A BOUNDED CONVERGENCE THEOREM FOR THE OPERATOR-VALUED FEYNMAN INTEGRAL

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1. Introduction

Fix $t > 0$. Denote by $C^t$ the space of $\mathbb{R}$-valued continuous functions $x$ on $[0, t]$. Let $C^t_0$ be the Wiener space $- C^t_0 = \{ x \in C^t : x(0) = 0 \}$ equipped with Wiener measure $m$. Let $F$ be a function from $C^t$ to $\mathbb{C}$. Given $\lambda > 0, \psi \in L^2(\mathbb{R})$ and $\xi \in \mathbb{R}$, let

$$ (K_\lambda(F)\psi)(\xi) = \int_{C^t_0} F(\lambda^{-\frac{1}{2}}x + \xi)\psi(\lambda^{-\frac{1}{2}}x(t) + \xi) \, dm(x). $$

**Definition.** The operator-valued function space integral $K_\lambda(F)$ exists for $\lambda > 0$ if (1.1) defines $K_\lambda(F)$ as a bounded linear operator on $L^2(\mathbb{R})$. If, in addition, the operator-valued function $K_\lambda(F)$, as a function of $\lambda$, has an extension to an analytic function in $\mathbb{C}_+ = \{ \lambda \in \mathbb{C} : Re\lambda > 0 \}$ and a strongly continuous function in $\hat{\mathbb{C}}_+ = \{ \lambda \in \mathbb{C} : Re\lambda \geq 0, \lambda \neq 0 \}$, we say that $K_\lambda(F)$ exists for $\lambda \in \hat{\mathbb{C}}_+$. When $\lambda$ is purely imaginary, $K_\lambda(F)$ is called the operator-valued Feynman integral of $F$.

For $s > 0, \lambda \in \hat{\mathbb{C}}_+$ and $\psi \in L^2(\mathbb{R})$, let

$$ (exp[-s(H_0/\lambda)]\psi)(\xi) = \frac{\lambda}{2\pi s}^{\frac{1}{2}} \int_{\mathbb{R}} \psi(u)exp(-\frac{\lambda(u - \xi)^2}{2s}) \, du. $$

The integral in (1.2) exists as an ordinary Lebesgue integral for $\lambda \in \mathbb{C}_+$, but, when $\lambda$ is purely imaginary and $\psi$ is not integrable, the integral

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should be interpreted in the mean as in the theory of the Fourier-Plancherel transform.

In this paper, \( \theta \) is a bounded Borel measurable and everywhere defined real valued function on \((0, t) \times \mathbb{R}\) and we will let \( M := ||\theta||_{\infty} \).

Let \( \eta \) be a finite signed Borel measure on \((0, t)\). Then \( \eta \) has a unique decomposition \( \eta = \mu + \eta_d \) into a continuous part \( \mu \) and a discrete part \( \eta_d[8] \). The case where \( \eta_d \) has a finite support is most likely to be of interest. So, let

\[
\eta_d = \sum_{j=1}^{N} \omega_j \delta_{\tau_j}
\]

where \( \delta_{\tau_j} \) is as usual the Dirac measure at \( \tau_j \in (0, t) \), \( 0 < \tau_1 < \cdots < \tau_N < t \) and \( \omega_j \in \mathbb{R} \) for \( j = 1, 2, \cdots, N \).

Let \( \mathcal{M}(\mathbb{R}) \) be the space of complex Borel measures on \( \mathbb{R} \). The Fourier transform of \( \nu \in \mathcal{M}(\mathbb{R}) \) is the function \( \hat{\nu} \) defined by

\[
\hat{\nu}(u) = \int_{\mathbb{R}} e^{-iu} d\nu(v), \quad u \in \mathbb{R}.
\]

Consider the functional

\[
F(x) = \hat{\nu}\left( \int_{(0,t)} \theta(s, x(s)) d\eta(s) \right), \quad x \in C^t.
\]

