▩ 연구논문

Estimator of Mean Residual Life for Some Parametric Families Using Censored Data[+]

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Abstract

In this paper we consider a new estimator of mean residual life(MRL) under the random censorship model, based on the partial moment of the distribution. The parameters of a partial moment are estimated by its maximum likelihood estimators when the underlying distribution is known. Though the new estimator is not a consistent estimator of the MRL, it is shown to have smaller mean squared error than the well known empirical MRL estimator for a parametric family.

We also compare the proposed estimator with some other estimators in terms of MSE for exponential and lognormal distributions using censored data.

1. Introduction

Let $X_1, X_2, \ldots, X_n$ be a random sample from a right continuous distribution function $F(x)$, and survival function $\overline{F}(x) \equiv 1 - F(x)$. we assume that $F(0) = 0$ and mean $\mu = E(X) = \int_0^\infty \overline{F}(x)dx$ is finite.

The mean residual life (MRL) function or remaining life expectancy function at

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age t is defined as

$$m(t) = E(X-t \mid X>t)$$
$$= \begin{cases} \int_t^\infty \overline{F}(x)dx/\overline{F}(t) & \text{if} \quad \overline{F}(t)>0 \\ 0 & \text{if} \quad \overline{F}(t)=0. \end{cases} \tag{1.1}$$

In reliability theory the MRL function arises naturally and is of practical interest in many applications. Guess and Proschan(1988) and Park(1992) provided excellent references on the properties of MRL function and its applications. In some situations one cannot observe subjects entirely. The subject may leave the study or survive to the closing time. Then we may have the right censored lifetime.

Let $C_1, C_2, \cdots C_n$ be a random sample from a censoring distribution function $G(c)$. We assume that C_i is independent of X_i for each i. In the random censorship model, the X_i may be censored on the right by the C_i, so that we only observe the pairs (Y_i, δ_i), $i=1, 2, \cdots, n$,

$$Y_i = \min(X_i, C_i)$$

and

$$\delta_i = \begin{cases} 1 & \text{if} \quad X_i \le C_i \\ 0 & \text{if} \quad X_i > C_i. \end{cases}$$

Under the random censoring model, Yang(1977) and Kumazawa(1987) proposed estimators for $m(t)$ based on Nelson-Aalen estimator and Kaplan-Meier estimator as estimators for survival function, respectively. Also, Park et al.(1993) proposed an estimator for $m(t)$ based on Susarla-Van Ryzin estimator of $\overline{F}$.

In this paper we propose an estimator of the MRL function for some parametric families using randomly censored data. Our approach for estimating the MRL function is based on the partial moment estimator, proposed recently by Choi and Nam(1994) in the case of uncensored data. When the underlying distribution is known, the parameters in the expression of the partial moment approximation of MRL function are replaced by its maximum likelihood estimators(MLE) and as a result, a new parametric estimator of the MRL function can be obtained.

In Section 2, we give the partial moment estimator and the empirical estimator of MRL function for completeness of our discussions and we present the new extimator of the MRL function, using the MLE's of the parameters. In Section 3, we compare the proposed estimator mumerically with Park et al (1993)'s estimator for various combinations of censoring rate and sample size when the life distribution is exponential and the censoring distribution is uniform. In Section 4 the new estimator is compared with the empirical MRL estimator for the lognormal survival distribution in terms of mean squared error.

2. Estimator of MRL Function under Random Censorship Model

The MRL function of (1.1) can be rewritten as

$$\begin{aligned} m(t) &= \overline{F}^{-1}(t) \int_t^{\infty} x dF(x) - t \\ &= \overline{F}^{-1}(t)\{\mu - \int_0^t x dF(x)\} - t \qquad (2.1) \\ &= (1-p)^{-1}\{\mu - \int_0^t x dF(x)\} - t, \end{aligned}$$

where $p = F(t) = P(X \le t)$ and $\int_0^t x dF(x)$ is defined as the first partial moment of X about the origin over $(0, t)$ for fixed t. (cf. Choobineh and Branting, 1986). Choobineh and Park (1990) proposed $p(\mu - (\frac{1-p}{p})^{\frac{1}{2}} \sigma)$ as an approximation to $\int_0^t x dF(x)$ and studied its properties. Thus, by replacing $\int_0^t x dF(x)$ of (2.1) by its partial moment approximation, the approximation of $m(t)$ is obtained as follows.

