Pusan Kyŏngnam Math J 12(1996), No 2, pp. 235–241

A NOTE ON SEVERAL CONTINUOUS FUNCTIONS ON FUZZY CONVERGENCE SPACES

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1. Introduction

The convergence function between the filters on a given set S and the subsets of S was introduced by D.C.Kent ([9]) in 1964 and it may be regarded as a generation of a topological space and further studied by many authors.

After Zadech created fuzzy sets in his classical paper ([10]) , Chang ([3]) used them to introduce the concept of a fuzzy sets using metric defined as the Hausdorff metric between the supported endographs. Recently, B.Y.Lee and J.H.Park ([12]) defined a new structure, called by fuzzy convergence structure, using prefilter.

We introduced the several continuous functions, that is, fuzzy super continuity, fuzzy δ-continuity, and fuzzy weakly δ-continuity in fuzzy convergence spaces ([5]).

In this paper, we introduce new continuities in fuzzy convergence spaces, that is, fuzzy θ-continuity, fuzzy strongly θ-continuity, fuzzy almost continuity, and fuzzy weakly almost continuity. And we study the relationships between them.

2. Preliminaries

The reader is asked to refer to [3], [5], [10], [15], [17] and [21], for fuzzy sets fuzzy convergence spaces, however, a brief review of basic terms will be given in here.

Let X be a nonempty set and I the unit closed interval $I = [0, 1]$. A fuzzy set A in X is an element of the set $F(X)$ of all functions from X into I and the elements of $F(X)$ are called fuzzy subsets ([10]). For fuzzy set A and B in X, $A \subseteq B$ if $A(x) \leq B(x)$ for all x in X. The

Received October 17,1996

This research supported by the Rearch Foundation Grant of Kyungnam University

symbol $\emptyset$ is used to denote the empty fuzzy set $\emptyset(x) = 0$ for all $x \in X$ and for X we have the definition $X(x) = 1$ for all $x \in X$.

A fuzzy point p in X is fuzzy set in X defined by $p(x) = \lambda$ $(0 < \lambda \leq 1)$ for $x = x_p$ and $p(x) = 0$ for $x \neq x_p$. Then, we call x_p the support of p and λ the value of p. A fuzzy point $p \in A$, where A is a fuzzy set in X, if $p(x_p) \leq A(x_p)$.

A fuzzy point p is said to be quasi coincident with A, denoted by pqA, if $p(x_p) + A(x_p) > 1$ for a fuzzy point p and a fuzzy set A (see in [21]). A fuzzy set A is said to be quasi coincident with a fuzzy set B, denoted by AqB, if there exists some x in X such that $A(x)+B(x) > 1$.

Let f is a function from a set X into a set Y and A, B be the fuzzy sets in X, Y, respectively. Then we define $f^{-1}(B)$ and $f(A)$ as follows:

$$f^{-1}(B)(x) = B(f(x))$$

and

$$f(A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

In here, we introduce fuzzy convergence spaces using prefiters, and we define the set functions Γ_c, I_c and introduce their properties.

DEFINITION 2.1. ([2]) A *prefilter* on X is a nonempty subset $\mathcal{F}$ of the set I^X of functions from X into closed interval $I = [0, 1]$ with the properties:

(1) If $A, B \in \mathcal{F}$, then $A \cap B \in \mathcal{F}$
(2) If $A \in \mathcal{F}$ and $A \subseteq B$, then $B \in \mathcal{F}$
(3) $\emptyset \notin \mathcal{F}$

If $\mathcal{F}$ and $\mathcal{G}$ are prefilters on X, $\mathcal{F}$ is said to be finer than $\mathcal{G}$ ($\mathcal{G}$ is coarser than $\mathcal{F}$) if and only if $\mathcal{G} \subseteq \mathcal{F}$. A prefilter $\mathcal{F}$ on X is said to be *ultra prefilter* if it is no other prefilter finer than $\mathcal{F}$ (i.e., it is maximal for the inclusion relation among prefilters).