Then, by [1], \( K_\lambda(F) \) exists for \( \lambda > 0 \). Also \( K_\lambda(F') \) exists for \( \lambda \in \tilde{C}_+ \) and is given by the generalized Dyson series, provided that

\[
\int_{\mathbb{R}^d} e^{M||\eta||||u||} d|\nu|(u) < \infty,
\]

i.e. for all \( \lambda \in \tilde{C}_+ \), the following expansion of \( K_\lambda(F) \) hold:

\[
K_\lambda(F) = \sum_{n=0}^{\infty} n! a_n \sum_{q_0 + \cdots + q_N = n} \frac{\omega_{q_1} \cdots \omega_{q_N}}{q_1! \cdots q_N!} \sum_{k_1 + \cdots + k_{N+1} = q_0} \int_{\Delta_{q_0, k_1, \cdots, k_{N+1}}} L_0 L_1 \cdots L_N d\mu(s_1) \cdots d\mu(s_{q_0})
\]

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where \( q_0, \cdots, q_N, k_1, \cdots, k_{N+1} \) are nonnegative integers.

\[
\Delta_{q_0;k_1,\cdots,k_{N+1}} = \{ (s_1, \cdots, s_{q_0}) \in (0, t)^{q_0} : 0 < s_1 < \cdots < s_{k_1} < \tau_1 < s_{k_1+1} < \cdots < s_{k_1+k_2} < \tau_2 < s_{k_1+k_2+1} < \cdots < s_{k_1+\cdots+k_N} < \tau_N < s_{k_1+\cdots+k_{N+1}} < \cdots < s_{q_0} < t \}
\]

and, for \((s_1, \cdots, s_{q_0}) \in \Delta_{q_0;k_1,\cdots,k_{N+1}} \) and \( r \in \{0, 1, \cdots, N\} \)

\[
L_r = [\theta(\tau_r)]^{q_r} e^{-\left(s_{k_1+\cdots+k_r+1} - \tau_r\right)(H_0/\lambda)} \theta(s_{k_1+\cdots+k_r+1}) \cdot \cdots \cdot e^{-\left(s_{k_1+\cdots+k_r+2} - s_{k_1+\cdots+k_r+1}\right)(H_0/\lambda)} \theta(s_{k_1+\cdots+k_r+2}) \cdot \cdots \cdot \theta(s_{k_1+\cdots+k_r+1}) e^{-\left(\tau_{r+1} - s_{k_1+\cdots+k_r+1}\right)(H_0/\lambda)}
\]

and

\[
a_n = \frac{1}{n!} \int_{\mathbb{R}} (-i)^n u^n \, d\nu(u).
\]

We use the conventions \( \tau_0 = 0, \tau_{N+1} = t \) and \( [\theta(\tau_0)]^{q_0} = 1 \).

## 2. A stability theorem

We begin with a lemma which will be useful in the main theorems.

**Lemma.** Let \( \{F_n(x)\} \) be a sequence of Borel measurable functionals such that \(|F_n(x)| \leq B\) for some constant \( B > 0 \) and for all \( n = 1, 2, 3, \cdots \). Further suppose that for every \( \lambda > 0 \)

\[
F_n(\lambda^{-\frac{1}{2}} x + \xi) \to F(\lambda^{-\frac{1}{2}} x + \xi) \quad \text{as} \quad n \to \infty
\]

for \( m \times \text{Leb. - a.e.} \ (x, \xi) \). Then for every \( \lambda > 0 \)

\[
K_\lambda(F_n) \to K_\lambda(F) \quad \text{strongly as} \quad n \to \infty.
\]
Proof. Let $\lambda > 0$, $\psi \in L^2(\mathbb{R})$ and $\xi \in \mathbb{R}$ be given. By (2.1), for $m \times \text{Leb. - a.e. } (x, \xi)$,

$$
(2.2) \quad F_n(\lambda^{-\frac{1}{2}} x + \xi) \psi(\lambda^{-\frac{1}{2}} x(t) + \xi) \to F(\lambda^{-\frac{1}{2}} x + \xi) \psi(\lambda^{-\frac{1}{2}} x(t) + \xi).
$$

Note that for every $x \in C_0^1$, for a.e. $\xi \in \mathbb{R}$ and for all $n = 1, 2, 3, \cdots$

$$
(2.3) \quad |F_n(\lambda^{-\frac{1}{2}} x + \xi) \psi(\lambda^{-\frac{1}{2}} x(t) + \xi)| \leq B |\psi(\lambda^{-\frac{1}{2}} x(t) + \xi)|.
$$

