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ON VECTOR QUASIVARIATIONAL–LIKE INEQUALITY

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1. Introduction and Preliminaries

Recently, Giannessi [1] introduced a vector variational inequality for vector-valued functions in an Euclidean space. Since then, Chen et al. [2-6], Lee et al. [7], and Yang [8] have intensively studied vector variational inequalities for vector-valued functions in abstract spaces. In particular, Lee et al. [9,10] have obtained the existence theorem for solutions of generalized vector variational inequalities for set-valued maps with vector values in abstract spaces. Very recently, Yang [11] has established the existence theorem of a vector variational-like inequality for vector-valued functions and obtain the relationship between a vector optimization problem and a vector variational-like inequality.

Our motivation of this paper is to obtain existence theorems for solutions of vector quasivariational-like inequalities for vector-valued functions.

Let E be a topological vector space and C a nonempty, compact and convex subset of E, Y a topological vector space with a convex cone P such that $int\ P \neq \emptyset$ and $P \neq Y$, where int denotes the interior. Let $T : C \to L(E, Y)$ be a function where $L(E, Y)$ is the space of all linear continuous functions from E into Y, $\eta : C \times C \to E$ a function and $K : C \to 2^C$ a set-valued map.

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We will use the following conventions ; for any $x, y \in Y$,

1. $x \leqq_P y$ if and only if $x - y \in -P$.
2. $x \leq_P y$ if and only if $x - y \in -P \setminus \{0\}$.
3. $x \nleq_P y$ if and only if $x - y \notin -P \setminus \{0\}$.
4. $x <_P y$ if and only if $x - y \in -int\ P$.
5. $x \nless_P y$ if and only if $x - y \notin -int\ P$.

Consider the following vector quasivariational-like inequality $(VQV - LI)$:

$$(VQV - LI) \ : \text{Find } \overline{x} \in C \ \text{ such that } \overline{x} \in K(\overline{x}) \text{ and}$$
$$\langle T(\overline{x}),\ \eta(x, \overline{x})\rangle \nleq_P 0 \ \text{ for any } \ x \in K(\overline{x}).$$

and the following weak vector quasivariational-like inequality $(WVQV - LI)$:

$$(WVQV - LI) \ : \text{Find } \overline{x} \in C \ \text{ such that } \overline{x} \in K(\overline{x}) \text{ and}$$
$$\langle T(\overline{x}),\ \eta(x, \overline{x})\rangle \nless_P 0 \ \text{ for any } \ x \in K(\overline{x}).$$

where $\langle s, y\rangle$ denotes the evaluation of s at y.

We notice that $(WVQV - LI)$ is a weak form of $(VQV - LI)$.

When $Y = \mathbb{R}^n$, $P = \mathbb{R}^n_+$, $T = (T_1, \cdots, T_n) : C \to L(E, \mathbb{R}^n)$, where $T_i : C \to L(E, \mathbb{R})$ for each i, $\eta : C \times C \to E$, and $K : C \to 2^C$, $(WVQV - LI)$ reduce to the following vector variational-like inequality $(VV - LI)$ investigated by Yang [11];

$$(VV - LI) \ : \ \text{Find } \overline{x} \in C \ \text{ such that}$$
$$(\langle T_1(\overline{x}),\ \eta(x, \overline{x})\rangle, \cdots, \langle T_n(\overline{x}),\ \eta(x, \overline{x})\rangle) \nless_P 0 \ \text{ for any } \ x \in K(\overline{x}).$$

When $E = \mathbb{R}^m$, $C \subset E$, $Y = \mathbb{R}$, $P = \mathbb{R}_+$, $T : C \to \mathbb{R}^m$, $\eta : C \times C \to \mathbb{R}^m$ and $K : C \to 2^C$, $(VQV - LI)$ and $(WVQV - LI)$ reduce to the following quasivariational-like inequality $(QV - LI)$ investigated by Parida et al. [12].

$$(QV - LI) \ : \ \text{Find } \overline{x} \in C \ \text{ such that } \overline{x} \in K(\overline{x}) \text{ and}$$
$$T(\overline{x})^t \eta(x, \overline{x}) \geqq 0 \ \text{ for any } \ x \in K(\overline{x}).$$

When $E = \mathbb{R}^m$, $C \subset E$, $Y = \mathbb{R}^n$, $P = \mathbb{R}^n_+$, $T = (T_1, \cdots, T_n) : C \to \mathbb{R}^{n\times m}$, $\eta(x, \overline{x}) = x - \overline{x} \in \mathbb{R}^m$ and for each $x \in C, K(x) \equiv C, (VQV-LI)$ becomes the following vector variational inequality (VVI) introduced by Giannessi [1].

