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REGULARITY FOR SOLUTIONS OF BIHARMONIC EQUATION ON LIPSCHITZ DOMAIN

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1. Introduction

Let Ω be a Lipschitz domain in R^n. In this paper, we study the behavior near the boundary point of solutions $u \in W_0^{2,2}(\Omega)$, the closure of the space $C_0^\infty(\Omega)$ in the norm $\|\nabla\nabla u\|_{L^2(\Omega)}$, to the equation

$$\Delta\Delta u = f, \quad f \in C^\infty(\overline{\Omega}). \tag{1.1}$$

When $f \in C_0^\infty(\Omega)$, C^∞ function with compact support in Ω, and $n = 5, 6, 7$, Maz'ya ([M]) showed that if $u \in W_0^{2,2}(\Omega)$ is the solution to (1.1), then $u \in C^{0,\alpha}(\overline{\Omega}), \alpha > 0$. In this paper, we obtain the same Hölder regularity result as in [M] for $f \in L^\infty(\Omega)$, that is, f need not be compactly supported in Ω.

When $n \geq 8$, the boundedness of the solution $u \in W_0^{2,2}(\Omega)$ of (1.1) is still unknown. In [M,N,P], V.G.Maz'ya , S.A. Nazarov, and B.A. Plamenevskii studied deeply the singularities of solutions of the Dirichlet problem for strongly elliptic differential systems of order 2m outside a slender cone $\mathcal{K}_\epsilon = \{(x,t) \in R^n; \epsilon t > |x|\}$. In there, when $n = 8$ these authors found that for sufficiently small ϵ which is the angle vertex of the cone $\mathcal{K}_\epsilon$ there is $u \in W_0^{2,2}(B \setminus \overline{\mathcal{K}_\epsilon})$ such that u is *not bounded near origin* and u satisfies the equation $\Delta\Delta u + 100\left(\frac{\partial}{\partial x_n}\right)^4 u = f \quad \text{in } B\setminus\overline{\mathcal{K}_\epsilon}$ where f is smooth in $\overline{B \setminus \mathcal{K}_\epsilon}$. Here B is the unit ball in R^n.

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When Ω is a C^1 domain in R^n, $n \geq 2$, it is not so hard to show Hölder regularity near boudary from the results in [V1](Theorem, page 867). Indeed, from [V1] and standard arguments the solution $u \in W_0^{2,2}(\Omega)$ of (1.1) also lies in the Sobolev space $W^{1,p}(\Omega)$ for all $1 < p < \infty$ and therefore from the standard Sobolev imbedding u is Hölder continuous in $\overline{\Omega}$. But these arguments([V1]) do not work for the case where Ω is only a Lipschtz domain.

Our main results are as follows:

THEOREM 1. *Let $5 \leq n \leq 7$. Let Ω be a bounded Lipschitz domain in R^n. For $f \in L^\infty(R^n)$, the solution $u \in W_0^{2,2}(\Omega)$ of the equation $\Delta\Delta u = f$ in Ω is Hölder continuous up to the boundary of Ω, that is, $u \in C^{0,\alpha}(\overline{\Omega}), \alpha > 0$.*

Throughout this paper constants C may differ in each occurance but C depends only on the dimension n and the domain Ω.

2. The Preliminaries

Capital letters X, Y, Z will denote points in Ω, while P, Q will be reserved for points in $\partial\Omega$. We will denote $B_r(X) = \{Y \in R^n : |X-Y| < r\}$, $B_r = B_r(0)$. For $X \in \Omega$ we will write d_X to mean the distance of X to $\partial\Omega$.

DEFINITION. A bounded open connected domain $\Omega \subset R^n$ is called a Lipschitz domain if for each $P \in \partial\Omega$ there is an open, right circular, doubly truncated cylinder $Z(P,r)$ centered at P, with radius equal to r, whose bottom and top are at a positive distance(usually a multiple of r) from $\partial\Omega$, such that there is a coordinate system (x,s), $x \in R^{n-1}$, $s \in R$, with s-axis containing the axis of Z and a Lipschitz function $\phi : R^{n-1} \to R$ such that $Z \cap \Omega = \{(x,s) \in R^{n-1} \times R : s > \phi(x)\} \cap Z$, and $Z \cap \partial\Omega = \{(x,s) \in R^{n-1} \times R : s = \phi(x)\} \cap Z$.