A *prefilterbase* on X is the nonempty subset β of I^X with the properties:

(1) If $A, B \in \beta$, there exists $C \in \beta$ such that $C \subseteq A \cap B$.
(2) $\emptyset \notin \beta$.

If β is a prefilterbase then $<\beta>=\{A \in I^X : B \subseteq A$ for some $B \in \beta\}$ is a prefilter. If $<\beta>=\mathcal{F}$, we say that β is a prefilterbase for the prefilter $\mathcal{F}$, or that β generates $\mathcal{F}$.

We define convergence structure by prefilter, called fuzzy convergence structure. For nonempty universal set X, $P(X)$ denotes the set of all prefilters on X and $F(X)$ the set of all fuzzy sets on X. For each fuzzy point p in X, $\dot{p}$ is denoted by

$$\{A \in I^X : pqA\}$$

Let f be a function from X into Y. Then for a fuzzy point p in fuzzy set A in X, $f(p) \in f(A)$ and for two prefilters $\mathcal{F}, \mathcal{G}$ on X, $f(\mathcal{F} \cap \mathcal{G}) = f(\mathcal{F}) \cap f(\mathcal{G})$ and so $f(\mathcal{F} \cap \dot{p}) = f(\mathcal{F}) \cap f(\dot{p})$ and $\dot{f}(p) = f(\dot{p})$. For a fuzzy prefilter $\mathcal{F}$ on X, $f(\mathcal{F})$ is said to be the prefilter on Y generated by $\{f(A) : A \in \mathcal{F}\}$.

DEFINITION 2.2. ([12]) A *fuzzy convergence structure* on X is a function C_X from $P(X)$ into $F(X)$ satisfying the following conditions:

(FC1) For each fuzzy point p in X, $p \in C_X(\dot{p})$.
(FC2) For $\mathcal{F}, \mathcal{G} \in P(X)$, if $\mathcal{F} \subseteq \mathcal{G}$ then $C_X(\mathcal{F}) \subseteq C_X(\mathcal{G})$.
(FC3) If $p \in C_X(\mathcal{F})$, then $p \in C_X(\mathcal{F} \cap \dot{p})$.

Then the pair (X, C_X) is said to be *fuzzy convergence space*. If $p \in C_X(\mathcal{F})$, we say that $\mathcal{F}$ *C_X-converges* to a fuzzy point p. The prefilter $\mathcal{V}_{C_X}(p)$ obtain by intersecting all prefilters which C_X-converge to p is said to be the *C_X-neighborhood prefilter* at p. If $\mathcal{V}_{C_X}(p)$ C_X-convergences to p for each fuzzy point p in X, then C_X is called a *fuzzy pretopological structure*, and (X, C_X) a *fuzzy pretopological space*. The fuzzy pretopological structure C_X is said to be *fuzzy topological structure* and (X, C_X) is said to be *fuzzy topological space*, if for each fuzzy point p in X, the prefilter $\mathcal{V}_{C_X}(p)$ has a prefilterbase $\beta_{C_X}(p) \subseteq \mathcal{V}_{C_X}(p)$ with the following property:

$$rq\sqcup \in \beta_{C_X}(p) \text{ implies } \sqcup \in \beta_{C_X}(r)$$

Throughout this paper, let $C(X)$ be the set of all fuzzy convergence structures on X. Then we define that $C_1 \leq C_2$ for $C_1, C_2 \in C(X)$ if and only if $C_2(\mathcal{F}) \subseteq C_1(\mathcal{F})$ for all $\mathcal{F} \in P(X)$. If $C_1 \leq C_2$ for

$C_1, C_2 \in C(X)$, we say that C_2 is *finer* than C_1 , also that C_1 is *coarser* than C_2.