In view of (2.2), (2.3), and the Dominated Convergence Theorem for Wiener integrals,

$$
(2.4) \quad (K_\lambda(F_n)\psi)(\xi) \to (K_\lambda(F)\psi)(\xi) \text{ for Leb. - a.e. } \xi.
$$

Moreover, by (2.3) and Wiener's integration formula

$$
(2.5) \quad |(K_\lambda(F_n)\psi)(\xi)| \leq \int_{C_0^1} |F_n(\lambda^{-\frac{1}{2}} x + \xi) \psi(\lambda^{-\frac{1}{2}} x(t) + \xi)| \, dm(x)
$$

$$
\leq B \int_{C_0^1} |\psi(\lambda^{-\frac{1}{2}} x + \xi)| \, dm(x)
$$

$$
= B(e^{-t(H_0/\lambda)}|\psi|)(\xi)
$$

for every $n = 1, 2, \cdots$ and a.e. $\xi \in \mathbb{R}$.

Since $e^{-t(H_0/\lambda)}|\psi| \in L^2(\mathbb{R})$, using (2.4), (2.5) and the Lebesgue Dominated Convergence Theorem, we have

$$
(2.6) \quad K_\lambda(F_n) \to K_\lambda(F)
$$

in $L^2(\mathbb{R})$.

The first theorem treats the case $\lambda > 0$.

Theorem 1. Let $\eta$ be a finite signed Borel measure on $(0, t)$ and let $\nu \in M(\mathbb{R})$. Suppose that $\theta$ and $\theta_m$, $m = 1, 2, \cdots$ are all bounded by $M$ on $(0, t) \times \mathbb{R}$. Let $F$ be defined as (1.5) and $F_m$ be defined as (1.5) except with $\theta$ replaced by $\theta_m$. Assume that

$$
(2.7) \quad \theta_m \to \theta
$$
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at each point of \((0, t) \times \mathbb{R}\) as \(m \to \infty\). Then for all \(\lambda > 0\),

\[ (2.8) \quad K_\lambda(F_m) \to K_\lambda(F) \quad \text{strongly as} \quad m \to \infty. \]

Proof. Let \(\lambda > 0, x \in C^t_0\) and \(\xi \in \mathbb{R}\) be given. Since \(\theta_m\) is bounded by \(M\) for all \(m = 1, 2, \cdots\), by (2.7),

\[ (2.9) \quad \int_{(0, t)} \theta_m(s, \lambda^{-\frac{1}{2}} x(s) + \xi) \, d\eta(s) \to \int_{(0, t)} \theta(s, \lambda^{-\frac{1}{2}} x(s) + \xi) \, d\eta(s) \]

Since \(\hat{\nu}\) is continuous,

\[ (2.10) \quad \hat{\nu}\left(\int_{(0, t)} \theta_m(s, \lambda^{-\frac{1}{2}} x(s) + \xi) \, d\eta(s)\right) \to \hat{\nu}\left(\int_{(0, t)} \theta(s, \lambda^{-\frac{1}{2}} x(s) + \xi) \, d\eta(s)\right) \]

\(i.e.\ \text{\(F_m(\lambda^{-\frac{1}{2}} x + \xi) \to F(\lambda^{-\frac{1}{2}} x + \xi)\).\) Note that

\[ (2.11) \quad |F_m(x)| = |\hat{\nu}\left(\int_{(0, t)} \theta_m(s, x(s)) \, d\eta(s)\right)| \leq ||\hat{\nu}|| \]

for all \(x \in C^t\) and for all \(m = 1, 2, \cdots\). Hence (2.11) and Lemma give the result for \(\lambda > 0\).

We now obtain a stability result for \(\lambda \in \mathring{C}_+\) under the assumption that the measure \(|\nu|\) dies off rapidly at \(\infty\).