$(\,)^*$ denote the nontangential maximal function defined for every $P \in \partial\Omega$ by

$$(w)^*(P) = \sup_{X \in \gamma(P)} |w(X)|$$

where $\gamma(P)$ is the nontangential cone for Ω at $P \in \partial\Omega$ defined by

$$\gamma(P) = \{X \in \Omega : |X - P| < 1 + M\text{dist}(X, \partial\Omega)\}.$$

Here the constant M is the Lipschitz constant of the domain Ω. The space $L_1^p(\partial\Omega)$ is the space of $L^p(\partial\Omega)$ functions with first distributional derivatives in $L^p(\partial\Omega)$. $N = N(P)$ denotes the outer unit normal vector at $P \in \partial\Omega$. Dahlberg, Kenig, and Verchota showed the following theorem by using layer potential technique.

THEOREM 2[D,K,V]. *Let Ω be a bounded Lipschitz domain in R^n. Let $f \in L_1^2(\partial\Omega)$, $g \in L^2(\partial\Omega)$. Then there exists a unique function, u, in Ω such that*

$$(2.1) \qquad \begin{cases} \Delta\Delta u = 0 \quad \text{in } \Omega \\ \lim\limits_{\substack{X \to P \\ X \in \gamma(P)}} u(X) = f(P) \quad \text{almost everywhere } P \in \partial\Omega \\ \lim\limits_{\substack{X \to P \\ X \in \gamma(P)}} \langle N(P), \nabla u(X)\rangle = g(P) \quad \text{for a.e. } P \in \partial\Omega \\ \|(\nabla u)^*\|_{L^2(\partial\Omega)} < \infty. \end{cases}$$

In fact,

$$\|(u)^*\|_{L^2(\partial\Omega)} + \|(\nabla u)^*\|_{L^2(\partial\Omega)} \le C\{\|f\|_{L_1^2(\partial\Omega)} + \|g\|_{L^2(\partial\Omega)}\}.$$

Let $\Gamma(X)$ denote the fundamental solution for $\Delta\Delta$, that is,

$$\Gamma(X) = \frac{|X|^{4-n}}{2(n-4)(n-2)\omega_n} \quad \text{if } n > 4$$

where ω_n denotes the surface area of the unit sphere in R^n. We define the Green's function for $\Delta\Delta$ by

$$(2.2) \qquad G(X,Y) = \Gamma(X - Y) - W^X(Y) \qquad X, Y \in \Omega$$

where $W^X(\cdot)$ is the solution of (2.1) with data $(f, g) = \left(\Gamma(X - \cdot), \frac{\partial}{\partial N}\Gamma(X - \cdot)\right)$.

From the standard argument,

$$G(X,Y) = G(Y,X) \quad \text{for all } X, Y \in \Omega.$$

We will recall the important results done by Maz'ya ([M]).

LEMMA 3[M]. *Let $5 \le n \le 7$ and let $0 < r < 2r < R$. Suppose a function $u \in W_0^{2,2}(\Omega)$ satisfies the equation*

$$\Delta\Delta u = 0 \quad \text{in } \Omega \cap B_{2R}.$$

Then for all points $X \in B_{\frac{r}{2}} \cap \Omega$

$$\begin{aligned} u(X)^2 &+ \int_{\Omega\cap B_r} \{|\nabla\nabla u|^2 + |Z-X|^{-2}|\nabla u|^2\}\Gamma(Z-X)dZ \\ &\le C\frac{1}{r^n}\int_{\Omega\cap(B_{2r}\setminus B_r)} u^2 dZ \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} u(X)^2 &+ \int_{\Omega\cap B_r} \{|\nabla\nabla u|^2 + |Z-X|^{-2}|\nabla u|^2\}\Gamma(Z-X)dZ \\ &\le C\int_{\Omega\cap(B_{2r}\setminus B_r)} \{|\nabla\nabla u|^2 + |Z-X|^{-2}|\nabla u|^2\}\Gamma(Z-X)dZ. \end{aligned} \tag{2.4}$$