Let $F(X)$ be the set of all fuzzy sets in X and A a fuzzy set in X. The set function Γ_{C_X}(resp. I_{C_X}) from $F(X)$ into $F(X)$ is given by $\Gamma_{C_X}(A)$=$\{p$: p is fuzzy point in X and $p \in C_X(\mathcal{F})$ for some ultra prefilter $\mathcal{F}$ with $A \in \mathcal{F}$ $\}$ (resp. $I_{C_X}(A) = \{p : A \in \mathcal{V}_{C_X}(p)$ and p is a fuzzy point in X $\}$). Then $\Gamma_{C_X}(A)$ (resp. $I_{C_X}(A)$) is called *fuzzy closure* of fuzzy set A (resp. *fuzzy interor* of A).

For a prefilter $\mathcal{F}$ on X, $\Gamma_{C_X}(\mathcal{F})$ and $I_{C_X}(\mathcal{F})$ are the prefilters on X generated by $\{\Gamma_{C_X}(A) : A \in \mathcal{F}\}$ and $\{I_{C_X}(A) : A \in \mathcal{F}\}$, respectively.

DEFINITION 2.3. The fuzzy convergence space (X, C_X) is called *fuzzy regular* (resp. *fuzzy semi-regular*) if $\Gamma_{C_X}(\mathcal{F})$ (resp. $I_{C_X}(\Gamma_{C_X}(\mathcal{F}))$) C_X-converges to p, whenever fuzzy prefilter $\mathcal{F}$ C_X-converges to fuzzy point p.

From definition of set functions Γ_{C_X} and I_{C_X} , we can obtain the followings : $\Gamma_{C_X}(A) \supseteq A$ and $I_{C_X}(A) \subseteq A$ for each fuzzy set A in X.

DEFINITION 2.4. A function f from (X, C_X) to (Y, C_Y) is *continuous* at p if $f(\mathcal{F})$ C_Y-converges to $f(p)$, whenever a prefilter $\mathcal{F}$ on X C_X-converges to p.

3. θ - continuity and almost continuity on fuzzy convergence spaces

In this section, we define θ - continuity, strongly θ-continuity, almost continuity, weakly almost continuity on fuzzy convergence spaces and investigate the relationships among them.

Throught this section, let (X, C_X) and (Y, C_Y) be the fuzzy convergence spaces and p a fuzzy point in X.

DEFINITION 3.1. A function f from (X, C_X) to (Y, C_Y) is *fuzzy θ-continuous* at p in X if $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\Gamma_{C_X}\mathcal{F})$ whenever a prefilter $\mathcal{F}$ on X C_X -converges to p.

DEFINITION 3.2. A function f from (X, C_X) to (Y, C_Y) is *fuzzy strongly θ-continuous* at p in X if $\mathcal{V}_{C_Y}(f(p)) \subseteq f(\Gamma_{C_X}(\mathcal{F}))$ whenever a prefilter $\mathcal{F}$ on X C_X-converges to p.

THEOREM 3.2. *A function f from (X, C_X) to (Y, C_Y) is fuzzy strongly θ -continuous at p in X, then f is fuzzy θ-continuous at p in X.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to fuzzy point p in X. Then $\mathcal{V}_{C_Y}(f(p)) \subseteq f(\Gamma_{C_X}(\mathcal{F}))$ by definition 3.2. Since $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq \mathcal{V}_{C_Y}(f(p))$, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\Gamma_{C_X}(\mathcal{F}))$. Accordingly, f is fuzzy θ-continuous at p in X.

THEOREM 3.4. *Let a f from (X, C_X) to (Y, C_Y) be a function and (X, C_X) regular convergence space. If f is fuzzy continuous at p in X, then f is fuzzy strongly θ-continuous at p in X.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to fuzzy point p in X. Then, since (X, C_X) is regular $\Gamma C_X(\mathcal{F})$ C_X-converges to p. Since f is fuzzy continuous, $f(\Gamma_{C_X}(\mathcal{F}))$ C_Y -converges to $f(p)$ in Y, and so $\mathcal{V}_{C_Y}(f(p)) \subseteq f(\Gamma_{C_X}(\mathcal{F}))$. Accordingly, f is fuzzy strongly θ-continuous at p in X.