$$m_p(t) = \mu + [\frac{p}{1-p}]^{1/2} \sigma - t, \qquad (2.2)$$

where μ and σ^2 are the mean and variance of X, respectively. Using (2.2), Choobineh and Park(1990) obtained a new estimator of MRL by substituting $\overline{X}$, S and $\hat{p}$, for μ, σ and p, respectively, in (2.2), where $\overline{X}$ and S^2 are the sample mean and variance, and $\hat{p} = \frac{1}{n} \sum I(X_i \le t)$. Also Choi and Nam(1994) proposed a new estimator(MLE estimator) of MRL by replacing μ, σ and p in (2.2) by their MLE's. When the underlying distribution is known to be either a gamma distribution with large shape parameter or a Weibull distribution and when only a small sample is available, Choi and Nam(1994) showed that the MLE estimator was superior to other estimatiors. Choobineh and Park(1990) and Choi and Nam(1994)s' estimators were proposed in the case of uncensoring model. Now we consider the MLE estimator under random censorship model. Our new estimator of MRL under random censoring model, $\hat{m}_p(t)$, is defined as

$$\hat{m}_p(t) = \hat{\mu} + [\frac{\hat{p}}{1-\hat{p}}]^{\frac{1}{2}}\hat{\sigma} - t \qquad \text{for} \quad 0 \le t < \infty, \tag{2.3}$$

where $\hat{\mu}$ and $\hat{\sigma}^2$ are MLEs of mean and variance of the underlying distribution, respectively, and $\hat{p} = \hat{F}(t) = \int_0^t f(y;\hat{\mu},\hat{\sigma})dy$ is MLE of $F(t)$. Throughout this paper, $\hat{m}_p(t)$ is referred to as the MLE of $m(t)$. The MLE is not a consistent estimator of $m(t)$, but the estimator is somewhat smoothing out the empirical estimator. This fact suggests that the mean squared error of $\hat{m}_p(t)$ is likely to be smaller than that of the empirical estimator, especially when the sample size is small.

For completeness of our discussion, we present a known estimator of the MRL function. The empirical MRL estimator, denoted by $\hat{m}_e(t)$, is obtained by replacing $\overline{F}(y)$ of MRL function of (1.1) by its consistent empirical estimator as follows.

$$\hat{m}_e(t) = \begin{cases} (n-k)^{-1} \sum_{i=k+1}^{n} (Y_{(i)} - t) & \text{for} \quad Y_{(k)} \le t < Y_{(k+1)} \\ 0 & \text{for} \quad t \ge Y_{(n)} \end{cases} \tag{2.4}$$

for $k = 0, 1, 2, \cdots, n-1$; $Y_{(0)} \equiv 0$ and $Y_{(1)} < Y_{(2)} < \cdots < Y_{(n)}$ is the corresponding order statistics of a random sample $Y_1, Y_2, \cdots, Y_n$ from $F(y)$.

3. Estimation of MRL for Exponential Distribution

In this section, we consider the exponential distribution with the probability density function

$$f(x;\lambda) = \lambda \exp(-\lambda x),\ x \ge 0,$$

as a life distribution under the random censoring model. In this case, $\mu = \lambda^{-1}$, $\sigma^2 = \lambda^{-2}$. The objective for considering the exponential distribution is to compare $\hat{m}_p(t)$ with Park et al.(1993)'s estimator $\hat{m}_{s_n}(t)$ in terms of bias and MSE. Note that the MRL of the exponential distribution is known to be equal to λ^{-1}. By the maximum likelihood method, we obtain the following MLE's

$$\hat{\mu} = \hat{\lambda}^{-1} = \frac{1}{n_1} \sum_{i=1}^{n} y_i$$

$$\hat{\sigma}^2 = \left(\frac{1}{n_1} \sum_{i=1}^{n} y_i \right)^2$$

$$\hat{p} = 1 - \exp\{ -\frac{n}{\sum y_i} t \},$$

where n_1 is the number of uncensored observations. Then

$$\hat{m}_p(t) = \frac{1}{n_1} \sum_{i=1}^{n} y_i [1 + \{ \exp (\frac{n_1}{\sum y_i} t) - 1 \}^{1/2}] - t .$$