COROLLARY 4. *Let $5 \le n \le 7$ and let a function $u \in W_0^{2,2}(\Omega)$ satisfies the equation*

$$\Delta\Delta u = 0 \quad \text{in } \Omega \cap B_{2R}.$$

Then for $0 < r < \frac{R}{2}$

$$\sup_{X\in B_{\frac{r}{4}}\cap\Omega} |u(X)|^2 \le C(\frac{r}{R})^\delta \left(\frac{1}{R^n}\int_{\Omega\cap(B_{2R}\setminus B_R)} u^2(Y)dY\right) \tag{2.5}$$

for some $\delta > 0$.

Proof. We define

$$\psi(s) = \int_{\Omega\cap B_s} [|\nabla\nabla u|^2 + |Z|^{-2}\nabla u|^2]\Gamma(Z)dZ.$$

Then from (2.4) for $0 < 4r < R$,

$$\psi(r) \le C(\psi(2r) - \psi(r)).$$

Hence $\psi(r) \le \theta\psi(2r)$ where $\theta = \frac{C}{1+C} < 1$. ¿From the standard iteration we obtain

$$\psi(r) \le C\left(\frac{r}{R}\right)^{\delta}\psi(R) \quad \text{for some } \delta > 0.$$

Using (2.3) and (2.4) we complete the proof.

LEMMA 5[M]. *Let $5 \le n \le 7$. Then for $X, Y \in \Omega$*

$$|G(X,Y)| \le C|X-Y|^{4-n} \qquad \textit{if } |X-Y| > d_X$$

(Recall $d_X = \text{dist}(X, \partial\Omega)$).

3. The proof of Theorem 1

LEMMA 6. *Let $n \ge 5$. For $X \in \Omega$,*

$$\int_{\Omega\cap B_{10d_X}(X)} |G(X,Y)|dY \le Cd_X^4$$

where the constant C only depends on the dimension n and Ω.

Proof. It suffice to consider the case when d_X is small. If d_X is sufficiently small, from the definition of Lipschitz domain, after change of a rectangular coordinate system for R^n, there is a Lipschitz function $\phi : R^{n-1} \to R$ such that

$$\Omega \cap B_{10d_X}(X) = \{Y = (y', y_n) \in B_{10d_X}(X); y_n > \phi(y')\}$$
$$y' = (y_1, y_2, ..., y_{n-1}).$$

For simplicity, we will write $\mathcal{E}_X = \Omega \cap B_{10d_X}(X)$. We will denote the projection of Y on $\partial\Omega$ by $Q_Y = (y', \phi(y'))$. Since $G(X, Q_Y) = 0$, from

(2.2)

$$\begin{aligned}\int_{\mathcal{E}_X} |G(X,Y)|dY =& \int_{\mathcal{E}_X} |G(X,Y) - G(X,Q_Y)|dY \\ \leq& \int_{\mathcal{E}_X} |\Gamma(X,Y) - \Gamma(X,Q_Y)|dY \\ &+ \int_{\mathcal{E}_X} |W^X(Y) - W^X(Q_Y)|dY \\ \leq& Cd_X^4 + C\left(d_X^n \int_{\mathcal{E}_X} |W^X(Y) - W^X(Q_Y)|^2 dY\right)^{\frac{1}{2}}.\end{aligned}$$

From Theorem 2([D,K,V]), we obtain

$$\begin{aligned}&\left(d_X^n \int_{\mathcal{E}_X} |W^X(Y) - W^X(Q_Y)|^2 dY\right)^{\frac{1}{2}} \\ &\leq C\left(d_X^{n+3} \int_{B_{10d_X}(X)\cap\partial\Omega} (\nabla W^X(Q))^{*2} d\sigma_Q\right)^{\frac{1}{2}} \\ &\leq C\left(d_X^{n+3} \int_{\partial\Omega} |\nabla\Gamma(X-Q)|^2 d\sigma_Q\right)^{\frac{1}{2}} \\ &\leq Cd_X^{\frac{9}{2}}.\end{aligned}$$

The first inequality follows from Mean value theorem and the definition of the nontangential maximal function (see Section 2).