Theorem 2. Let \(\theta\) and \(\theta^{(m)}, m = 1, 2, \cdots\) be everywhere defined \(\mathbb{R}\)-valued and Borel measurable functions bounded by \(M\) on all of \((0, t) \times \mathbb{R}\). Let \(\eta = \mu + \eta_d\) be a finite signed Borel measure on \((0, t)\) where \(\eta_d\) is given by (1.3), and let \(\nu \in \mathcal{M}(\mathbb{R})\) be such that

\[ (2.12) \quad \int_{\mathbb{R}} e^{M||\eta||u} \, d|\nu||(u) < \infty. \]

Assume that

\[ (2.13) \quad \theta^{(m)} \to \theta \quad \text{as} \quad m \to \infty \quad \eta \times \text{Leb. - a.e. on} \quad (0, t) \times \mathbb{R}. \]
Let $F$ and $F^{(m)}$ be defined as in Theorem 1. Then for all $\lambda \in \mathbb{C}_+$

\begin{equation}
(2.14) \quad K_\lambda(F^{(m)}) \to K_\lambda(F) \text{ strongly as } m \to \infty.
\end{equation}

Further, the operator $K_\lambda(F)$ preserves the form of the operator $K_\lambda(F^{(m)})$; to be more specific,

\begin{equation}
(2.15) \quad K_\lambda(F^{(m)}) = \sum_{n=0}^{\infty} n! a_n \sum_{q_0+\cdots+q_N=n} \frac{\omega_{q_1}^{q_1} \cdots \omega_{q_N}^{q_N}}{q_1! \cdots q_N!} \sum_{k_1+\cdots+k_{N+1}=q_0} \int_{\Delta_{q_0;k_1,\ldots,k_{N+1}}} L_0^{(m)} L_1^{(m)} \cdots L_{\lambda}^{(m)} d\mu(s_1) \cdots d\mu(s_{q_0})
\end{equation}

\rightarrow

\begin{equation}
K_\lambda(F) = \sum_{n=0}^{\infty} n! a_n \sum_{q_0+\cdots+q_N=n} \frac{\omega_{q_1}^{q_1} \cdots \omega_{q_N}^{q_N}}{q_1! \cdots q_N!} \sum_{k_1+\cdots+k_{N+1}=q_0} \int_{\Delta_{q_0;k_1,\ldots,k_{N+1}}} L_0 L_1 \cdots L_N d\mu(s_1) \cdots d\mu(s_{q_0})
\end{equation}

strongly as $m \to \infty$;

where $q_0, \ldots, q_N, k_1, \ldots, \ldots, k_{N+1}$ are nonnegative integers and $\Delta_{q_0;k_1,\ldots,k_{N+1}}$ is given by (1.8) and, for $(s_1, \ldots, s_{q_0}) \in \Delta_{q_0;k_1,\ldots,k_{N+1}}$ and $r \in \{0, 1, \cdots, N\}$, $L_r$ is given by (1.9), and $L_r^{(m)}$ is given as in (1.9) except with $\theta$ replaced by $\theta^{(m)}$ and $a_n$ is given by (1.10).

Proof. Let $\psi \in L^2(\mathbb{R})$ be given. Let $\theta^{(m)}(s)$ denote the operator of multiplication by $\theta^{(m)}(s, \cdot)$ so that $(\theta^{(m)}(s)\psi)(\xi) = \theta^{(m)}(s, \xi)\psi(\xi)$ for all $\xi \in \mathbb{R}$. So, by (2.13)

\begin{equation}
(2.16) \quad (\theta^{(m)}(s)\psi)(\xi) \to (\theta(s)\psi)(\xi) \text{ as } m \to \infty
\end{equation}

Leb. $- a.e.$ for $\eta - a.e. s \in (0, t)$. But

\begin{equation}
(2.17) \quad |(\theta^{(m)}(s)\psi)(\xi) - (\theta(s)\psi)(\xi)|^2 \\
\leq (|\theta^{(m)}(s, \xi)||\psi(\xi)| + |\theta(s, \xi)||\psi(\xi)||^2 \\
\leq 4M^2|\psi(\xi)|^2.
\end{equation}
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Since \( \psi \in L^2(\mathbb{R}) \), next using (2.16) and the Lebesgue Dominated Convergence Theorem, we have

\[
\| \theta^{(m)}(s)\psi - \theta(s)\psi \|_2 \rightarrow 0 \quad \text{as} \quad m \rightarrow \infty;
\]

i.e. \( \theta^{(m)}(s) \rightarrow \theta(s) \) strongly as \( m \rightarrow \infty \) for \( \eta - a.e. \ s \).