(VVI) : Find $\overline{x} \in C$ such that there is no $x \in C$ such that $\langle T_i(\overline{x}),\ x - \overline{x}\rangle \leqq 0,\ \ i = 1, \cdots, n$ and $\langle T_i(\overline{x}),\ x - \overline{x}\rangle \neq 0$ for at least one i.

Our purpose of this paper is to obtain the existence theorems of solutions for $(VQV-LI)$ and $(WVQV-LI)$ by the scalarization method and obtain the relationship between a vector optimization problem and a vector variational-like inequality. In Section 2, we prove a generalized Ky Fan inequality and the existence theorems of solutions for $(VQV-LI)$ and $(WVQV-LI)$ by the scalarization method. In Section 3, we establish the relationship between a vector optimization problem and a vector variational-like inequality.

DEFINITION 1.1. Let F be a set valued map from a toplogical space E into a topological space Y.

1. F is said to be upper semicontinuous (shortly u.s.c.) at $x_0 \in E$ if for every open set V containing $F(x_0)$, there exists a neighborhood $N(x_0)$ of x_0 such that $F(x) \subset V$ for all $x \in N(x_0)$. F is called u.s.c. on E if F is u.s.c. at every $x_0 \in E$.
2. F is said to closed if the graph of F : $\{(x, y)\ :\ y \in F(x)\}$ is closed.
3. F is said to be lower semicontinuous (shortly l.s.c.) at $x_0 \in E$ if for every open set V which intersects $F(x_0)$, there exists a neighborhood $N(x_0)$ of x_0 such that $F(x) \cap V \neq \emptyset$ for all $x \in N(x_0)$. F is called l.s.c. on E if F is l.s.c. at every $x_0 \in E$.
4. F is said to be continuous on E if F is u.s.c. and l.s.c.

The following theorem is a generalized form of Kakutani's fixed point theorem in [13, p.218].

THEOREM 1.1. *Let E be a locally convex topological vector space and C a nonempty compact convex subset of E. If $F : C \to 2^C$ is u.s.c.*

such that for each $x \in C, F(x)$ is a nonempty compact convex subset of C, then there exists $\overline{x} \in C$ such that $\overline{x} \in F(\overline{x})$.

2. Existence Theorems

First, we give definitions for the existence theorems for vector quasi-variational-like inequalities.

DEFINITION 2.1. Let E be a vector space and C a convex subset of E. Let Y be a topological vector space with a closed convex cone P such that $int\ P \neq \emptyset$ and $P \neq Y$ and $g : C \to Y$ a function.

1. g is said to be P-convex if for any $x, y \in C$ and $\alpha \in [0, 1]$,

$$g(\alpha y + (1-\alpha)x) \leqq_P \alpha g(y) + (1-\alpha)g(x).$$

2. [14, 15] g is said to be natural quasi P-convex if for any $x, y \in C$ and $\alpha \in [0, 1]$,

$$g(\alpha y + (1-\alpha)x) \in co\{g(y),\ g(x)\} - P.$$

REMARK 2.1. 1. Every P-convex function is natural quasi P-convex. 2. g is natural quasi P-convex if and only if there exists $\mu \in [0, 1]$ such that

$$g(\alpha y + (1-\alpha)x) \leqq_P \mu g(y) + (1-\mu)g(x).$$

LEMMA 2.1 [16]. *Let Y be a topological vector space with dual space Y^* and P a convex cone in Y with $int\ P \neq \emptyset$. Then we have*

$$int\ P = \{x \in Y\ :\ \langle x^*, x\rangle > 0 \text{ for all } x^* \in P^* \setminus \{0\}\}.$$

Also if Y is a reflexive locally convex topological vector space with dual space Y^* and P is a closed and convex cone with $int\ P^* \neq \emptyset$, then

$$int\ P^* = \{x^* \in Y^*\ :\ \langle x^*, x\rangle > 0 \text{ for all } x \in P \setminus \{0\}\},$$

where $P^* = \{x^* \in Y^*\ :\ \langle x^*, x\rangle \geqq 0 \text{ for all } x \in P\}$.