Now, we compare the behavior of $\hat{m}_p(t)$ and Park et al.(1993)'s estimator ($\hat{m}_{sv}(t)$) The simulation scheme is designed with various combinations of censoring rate (10%, 20%, 30%), different sample sizes(n = 10, 20, 30) and uniform distribution on $[0, \alpha]$ for a fixed $\alpha > 0$, as the censoring distribution. The number of replications for all cases were 1000 times and the estimators and its mean square errors (MSE's) were computed.

〈 Table 1 〉 The values of $\hat{m}_p(t)$ and $\hat{m}_{sv}(t)$ when $\bar{F}(x)$ = Exp(1) and $\bar{G}(x)$ = Uniform(α)

(censoring rate = 10%, α = 10)

t		0.105	0.223	0.357	0.511	0.693	0.916	1.204	1.610
$n=10$	$\hat{m}_p(t)$	1.038(.091)	1.076(.105)	1.091(.114)	1.097(.119)	1.102(.118)	1.119(.112)	1.174(.106)	1.351(.175)
	$\hat{m}_{sv}(t)$	.943(.103)	.942(.121)	.943(.147)	.941(.174)	.926(.198)	.924(.270)	.920(.340)	.907(.393)
$n=20$	$\hat{m}_p(t)$	1.024(.039)	1.062(.047)	1.076(.052)	1.080(.054)	1.082(.054)	1.094(.052)	1.138(.052)	1.284(.095)
	$\hat{m}_{sv}(t)$	.976(.062)	.947(.069)	.970(.082)	.972(.101)	.970(.119)	.965(.159)	.968(.226)	.967(.287)
$n=30$	$\hat{m}_p(t)$	1.029(.028)	1.067(.035)	1.080(.039)	1.084(.041)	1.086(.040)	1.096(.040)	1.137(.042)	1.265(.085)
	$\hat{m}_{sv}(t)$	.977(.040)	.978(.046)	.978(.052)	.976(.062)	.977(.145)	.972(.145)	.927(.203)	.930(.329)

(censoring rate = 20%, α = 5)

t		0.105	0.223	0.357	0.511	0.693	0.916	1.204	1.610
$n=10$	$\hat{m}_p(t)$	.986(.106)	1.021(.118)	1.034(.126)	1.040(.130)	1.047(.129)	1.070(.119)	1.141(.108)	1.364(.224)
	$\hat{m}_{sv}(t)$	.888(.122)	.874(.138)	.866(.154)	.850(.193)	.839(.230)	.841(.283)	.855(.302)	.829(.343)
$n=20$	$\hat{m}_p(t)$	.958(.044)	.991(.047)	1.003(.050)	1.006(.051)	1.010(.050)	1.027(.046)	1.084(0.39)	1.268(.087)
	$\hat{m}_{sv}(t)$	.914(.057)	.910(.067)	.905(.079)	.900(.096)	.887(.120)	.875(.154)	.854(.206)	.849(.296)
$n=30$	$\hat{m}_p(t)$	.957(.030)	.990(.032)	1.002(.034)	1.004(.035)	1.007(.034)	1.023(.031)	1.076(.027)	1.250(.070)
	$\hat{m}_{sv}(t)$	.949(.041)	.948(.048)	.943(.056)	.934(.068)	.922(.083)	.903(.105)	.904(1.56)	.862(.215)