Using (2.18) and the fact that the composition of operator is jointly continuous in the strong operator topology when the operators involved are uniformly bounded we see that

\[
L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)} \rightarrow L_0 L_1 \cdots L_N
\]

strongly \( \mu \times \cdots \times \mu - a.e. \) in \( \Delta_{q_0; k_1, \ldots, k_{N+1}} \).

Note that \( L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)}(\lambda; s_1, \ldots, s_{q_0}) \) is strongly measurable [7].

Since \( \theta^{(m)} \) is bounded by \( M \) and \( \|e^{-s(H_0/\lambda)}\| \leq 1 \)

\[
\|L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)} \psi\| \leq M^n \|\psi\|_2.
\]

Further,

\[
\sum_{k_1 + \cdots + k_{N+1} = q_0} \int_{\Delta_{q_0; k_1, \ldots, k_{N+1}}} \|L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)} \psi\| d|\mu|(s_1) \cdots d|\mu|(s_{q_0})
\]

\[\leq M^n \|\psi\|_2 \int_{\Delta_{q_0}} d|\mu|(s_1) \cdots d|\mu|(s_{q_0})
\]

\[\leq M^n \|\psi\|_2 \frac{\|\mu\|_{q_0}}{q_0!} < \infty.
\]

So, \( L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)} \) is Bochner integrable over \( \Delta_{q_0; k_1, \ldots, k_{N+1}} \). Note that since \( \mu \) is a finite signed Borel measure on \((0, t)\)

\[
M^n \|\mu\| \in L_1(\Delta_{q_0; k_1, \ldots, k_{N+1}}; \mu \times \cdots \times \mu).
\]
Therefore, using (2.19) and the Dominated Convergence Theorem for the Bochner integral [3], we have that $L_0 L_1 \cdots L_N$ is Bochner integrable and

\begin{equation}
(2.23) \quad \int_{\Delta_{q_0:k_1,\ldots,k_{N+1}}} L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)} \, d\mu(s_1) \cdots d\mu(s_{q_0})
\end{equation}

\rightarrow

\begin{equation}
\int_{\Delta_{q_0:k_1,\ldots,k_{N+1}}} L_0 L_1 \cdots L_N \, d\mu(s_1) \cdots d\mu(s_{q_0})
\end{equation}

in $L^2(\mathbb{R})$. Set

\begin{equation}
(2.24) \quad \mathcal{L}_n^{(m)} := \sum_{q_0 + \cdots + q_N = n} \left( \frac{\omega_1^{q_1} \cdots \omega_N^{q_N}}{q_1! \cdots q_N!} \right) \sum_{k_1 + \cdots + k_{N+1} = q_0} \int_{\Delta_{q_0:k_1,\ldots,k_{N+1}}} L_0^{(m)} L_1^{(m)} \cdots L_N^{(m)} \, d\mu(s_1) \cdots d\mu(s_{q_0})
\end{equation}

and

\begin{equation}
(2.25) \quad \mathcal{L}_n := \sum_{q_0 + \cdots + q_N = n} \left( \frac{\omega_1^{q_1} \cdots \omega_N^{q_N}}{q_1! \cdots q_N!} \right) \sum_{k_1 + \cdots + k_{N+1} = q_0} \int_{\Delta_{q_0:k_1,\ldots,k_{N+1}}} L_0 L_1 \cdots L_N \, d\mu(s_1) \cdots d\mu(s_{q_0}).
\end{equation}