The following lemma is the special case of Theorem 1 in Chang et al. [17]. However, for the completeness, we prove it by using the results of Aubin [18].

LEMMA 2.2. *Let C be a compact convex subset of a locally convex topological vector space E, $G : C \times C \to \mathbb{R}$ a function and $K : C \to 2^C$ a set-valued map, where $\mathbb{R}$ is the real number system. Suppose that the following conditions are satisfied;*

(1) *for each $x \in C$, $G(x,x) \geqq 0$;*
(2) *G is continuous on $C \times C$;*
(3) *for each $x \in C$, $G(x,\cdot)$ is quasiconvex;*
(4) *K is continuous on C;*
(5) *for each $x \in C$, $K(x)$ is compact and convex.*

Then there exists $\overline{x} \in C$ such that $\overline{x} \in K(\overline{x})$ and $0 \leqq G(\overline{x},y)$ for any $y \in K(\overline{x})$.

Proof. As in [12], for each $x \in C$, define a set-valued map $V : C \to 2^C$ by for any $x \in C$,

$$V(x) = \left\{ s \in K(x) \ : \ G(x,s) = \inf_{y \in K(x)} G(x,y) \right\}.$$

Then, since for each $x \in C, G(x,\cdot)$ is continuous and $K(x)$ is compact, then $V(x) \neq \emptyset$ and $V(x)$ is compact. By (3), for each $x \in C, V(x)$ is convex. By the conditions (2) and (4), and Theorem 3 in [18, p.70], V is u.s.c.. By Theorem 1.1, there exists $\overline{x} \in C$ such that $\overline{x} \in V(\overline{x})$. Thus there exists $\overline{x} \in C$ such that $\overline{x} \in K(\overline{x})$ and $G(\overline{x},\overline{x}) \leqq G(\overline{x},y)$ for any $y \in K(\overline{x})$. By the condition (1), we have

$$0 \leqq G(\overline{x},y) \ \text{for any} \ y \in K(\overline{x}).$$

Now we prove existence theorems for $(VQV-LI)$ and $(WVQV-LI)$ by the scalarization method.

THEOREM 2.1. *Let C be a compact convex subset of a locally convex topological vector space E, Y a reflexive locally convex topological vector space with a closed convex cone P such that int $P^* \neq \emptyset$. Let $T : C \to L(E,Y)$ be a function, $\eta : C \times C \to E$ a function and $K : C \to 2^C$ a set-valued map. Suppose that the following conditions are satisfied;*

(1) *for each $x \in C$, $0 \leqq_P \langle T(x),\ \eta(x,x)\rangle$;*

(2) $\langle T(\cdot),\ \eta(\cdot,\cdot)\rangle$ *is continuous on* $C \times C$*;*
(3) *for each* $x \in C$, $\langle T(x),\ \eta(\cdot, x)\rangle$ *is natural quasi* P*-convex;*
(4) K *is continuous on* C*;*
(5) *for each* $x \in C$, $K(x)$ *is compact and convex.*
Then the vector quasivariational-like inequality $(VQV - LI)$ *is solvable.*

Proof. Define a function $G : C \times C \to Y$ by for any $x, y \in C$

$$G(x, y) = \langle T(x),\ \eta(y, x)\rangle .$$

Choose $s \in int\ P^*$ and define $g(x, y) = (s \cdot G)(x, y)$ for any $x, y \in C$. Then $g : C \times C \to \mathbb{R}$. By (1), for each $x \in C$, $g(x, x) \geqq 0$. By (2), g is continuous. By (3) and Remark 1.1, for each $x \in C$, $g(x, \cdot)$ is quasiconvex. Hence by Lemma 2.2, there exists $\overline{x} \in K(\overline{x})$ such that

$$g(\overline{x}, y) \geqq 0 \text{ for any } y \in K(\overline{x}) \tag{2.1}$$

If there exists $y^* \in K(\overline{x})$ such that $G(\overline{x}, y^*) \leq_P 0$, by Lemma 2.1, we have

$$(s \cdot G)(\overline{x}, y^*) < 0,$$

which contradicts (2.1). Hence we have

$$G(\overline{x}, y) \not\leq_P 0 \text{ for any } y \in K(\overline{x}).$$

Thus, we have

$$\langle T(\overline{x}),\ \eta(x, \overline{x})\rangle \not\leq_P 0 \text{ for any } x \in K(\overline{x}).$$