(censoring rate = 30%, α = 10/3)

t		0.105	0.223	0.357	0.511	0.693	0.916	1.204	1.610
$n=10$	$\hat{m}_p(t)$	.974(.169)	1.009(.185)	1.021(.198)	1.027(.205)	1.036(.205)	1.062(.195)	1.143(.182)	1.390(.316)
	$\hat{m}_{sv}(t)$	.835(.120)	.828(.140)	.821(.165)	.796(.198)	.769(.243)	.773(.261)	.747(.291)	.736(.329)
$n=20$	$\hat{m}_p(t)$	.931(.049)	.963(.051)	.974(.054)	.977(.055)	.981(.054)	1.001(.047)	1.065(.038)	1.267(.088)
	$\hat{m}_{sv}(t)$	.871(.070)	.857(.081)	.844(.094)	.833(.113)	.812(.139)	.799(.180)	.769(.240)	.748(.278)
$n=30$	$\hat{m}_p(t)$	.936(.037)	.968(.038)	.979(.040)	.982(.041)	.985(.040)	1.003(.035)	1.062(.028)	1.251(.073)
	$\hat{m}_{sv}(t)$	.890(.048)	.879(.054)	.868(.064)	.853(.079)	.846(.099)	.809(.127)	.773(.174)	.726(.239)

* $m(t) = 1$
* Values in the parenthesis are MSE's
* The t's were obtained by the inverse of $\overline{F}$, i.e,

$$t = \overline{F}^{-1}(0.9), \overline{F}^{-1}(0.8), \overline{F}^{-1}(0.2).$$

From ⟨Table 1⟩, we can conclude that the MSE's of the proposed estimator $\hat{m}_p(t)$ are consistently smaller than those of the $\hat{m}_{sv}(t)$ for all cases. Particularly, it shows that $\hat{m}_p(t)$ becomes much better in terms of both bias and MSE as censoring rate increases.

4. Estimation of MRL for Lognormal Distribution

To study the accuracy of the approximation $m_p(t)$ of (2.2) for lognormal distribution, we compare $m(t)$ and $m_p(t)$ for $\xi = 1$ and for various choices of t and θ.

⟨Table 2⟩ shows that the approximation $m_p(t)$ is very good for smaller values of t, regardless of the values of θ. However, for fixed t, the approximation gets better as θ decreases. It implies that the approximation works best when the underlying distribution is symmetric and t is relatively small. Note that for lognormal family the lognormal distribution becomes symmetric as θ decreases. It is apparent from ⟨Table 2⟩ that the approximation is not very good for the values of t beyond a certain bound.

Under the random censoring model, we consider the lognormal distribution with the probability density function (p.d.f)

$$f(x;\xi,\theta) = \frac{1}{x\sqrt{2\pi}\,\theta}\exp\left[-\frac{1}{2}\left(\frac{\log x-\xi}{\theta}\right)^2\right], \qquad t>0$$

with

$$E(X) = \exp(\xi + \frac{1}{2}\theta^2)$$

$$Var(X) = \exp(2\xi+\theta^2)\left[\exp(\theta^2)-1\right].$$

Then the log likelihood function is

$$\begin{aligned}\log L(\xi,\theta) = & -\frac{n_1}{2}\log 2\pi - n_1\log\theta \\ & -\sum_u \log y_i - \sum_u \frac{(\log y_i-\theta)^2}{2\theta^2} \\ & + \sum_c \log\left[1-\Phi\left(\frac{\log y_i-\xi}{\theta}\right)\right],\end{aligned}$$

where $\sum_u(\sum_c)$ denotes a summation over the uncensored(censored) obeservations and n_1 is the number of uncensored observations and $\Phi(\cdot)$ is the c.d.f of the standard normal distribution.

Taking the first and second derivatives of log L and equating them to 0, we obtain the MLE's of ξ and θ by using Newton-Raphson method as follows.

$$\frac{\partial \log L}{\partial \xi} = \frac{1}{\theta}\sum_u z_i + \frac{1}{\theta}\sum_c \frac{\phi(z_i)}{\Phi(-z_i)}$$

$$\frac{\partial \log L}{\partial \theta} = -\frac{n_1}{\theta} + \frac{1}{\theta}\sum_u z_i^2 + \frac{1}{\theta}\sum_c \frac{z_i\phi(z_i)}{\Phi(-z_i)}$$

$$\frac{\partial \log L}{\partial \xi^2} = -\frac{n_1}{\theta} + \frac{1}{\theta^2} \sum_c \left[\frac{\phi(z_i)(z_i \Phi(-z_i) - \phi(z_i))}{(\Phi(-z_i))^2} \right]$$