DEFINITION 3.5. A function f from (X, C_X) to (Y, C_Y) is *fuzzy almost continuous* at p in X if $I_{C_Y}(\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))) \subseteq f(\mathcal{F})$ whenever a prefilter $\mathcal{F}$ on X C_X-converges to p.

DEFINITION 3.6. A function f from (X, C_X) to (Y, C_Y) is *fuzzy weakly almost continuous* at p in X if $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\mathcal{F})$ whenever a prefilter $\mathcal{F}$ on X C_X -converges to p.

THEOREM 3.7. *If a function f from (X, C_X) to (Y, C_Y) is fuzzy almost continuous at p in X, then f is fuzzy weakly almost continuous at p*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to p in X. Then $I_{C_Y}(\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))) \subseteq f(\mathcal{F})$ by definition 3.5. Since $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq I_{C_Y}(\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))))$, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\mathcal{F})$. Accordingly f is fuzzy weakly almost continuous at p.

THEOREM 3.8. *If a function f from (X, C_X) to (Y, C_Y) is fuzzy θ-continuous at p in X , then it is fuzzy weakly almost continuous at p.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to p in X. Then $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\Gamma_{C_X}(\mathcal{F}))$ by definition 3.1. Since $f(\Gamma_{C_X}(\mathcal{F})) \subseteq$

$f(\mathcal{F})$, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\mathcal{F})$. Accordingly, f is fuzzy weakly almost continuous at p.

From definition 3.2 and theroem 3.8 , we obtain that if f is fuzzy strongly θ continuous then f is fuzzy weakly almost continuous. And by theorem 3.4 and 3.8, if (X, C_X) is regular space and f is fuzzy continuous at p in X, then f is fuzzy θ-continuous and fuzzy weakly almost continuous at p.

THEOREM 3.9. *If a function from (X, C_X) to (Y, C_Y) is fuzzy continuous at p in X, then f is fuzzy weakly almost continuous at p.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to p in X. Then $f(\mathcal{F})$ C_Y-converges to $f(p)$ in Y by definition 2.4. But $\mathcal{V}_{C_Y}(f(p)) \subseteq f(\mathcal{F})$ and $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq \mathcal{V}_{C_Y} f(p)$. And so $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\mathcal{F})$. Accordingly f is fuzzy weakly almost continuous at p.

THEOREM 3.10. *Let f from (X, C_X) to (Y, C_Y) be a function and (Y, C_Y) fuzzy regular pretopogical space. If f is fuzzy weakly almost continuous at p in X, then f is continuous at p.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to p in X. Since (Y, C_Y) is pretopological convergence space, $\mathcal{V}_{C_Y}(f(p))$ C_Y-converges to $f(p)$. And so $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))$ C_Y-converges to $f(p)$ in Y by definition of regular space. Thus $\mathcal{V}_{C_Y}(f(p)) \subseteq \Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))$ by definition of $\mathcal{V}_{C_Y}(f(p))$. But $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq \mathcal{V}_{C_Y}(f(p))$ by definition of Γ_{C_Y}, that is, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) = \mathcal{V}_{C_Y}(f(p))$.

Accordingly $\mathcal{V}_{C_Y}(f(p)) = \Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(\mathcal{F})$ and $\mathcal{V}_{C_Y}(f(p))$ C_Y-converges to $f(p)$ in Y. Hence $f(\mathcal{F})$ C_Y-converges to $f(p)$, and so f is fuzzy continuous.

From proof of theorem 3.10, we obtain the following.

COROLLAY 3.11. *Let f from (X, C_X) to (Y, C_Y) be a function and (Y, C_Y) regular pretopological convergence space. If f is fuzzy almost continuous at p in X, then f is fuzzy continuous at p.*

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