Then

\begin{equation}
(2.26) \quad \mathcal{L}_n^{(m)} \rightarrow \mathcal{L}_n \text{ strongly as } m \rightarrow \infty.
\end{equation}
Furthermore, (2.27)

\[
\|L^{(m)}_n \psi\| \leq \sum_{q_0 + \cdots + q_N = n} \frac{|\omega_1|^{q_1} \cdots |\omega_N|^{q_N}}{q_1! \cdots q_N!} \sum_{k_1 + \cdots + k_{N+1} = q_0} \int_{\Delta_{q_0,k_1,\ldots,k_{N+1}}} \|L^{(m)}_0 \cdot L^{(m)}_1 \cdots L^{(m)}_N \psi\| \, d|\mu|(s_1) \cdots d|\mu|(s_{q_0})
\]

\[
\leq \sum_{q_0 + \cdots + q_N = n} \frac{|\omega_1|^{q_1} \cdots |\omega_N|^{q_N}}{q_1! \cdots q_N!} M^n \|\psi\| \int_{\Delta_{q_0}} d|\mu|(s_1) \cdots d|\mu|(s_{q_0})
\]

\[
= \sum_{q_0 + \cdots + q_N = n} \frac{|\omega_1|^{q_1} \cdots |\omega_N|^{q_N}}{q_1! \cdots q_N!} M^n \|\psi\| \frac{||\mu||^{q_0}}{q_0!}
\]

\[
= M^n \|\psi\| \frac{1}{n!} \sum_{q_0 + \cdots + q_N = n} \frac{n!}{q_0! \cdots q_N!} ||\mu||^{q_0} |\omega_1|^{q_1} \cdots |\omega_N|^{q_N}
\]

\[
= M^n \|\psi\| \left( \frac{||\mu|| + |\omega_1| + \cdots + |\omega_N|}{n!} \right)^n
\]

\[
= M^n \|\psi\| \frac{||\eta||^n}{n!}.
\]

Similarly

(2.28)

\[
\|L_n \psi\| \leq M^n \frac{||\eta||^n}{n!} \|\psi\|.
\]

Let \( \epsilon > 0 \) be given. Using (2.12), take \( N_0 \) so large that

(2.29)

\[
\sum_{n = N_0 + 1}^{\infty} |a_n| M^n \frac{||\eta||^n}{n!} \| \psi \| < \frac{\epsilon}{4}.
\]

Now using (2.26), let \( N \) be so large that for \( m \geq N \)

(2.30)

\[
\sum_{n = 1}^{N_0} n! |a_n| \| L^{(m)}_n \psi - L_n \psi \| < \frac{\epsilon}{2}.
\]
Now, let \( m \geq N \). Then using (2.30), (2.27), (2.28) and (2.29), (2.31)

\[
||K_\lambda(F^{(m)})\psi - K_\lambda(F)\psi|| \\
= ||\sum_{n=0}^{\infty} n!a_n L_n^{(m)}\psi - \sum_{n=0}^{\infty} n!a_n L_n \psi|| \\
= ||\sum_{n=0}^{N_0} (n!a_n L_n^{(m)}\psi - n!a_n L_n \psi) + \sum_{n=N_0+1}^{\infty} (n!a_n L_n^{(m)}\psi - n!a_n L_n \psi)|| \\
\leq \sum_{n=0}^{N_0} n!|a_n|||L_n^{(m)}\psi - L_n \psi|| + \sum_{n=N_0+1}^{\infty} n!|a_n|||L_n^{(m)}\psi|| \\
+ \sum_{n=N_0+1}^{\infty} n!|a_n|||L_n \psi|| \\
< \frac{\epsilon}{2} + \sum_{n=N_0+1}^{\infty} n!|a_n|\left|\frac{M^n||\eta||^n}{n!}\right||\psi|| \\
+ \sum_{n=N_0+1}^{\infty} n!|a_n|\left|\frac{M^n||\eta||^n}{n!}\right||\psi|| \\
= \frac{\epsilon}{2} + 2 \sum_{n=N_0+1}^{\infty} |a_n|M^n||\eta||^n||\psi|| \\
< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon \text{ as desired.}
\]

We can obtain a corollary immediately from a simple standard result of functional analysis.

**COROLLARY 1.** Let the hypotheses of Theorem 2 be satisfied and suppose that \( ||\psi_m - \psi|| \to 0 \) as \( m \to \infty \). Then

\[
||K_\lambda(F^{(m)})\psi_m - K_\lambda(F)\psi|| \to 0 \quad \text{as} \quad m \to \infty.
\]

**References**

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