$$\frac{\partial \log L}{\partial \xi \partial \theta} = -\frac{2}{\theta^2} \sum_u z_i - \frac{1}{\theta^2} \sum_c \frac{\phi(z_i)}{\Phi(-z_i)} + \frac{1}{\theta^2} \sum_c \frac{\phi(z_i)(z_i^2 \Phi(-z_i) - \phi(z_i))}{(\Phi(-z_i))^2}$$

$$\frac{\partial^2 \log L}{\partial \theta^2} = \frac{n_1}{\theta^2} - \frac{3}{\theta^2} \sum_u z_i^2 - \frac{2}{\theta^2} \sum_c \frac{z_i \phi(z_i)}{\Phi(-z_i)}$$
$$+ \frac{1}{\theta^2} \sum_c \frac{z_i^2 \phi(z_i)(z_i \Phi(-z_i) - \phi(z_i))}{(\Phi(-z_i))^2},$$

where $z_i = \frac{\log y_i - \xi}{\theta}$ and $\phi(\cdot)$ is the p.d.f of the standard normal distribution.

By the invariance property of MLE, the MLE's of $E(X)$ and $Var(X)$ are expressed as

$$\hat{\mu} = \exp(\hat{\xi} + \frac{1}{2} \hat{\theta}^2)$$
$$\hat{\sigma}^2 = \exp(2\hat{\xi} + \hat{\theta}^2)[\exp(\hat{\theta}^2) - 1],$$

where $\hat{\xi}$ and $\hat{\theta}$ are the MLE's of ξ and θ, respectively. Thus we obtain the new parametric estimator, $\hat{m}_p(t)$, as

$$\hat{m}_p(t) = \exp(\hat{\xi} + \frac{1}{2} \hat{\theta}^2) + [\frac{\hat{p}}{1-\hat{p}}]^{1/2} \{\exp(2\hat{\xi} + \hat{\theta}^2) \exp(\hat{\theta}^2) - 1)\}^{1/2} - t, \quad (4.1)$$

where $\hat{p} = \Phi(\frac{\log t - \hat{\xi}}{\hat{\theta}})$.

We carry out some simulations for lognormal distribution to see how our parametric estimator of (4.1) performs with respect to its mean squared error (MSE). As a competitor, we use the empirical MRL estimator given in (2.4).

The simulation scheme is as follows.

i) The censoring distribution is an exponential distribution with the probability density function

$$g(x;\lambda) = \lambda \exp(-\lambda x), \qquad x \geq 0, \lambda > 0.$$

ii) The censoring rates are 10% and 20%.

iii) The sample sizes are 5, 7, 10, and 15.

〈Table 2〉 Numerical comparison of $m(t)$ and $m_p(t)$ for lognormal distribution with $\xi=1$

θ	t	$m(t)$	$m_p(t)$	θ	t	$m(t)$	$m_p(t)$
0.2	0.5	2.27319	2.27319	1.0	0.3	4.24274	4.88446
	1.0	1.77319	1.77345		0.5	4.17893	5.26501
	1.5	1.27547	1.29545		0.7	4.16695	5.60256
	2.0	0.83633	0.91817		0.9	4.18750	5.89864
	3.0	0.45724	0.60709		1.0	4.02609	6.03351
0.3	0.5	2.34339	2.34345	1.1	0.2	4.82248	5.51131
	1.0	1.84432	1.86216		0.3	4.78976	5.84766
	2.0	1.04606	1.21465		0.4	4.77954	6.15869
	3.5	0.67971	1.08975		0.5	4.78571	6.44381
	3.8	0.68785	1.27922		0.6	4.08397	6.70563

Let $Se=E[(\hat{m}_e(t)-m(t)]^2$ and $Sp=E[(\hat{m}_p(t)-m(t)]^2$ be the MSE's of the empirical estimator $\hat{m}_e(t)$ and $\hat{m}_p(t)$, respectively. 〈Table 3〉 and 〈Table 4〉 show the values of Se and Sp for various combinations of θ, t, n, and censoring rates for lognormal distribution. The values in these tables are based on 1000 replications for each combination.