By Theroem 2.1, we can obtain the following corollary;

COROLLARY 2.1. *Let* C *be a compact convex subset of a Banach space* E, Y *a reflexive Banach space with a closed convex cone* P *such that* $int\ P^* \neq \emptyset$. *Let* $T : C \to L(E, Y)$ *be a function,* $\eta : C \times C \to E$ *a function and* $K : C \to 2^C$ *a set-valued map. Suppose that the following conditions are satisfied;*
(1) *the conditions (1), (3), (4) and (5) of Theorem 2.1 hold;*
(2) T *is continuous and* η *is continuous.*

Then the vector quasivariational-like inequality $(VQV-LI)$ *is solvable.*

Proof. Let $\{(x_n, y_n)\}$ be a sequence in $C \times C$ converging to (x, y). By (2), there exists $M > 0$ such that $\|\eta(y_n, x_n)\| \leq M$ for sufficiently large n

$$\begin{aligned} & \| \langle T(x_n),\ \eta(y_n, x_n)\rangle - \langle T(x),\ \eta(y, x)\rangle \| \\ = & \| \langle T(x_n) - T(x),\ \eta(y_n, x_n)\rangle + \langle T(x),\ \eta(y_n, x_n) - \eta(y, x)\rangle \| \\ \leqq & \|T(x_n) - T(x)\| \|\eta(y_n, x_n)\| + \|T(x)\| \|\eta(y_n, x_n) - \eta(y, x)\| \\ \leqq & M\|T(x_n) - T(x)\| + \|T(x)\| \|\eta(y_n, x_n) - \eta(y, x)\| \to 0 \text{ as } n \to \infty. \end{aligned}$$

Hence all the conditions of Theorem 2.1 hold. By Theorem 2.1, the result of Corollary 2.1 holds.

By choosing $s \in P^* \setminus \{0\}$ and using the similar method in the proof of Theorm 2.1, we can obtain the following theorem;

THEOREM 2.2. *Let* C *be a compact convex subset of a locally convex topological vector space* E, Y *a locally convex topological vector space with a closed convex cone* P *such that* $int\ P \neq \emptyset$ *and* $P \neq Y$. *Let* $T : C \to L(E, Y)$ *be a function,* $\eta : C \times C \to E$ *a function and* $K : C \to 2^C$ *a set-valued map. If all the conditions of Theorem 2.1 are satisfied, then the weak vector quasivariational-like inequality* $(WVQV - LI)$ *is solvable.*

By the method similar to the proof of Corollary 2.1, we can obtain the following corollary from Theorem 2.2.

COROLLARY 2.2. *Let* C *be a compact convex subset of a Banach space* E, Y *a Banach space with a closed convex cone* P *such that* $int\ P \neq \emptyset$ *and* $P \neq Y$. *Let* $T : C \to L(E, Y)$ *be a function,* $\eta : C \times C \to E$ *a function and* $K : C \to 2^C$ *a set-valued map. If all the conditions of Corollary 2.1 are satisfied, then the weak quasivariational-like inequality* $(WVQV - LI)$ *is solvable.*

REMARK 2.2. Corollary 2.2 is a generalization of Theorem 3.3 in Yang [11].

3. Vector Optimization Problem

Now, we give the relationship between a vector optimization problem and a vector variational-like inequality.

Let E be a Banach space, Y a Banach space with a closed convex cone P such that $int\ P \neq \emptyset$ and $P \neq Y$, and C a subset of E.

