3051	0.2966	0.2973	0.3314
		shape	1.0000	0.9354	0.8895	0.9052	1.0000	0.9369	0.8916	0.9109
	75th percentile	error rate	0.1350	0.1200	0.1030	0.1250	0.0880	0.1180	0.1050	0.1250
		length	0.3863	0.3874	0.3860	0.3855	0.3996	0.3743	0.3755	0.3733
		shape	1.0000	0.9613	0.9468	0.9451	1.0000	0.9705	0.9633	0.9652
50	25th percentile	error rate	0.1240	0.1220	0.1080	0.0900	0.1250	0.1220	0.1090	0.0910
		length	0.2385	0.2324	0.2256	0.2278	0.2403	0.2300	0.2228	0.2253
		shape	1.0000	0.9750	0.8775	0.8949	1.0000	0.9735	0.8768	0.8960
	50th percentile	error rate	0.1080	0.1000	0.0910	0.0910	0.0910	0.1000	0.0930	0.0500
		length	0.2370	0.2261	0.2280	0.2227	0.2416	0.2235	0.2252	0.2209
		shape	1.0000	1.0485	0.9649	0.9568	1.0000	1.0471	0.9630	0.9596
	75th percentile	error rate	0.1270	0.1180	0.1100	0.1240	0.1030	0.1170	0.1130	0.1250
		length	0.2195	0.2198	0.2234	0.2121	0.2299	0.2157	0.2186	0.2052
		shape	1.0000	1.1343	1.1912	1.1627	1.0000	1.1426	1.1918	1.1900
100	25th percentile	error rate	0.1150	0.1140	0.1050	0.0930	0.1140	0.1160	0.1070	0.0950
		length	0.1583	0.1522	0.1543	0.1567	0.1587	0.1515	0.1535	0.1558
		shape	1.0000	0.8944	0.9644	0.9496	1.0000	0.8944	0.9639	0.9490
	50th percentile	error rate	0.1020	0.0990	0.0990	0.0950	0.1000	0.1060	0.0970	0.0960
		length	0.1825	0.1836	0.1834	0.1830	0.1839	0.1827	0.1834	0.1809
		shape	1.0000	0.9762	1.0161	1.0118	1.0000	0.9792	0.9931	1.0030
	75th percentile	error rate	0.0960	0.0940	0.0930	0.1050	0.0890	0.1050	0.0990	0.1090
		length	0.1705	0.1688	0.1688	0.1644	0.1744	0.1670	0.1674	0.1623
		shape	1.0000	1.0330	1.0470	1.0372	1.0000	1.0333	1.0388	1.0448

A : asymptotic normal-theory B : percentile method

C : BC method D : BC_a method

〈 Table 3 〉 Achieved error rates, length, and shape for the bootstrap and normal-theory intervals under lifetime distribution Weib(1, 2)

			KME				NE			
sample size	reference time		A	B	C	D	A	B	C	D
10	25th percentile	error rate	0.0730	0.0870	0.1910	0.1530	0.0680	0.0840	0.1910	0.1530
		length	0.5096	0.5000	0.5000	0.5000	0.5470	0.4750	0.4741	0.4741
		shape	1.0000	1.5000	0.6667	0.6667	1.0000	1.5035	0.6695	0.6695
	50th percentile	error rate	0.1360	0.1930	0.1700	0.2000	0.1150	0.1700	0.1080	0.1340
		length	0.5096	0.5000	0.5000	0.5000	0.5470	0.4750	0.4741	0.4741
		shape	1.0000	1.5000	0.6667	0.6667	1.0000	1.5035	0.6695	0.6695
	75th percentile	error rate	0.2800	0.2570	0.2170	0.2160	0.0600	0.1240	0.1630	0.1630
		length	0.4226	0.4000	0.4000	0.3750	0.5713	0.3758	0.3758	0.3222
		shape	1.0000	1.6667	1.6667	1.5000	1.0000	1.5168	1.5168	1.1580
30	25th percentile	error rate	0.1540	0.1450	0.1140	0.0910	0.1260	0.1450	0.1160	0.0920
		length	0.2042	0.2000	0.2333	0.2333	0.2047	0.1967	0.2295	0.2295
		shape	1.0000	1.0000	0.4000	0.4000	1.0000	1.0000	0.4000	0.4000
	50th percentile	error rate	0.1180	0.1230	0.1210	0.1210	0.1120	0.1190	0.1130	0.1060
		length	0.2809	0.2748	0.2917	0.2995	0.2830	0.2700	0.2870	0.2945
		shape	1.0000	0.9703	0.8193	0.8056	1.0000	0.9715	0.8187	0.8055
	75th percentile	error rate	0.1280	0.1180	0.0930	0.1200	0.0810	0.1070	0.1000	0.1180
		length	0.3343	0.3309	0.3245	0.3313	0.3408	0.3229	0.3159	0.3245
		shape	1.0000	0.9790	0.7217	0.7458	1.0000	0.9851	0.7283	0.7520
50	25th percentile	error rate	0.1380	0.1240	0.1090	0.0900	0.1300	0.1250	0.1090	0.0900
		length	0.2307	0.2280	0.2240	0.2250	0.2323	0.2257	0.2217	0.2227
		shape	1.0000	1.0310	0.8405	0.8446	1.0000	1.0327	0.8405	0.8446
	50th percentile	error rate	0.1150	0.1070	0.1090	0.1090	0.1090	0.1150	0.1100	0.1090
		length	0.2314	0.2156	0.2217	0.2144	0.2353	0.2131	0.2194	0.2122
		shape	1.0000	1.0347	0.9301	0.9613	1.0000	1.0356	0.9326	0.9602
	75th percentile	error rate	0.1200	0.1170	0.1040	0.1210	0.0900	0.1070	0.1040	0.1140
		length	0.2062	0.2144	0.2139	0.1985	0.2199	0.2043	0.2013	0.1892
		shape	1.0000	1.1003	1.0258	1.0110	1.0000	1.1783	1.1017	1.0537
100	25th percentile	error rate	0.0970	0.0990	0.0960	0.0930	0.0970	0.0990	0.0900	0.0780
		length	0.1508	0.1500	0.1600	0.1600	0.1511	0.1493	0.1592	0.1592
		shape	1.0000	0.8750	0.7778	0.7778	1.0000	0.8750	0.7778	0.7778
	50th percentile	error rate	0.1030	0.0980	0.0950	0.0960	0.0930	0.0970	0.0960	0.0980
		length	0.1696	0.1708	0.1702	0.1702	0.1706	0.1698	0.1694	0.1691
		shape	1.0000	0.9966	1.0355	1.0355	1.0000	0.9971	1.0338	1.0369
	75th percentile	error rate	0.0870	0.0890	0.0800	0.0930	0.0800	0.0880	0.0850	0.0910
		length	0.1534	0.1564	0.1570	0.1528	0.1565	0.1549	0.1558	0.1521
		shape	1.0000	1.0778	1.2152	1.1847	1.0000	1.0765	1.2161	1.1951

A : asymptotic normal-theory B : percentile method

C : BC method D : BC_a method

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