〈Table 3〉 Numerical values of the MSE's of $\hat{m}_p(t)$ and $\hat{m}_e(t)$ for lognormal distribution $f(x;\theta,\xi=1)$ when censoring rate is 10%.

$\xi=1$		$\theta=0.2$, $\lambda=0.271$				$\xi=1$		$\theta=0.3$, $\lambda=0.185$			
t		1.0	1.5	2.0	2.5	t		1.0	1.5	2.0	2.5
$m(t)$		1.77319	1.27547	0.83633	0.56356	$m(t)$		1.84432	1.38090	1.04606	0.85063
$n=5$	sp	0.04336	0.04444	0.04861	0.04128	$n=5$	sp	0.10586	0.10414	0.11731	0.13491
	se	0.17320	0.09095	0.06871	0.06387		se	0.20327	0.14739	0.15144	0.19067
$n=7$	sp	0.03901	0.03776	0.04771	0.03881	$n=7$	sp	0.08154	0.10042	0.10380	0.10652
	se	0.20593	0.09427	0.0665	0.05938		se	0.19839	0.13975	0.14555	0.15473
$n=10$	sp	0.02718	0.03106	0.03553	0.03237	$n=10$	sp	0.62184	0.07011	0.08113	0.08627
	se	0.19169	0.09486	0.06124	0.04956		se	0.18837	0.12064	0.12753	0.13697
$n=15$	sp	0.02066	0.02339	0.02634	0.02160	$n=15$	sp	0.04778	0.04650	0.05705	0.06310
	se	0.19631	0.08348	0.04741	0.03792		se	0.16373	0.10401	0.10496	0.10305

〈 Table 4 〉 Numerical values of the MSE's of $\hat{m}_p(t)$ and $\hat{m}_e(t)$ for lognormal distribution $f(x;\theta,\xi=1)$ when censoring rate is 20%.

$\xi=1$		$\theta=0.2,\ \lambda=0.322$				$\xi=1$		$\theta=0.3,\ \lambda=0.236$			
t		1.0	1.5	2.0	2.5	t		1.0	1.5	2.0	2.5
$m(t)$		1.77319	1.27547	0.83633	0.56356	$m(t)$		1.84432	1.38090	1.04606	0.85063
$n=5$	sp	0.05182	0.05268	0.05309	0.05651	$n=5$	sp	0.09887	0.12832	0.13184	0.15080
	se	0.21269	0.10327	0.07349	0.70861		se	0.23234	0.17284	0.17093	0.19687
$n=7$	sp	0.04509	0.04322	0.06579	0.04895	$n=7$	sp	0.10345	0.09743	0.12180	0.14958
	se	0.24437	0.10722	0.07262	0.06700		se	0.22982	0.15907	0.15477	0.16618
$n=10$	sp	0.03698	0.03938	0.03750	0.04090	$n=10$	sp	0.07064	0.08084	0.09580	0.10099
	se	0.24644	0.10628	0.06640	0.05043		se	0.21872	0.14489	0.13084	0.14592
$n=15$	sp	0.02465	0.02516	0.02732	0.02629	$n=15$	sp	0.05488	0.05439	0.06110	0.07449
	se	0.23402	0.09907	0.05707	0.03962		se	0.22762	0.12533	0.12059	0.11694

〈Table 3〉 and 〈Table 4〉 show that Sp is smaller than Se, regardless of censoring rates and θ. It is also clear that the MSE converges to 0 as $n\rightarrow\infty$ for fixed t.

Since the MLE, $\hat{m}_p(t)$, is based on the approximation $m_p(t)$, it is not a consistent estimator of $m(t)$. However, 〈Table 2〉 shows the approximation is quite good when t is small. Thus the MLE would perform well when t is small and n is relatively small. Especially, when the underlying distribution is known to have a lognormal distribution with small scale parameter and when only a small sample is available, our MLE is recommended.

It is also emphasized that although we discuss only the lognormal family for applying the MLE of MRL function in this paper, the same method can be used to estimate the MRL function of other parametric families as long as its maximum likelihood estimators of mean and variance exist.

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