DEFINITION 3.1 [19, 20, 21]. 1. A set C is said to be η-connected if for any $x, y \in C$ and $\alpha \in [0, 1]$, $x + \alpha\eta(y, x) \in C$
2. A function $g : E \to Y$ is said to be preinvex over a η-connected subset C of E w.r.t. η if $\eta : C \times C \to E$ is a function and for any $x, y \in C$ and any $\alpha \in [0, 1]$,

$$g\,(x + \alpha\eta(y, x)) \leqq_P \alpha g(y) + (1 - \alpha)g(x).$$

3. A linearly Gâteaux differentiable function $g : E \to Y$ is said to be invex over a η-connected subset C of E w.r.t. η if $\eta : C \times C \to E$ is a function and for any $x, y \in C$,

$$\langle g'(x),\ \eta(y, x)\rangle \leqq_P g(y) - g(x),$$

where g' is the Gâteaux derivative of g at x.

PROPOSITION 3.1. *Let $g : E \to Y$ be a linearly Gâteaux differentiable function. If g is preinvex over C w.r.t. η, then g is invex over C w.r.t. η.*

Proof. Since g is preinvex , we have, for any $x, y \in C$

$$\lim_{\alpha \to 0+} \frac{g\,(x + \alpha\eta(y, x)) - g(x)}{\alpha} \leqq_P g(y) - g(x).$$

Hence we have

$$\langle g'(x),\ \eta(y, x)\rangle \leqq_P g(y) - g(x).$$

REMARK 3.1. Proposition 3.1 is the infinite-dimensional version of Theorem 2 in Ben-Israel and Mond [21].

Let C be a nonempty η-connected subset of E. Consider the following vector optimization problem;

$$(P) \qquad \text{Minimize} \quad g(x)$$
$$\text{subject to} \quad x \in C$$

where $g : C \to Y$ is a linearly Gâteaux differentiable function. We say that $\overline{x}$ is a weak minimum of (P) if for any $x \in C$, $g(x) \not<_P g(\overline{x})$.

THEOREM 3.1. *Let $T(\overline{x}) = g'(\overline{x})$ for any $x \in C$ and g invex over C w.r.t. η. If $\overline{x}$ is a solution of a vector variational-like inequality $(VV - LI)$: Find $\overline{x} \in C$ such that for any $x \in C$, $\langle T(\overline{x}),\ \eta(x, \overline{x})\rangle \not<_P 0$, then $\overline{x}$ is a weak minimum of (P).*

Proof. Since $\overline{x}$ is a solution of $(VV - LI)$, we have

$$\langle g'(\overline{x}),\ \eta(x, \overline{x})\rangle \not<_P 0 \text{ for any } x \in C. \tag{3.1}$$

Since g is invex, $g(x) - g(\overline{x}) - \langle g'(\overline{x}),\ \eta(x, \overline{x})\rangle \geqq_P 0$ for any $x \in C$. Hence $\overline{x}$ is a weak minimum of (P). In fact, suppose that there exists $z \in C$ such that $g(z) <_P g(\overline{x})$. Then we have

$$\begin{aligned} & - \langle g'(\overline{x}),\ \eta(z, \overline{x})\rangle \\ &= g(z) - g(\overline{x}) - \langle g'(\overline{x}),\ \eta(z, \overline{x})\rangle + g(\overline{x}) - g(z) \\ &\in P + int\ P \\ &= int\ P. \end{aligned}$$

Hence $\langle g'(\overline{x}),\ \eta(z, \overline{x})\rangle <_P 0$, which contradicts (3.1).

THEOREM 3.2. *If $\overline{x} \in C$ is a weak minimum of (P), then $\overline{x}$ is a solution of a vector variational-like $(VV - LI)$: Find $\overline{x} \in C$ such that $\langle g'(\overline{x}),\ \eta(x, \overline{x})\rangle \not<_P 0$ for any $x \in C$.*

Proof. Since $\overline{x} \in C$ is a weak minimum of (P), $g(x) - g(\overline{x}) \in Y \setminus (-int\ P)$ for any $x \in C$. Hence $[g(\overline{x} + \alpha\eta(x, \overline{x})) - g(\overline{x})]/\alpha \in Y \setminus (-int\ P)$ for any $\alpha \in (0, 1]$. Thus $\langle g'(\overline{x}),\ \eta(x, \overline{x})\rangle \not<_P 0$ for any $x \in C$.

REMARK 3.2. Theorem 3.1 and 3.2 are the infinite-dimensional versions of results in Yang [10].

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