This completes the proof.

MAIN LEMMA. *Let $n = 5, 6, 7$. Then there is a constant C and $\alpha > 0$ such that for all $X \in \Omega$*

$$\int_{\Omega} |G(X,Y)|dY \leq Cd_X^{\alpha}. \tag{3.1}$$

Proof. For notational simplicity, we will assume $0 \in \partial\Omega$ and $|X| = \text{dist}(X, \partial\Omega)$. Let $r = |X|$. We divide

$$\begin{aligned}\int_{\Omega} |G(X,Y)|dY =& \int_{\Omega\cap B_{8r}} |G(X,Y)|dY + \int_{\Omega\setminus B_{8r}} |G(X,Y)|dY \\ =& I_1 + I_2.\end{aligned}$$

From Lemma 6 we obtain

$$I_1 \leq Cr^4 = C|X|^4.$$

To handle I_2, we choose the integer N such that

$$2^{N-1}r \leq \mathrm{diam}(\Omega) \leq 2^N r.$$

Let

$$\mathcal{O}_i = \{Y \in \Omega : 2^{i-1}r \leq |Y| < 2^i r\}.$$

Then

$$\begin{aligned} I_2 \leq & C\sum_{i=4}^{N} \int_{\mathcal{O}_i} |G(X,Y)|dY \\ \leq & C\sum_{i=4}^{N} \mathrm{Sup}\{|G(X,Y)| : Y \in \mathcal{O}_i\}(2^i r)^n. \end{aligned} \tag{3.2}$$

Since for fixed i and fixed $Y \in \mathcal{O}_i$

$$\Delta\Delta G(\cdot, Y) = 0 \quad \text{in } B_{2^{i-1}r} \cap \Omega,$$

from Corollary 4 and Lemma 5([M]) we obtain for any $Y \in \mathcal{O}_i$

$$\begin{aligned} |G(X,Y)| &\leq C2^{-i\frac{\delta}{2}} \left(|\mathcal{O}_{i-2}|^{-1} \int_{\mathcal{O}_{i-2}} |G(Z,Y)|^2 dZ \right)^{\frac{1}{2}} \\ &\leq C2^{-i\frac{\delta}{2}} \left(|\mathcal{O}_{i-2}|^{-1} \int_{\mathcal{O}_{i-2}} |Z-Y|^{2(4-n)} dZ \right)^{\frac{1}{2}} \\ &\leq C2^{-i\frac{\delta}{2}} (2^i r)^{4-n} \end{aligned} \tag{3.3}$$

where the constant C is independent to Y. Therefore by applying (3.3) to (3.2), we obtain

$$\begin{aligned} \int_{\Omega \setminus B_{8r}} |G(X-Y)|dY &\leq C\sum_{i=3}^{N} (2^i r)^n (2^i)^{-\frac{\delta}{2}} (2^i r)^{4-n} \\ &\leq C\sum_{i=3}^{N} (2^i)^{4-\frac{\delta}{2}} r^4 \\ &\leq C(2^N)^{4-\frac{\delta}{2}} r^4 \\ &\leq Cr^{\frac{\delta}{2}}. \end{aligned} \tag{3.4}$$

(Note that $2^{N-1} \le r^{-1}\mathrm{diam}(\Omega) \le 2^N$.) We complete the proof.

Proof of Theorem 1. It is easy to see

$$u(X) = \int_\Omega G(X,Y)f(Y)dY.$$

Since $f \in L^\infty(R^n)$, it suffices to prove that there exist $\alpha > 0$ such that

$$\int_\Omega |G(X_1,Y) - G(X_2,Y)|dY \le C|X_1 - X_2|^\alpha \tag{3.5}$$

for all $X_1, X_2 \in \Omega$.

Let $r_i = \mathrm{dist}(X_i, \partial\Omega), i = 1,2$ and $r_1 < r_2$.

If $|X_1 - X_2| > \frac{r_2}{100}$, from Lemma 7

$$\begin{aligned} \int_\Omega |G(X_1,Y) - G(X_2,Y)|dY \le& C(r_1^\alpha + r_2^\alpha) \\ \le& C|X_1 - X_2|^\alpha. \end{aligned} \tag{3.6}$$

If $\frac{r_1}{10} < |X_1 - X_2| \le \frac{r_2}{10}$, then it must be $r_1 \ge \frac{r_2}{10}$ (otherwise $\mathrm{dist}(X_2, \partial\Omega) < \frac{r_2}{5}$) and therefore $|X_1 - X_2| > \frac{r_2}{100}$ which again gives us the estimate (3.6).

Finally suppose $|X_1 - X_2| \le \frac{r_1}{10}$. Then by mean value theorem

$$\begin{aligned} &\int_{\Omega\setminus B_{10r_1}} |G(X_1,Y) - G(X_2,Y)|dY \\ &\le |X_1 - X_2| \operatorname*{Sup}_{\substack{X = tX_1 + (1-t)X_2 \\ 0 \le t \le 1}} \int_{\Omega\setminus B_{10r_1}} |\nabla_X G(X,Y)|dY. \end{aligned} \tag{3.7}$$

By the standard interior estimates, we obtain

$$|\nabla_X G(X,Y)| \le C r_1^{-1-n} \int_{B_{\frac{1}{2}r_1}(X_1)} |G(Z,Y)|dZ. \tag{3.8}$$

for all $X = tX_1 + (1-t)X_2, 0 \le t \le 1$, and for all $Y \in \Omega \setminus B_{10r_1}$. Hence from (3.7), (3.8), and Main Lemma we obtain

(3.9)
$$
\begin{aligned}
&\int_{\Omega \setminus B_{10r_1}} |G(X_1,Y) - G(X_2,Y)| dY \\
&\qquad \le C|X_1 - X_2| \int_{\Omega \setminus B_{10r_1}} r_1^{-1-n} \int_{B_{\frac{1}{2}r_1}(X_1)} |G(Z,Y)| dZ dY \\
&\qquad \le C \frac{|X_1 - X_2|}{r_1} r_1^{-n} \int_{B_{\frac{1}{2}r_1}(X_1)} \left(\int_{\Omega \setminus B_{10r_1}} |G(Z,Y)| dY \right) dZ \\
&\qquad \le C \frac{|X_1 - X_2|}{r_1} r_1^{-n} \int_{B_{\frac{1}{2}r_1}(X_1)} r_1^{\alpha} dZ \\
&\qquad = C \frac{|X_1 - X_2|}{r_1} r_1^{\alpha} \le C|X_1 - X_2|^{\alpha}.
\end{aligned}
$$

Now it remains to prove

$$\int_{\Omega \cap B_{10r_1}} |G(X_1,Y) - G(X_2,Y)| dY \le C|X_1 - X_2|^{\alpha}.$$

The above inequality follows from

$$
\begin{aligned}
&\int_{\Omega \cap B_{10r_1}} |G(X_1,Y) - G(X_2,Y)| dY \\
&= \int_{\Omega \cap B_{10r_1}} |\Gamma(X_1,Y) - \Gamma(X_2,Y)| dY + \int_{\Omega \cap B_{10r_1}} |W^{X_1}(Y) - W^{X_2}(Y)| dY \\
&\quad \le C|X_1 - X_2| \operatorname*{Sup}_{X = tX_1 + (1-t)X_2, 0 \le t \le 1} \int_{\Omega \cap B_{10r_1}(X_1)} |X - Y|^{3-n} dY \\
&\qquad + C|X_1 - X_2| \operatorname*{Sup}_{X = tX_1 + (1-t)X_2, 0 \le t \le 1} \int_{\Omega \cap B_{10r_1}(X_1)} |\nabla_X W^X(Y)| dY \\
&\quad \le C|X_1 - X_2| \left(r_1^3 + \int_{\Omega \cap B_{10r_1}} r_1^{1-n} \int_{B_{\frac{1}{2}r_1}(X_1)} |W^Z(Y)| dZ dY \right)
\end{aligned}
$$

$$= C|X_1 - X_2| \left(r_1^3 + r_1^{1-n} \int_{B_{\frac{1}{2}r_1}(X_1)} \int_{\Omega \cap B_{10r_1}} |W^Z(Y)| dY dZ \right)$$

$$\leq C|X_1 - X_2| \left(r_1^3 + r_1^{\frac{9}{2}} \right)$$

The second inequality follow by the standard interior estimates. For the last inequality, by using Theorem 2 and the Schwartz inequality, for all $Z \in B_{\frac{1}{2}r_1}(X_1)$

$$\int_{\Omega \cap B_{10r_1}} |W^Z(Y)| dY \leq r_1^{\frac{n}{2}} \left(\int_{\Omega \cap B_{10r_1}} |W^Z(Y)|^2 dY \right)^{\frac{1}{2}}$$

$$\leq C r_1^{\frac{n}{2}+1} \left(\int_{\partial\Omega \cap B_{10r_1}} |(W^Z(Q))^*|^2 dY \right)^{\frac{1}{2}}$$

$$\leq C r_1^{\frac{n}{2}+1} \left(\int_{\partial\Omega} |W^Z(Q)|^2 + |\nabla_Q W^Z(Q)|^2 dY \right)^{\frac{1}{2}}$$

$$\leq C r_1^{\frac{n}{2}+1} \left(\int_{\partial\Omega} |Z - Q|^{6-2n} dY \right)^{\frac{1}{2}}$$

$$\leq C r_1^{\frac{n}{2}+1+\frac{5-n}{2}} = C r_1^{\frac{7}{2}}$$

This completes the proof.

4. Remark

Let $F \in C^2(R^n)$ and let u be a solution of (2.1) with data $(F, \dfrac{\partial F}{\partial N})$. G. Verchota[V2] showed that u satisfies the estimates

$$\|(\nabla\nabla u)^*\|_{L^{2+\epsilon}(\partial\Omega)} < \infty. \tag{4.1}$$

If $n = 3, 4$, it is not so hard to show from the above estimates (4.1) that $u \in C^{0,\alpha}(\bar{\Omega})$ for some $\alpha > 0$. We will prove Hölder regularity for

u in the case where $n = 5$. We denote $\Omega(r, x_0) = B_r(x_0) \cap \Omega$. ¿From the Sobolev imbedding theorem and Hölder inequality we obtain

$$\int_{\Omega(r,x_0)} |u - u_{\Omega(r,x_0)}| dx \leq C r^{4+\frac{1}{2}} \left(\int_{\Omega(r,x_0)} |u - u_{\Omega(r,x_0)}|^{10} \right)^{\frac{1}{10}}$$
$$\leq C r^{4+\frac{1}{2}} \|u\|_{W^{2,2}(\Omega(r,x_0))}$$
$$\leq C r^{4+\frac{1}{2}} r^{\frac{5\epsilon}{2(2+\epsilon)}} \|u\|_{W^{2,2+\epsilon}(\Omega(r,x_0))}$$

Now from the result (4.1) of [V2] the last term is

$$\leq r^{5+\frac{4\epsilon}{2(2+\epsilon)}} \left(\int_{\partial\Omega \cap B_r(x_0)} |(\nabla\nabla u)^*|^{2+\epsilon} \right)^{\frac{1}{2+\epsilon}}$$

Now the result follows from Morrey-Campanato space arguments(see for example [G]).

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