STEEPEST DESCENT METHOD FOR
LOCALLY ACCRETIVE MAPPINGS

C. E. CHIDUME

1. Introduction

Let $E$ be a real normed linear space, $K \subseteq E$. A mapping $A : K \rightarrow E$
is called strongly pseudocontractive if there exists $t > 1$ such that the inequality

$$
\|x - y\| \leq \|(1 + t)(x - y) - r t(Ax - Ay)\|
$$

holds for all $x, y \in K$ and $r > 0$. If $t = 1$ then $A$ is called pseudocontractive. The map $A$ is called locally strongly pseudocontractive if each point of $K$ has a neighbourhood $N$ for which (1) holds for each $x, y \in N$ and some $t > 1$. Pseudocontractive operators have been studied by various authors (see e.g., [1], [2], [4], [8-12], [14], [16], [17], [18], [19], [21], [22], [28], [29], [30], [32-33], [37]). Interest in such mappings stems mainly from the fact that they are firmly connected with the important class of nonlinear accretive operators. A mapping $U$ with domain $D(U)$ and range $R(U)$ in $E$ is called accretive (see e.g., [2], [15]) if the inequality

$$
\|x - y\| \leq \|x - y + t(Ux - Uy)\|
$$

holds for each $x, y \in D(U)$ and all $t > 0$. The accretive operators were introduced independently by Browder [3] and Kato [15]. If $E = H$, a Hilbert space, one of the earliest problems in the theory of accretive operators was to solve the equation $x + Ux = f$ for $x$, given an element $f$ of $H$ and an accretive operator $U$. We remark here that in Hilbert spaces, accretive operators are also called monotone. In [3],

Received April 18, 1994.

1991 AMS Subject Classification: 47H04, 47H05, 47H06, 47H10, 47H15, 47H17.

Key words and phrases: Local strong pseudo-contractions; $q$-uniformly smooth spaces; Mann iteration process; Ishikawa iteration process.
Browder proved that if $U$ is locally Lipschitzian and accretive then $U$ is $m$-accretive, that is, $(I + U)$ is surjective. This result was subsequently generalized by Martin [20] to the continuous accretive operators.

The firm connection between the pseudocontractive mappings and the accretive operators is that a mapping $U$ is pseudocontractive if and only if $(I - U)$ is accretive [3, Proposition 1]. Consequently, the mapping theory for accretive operators is closely related to the fixed point theory of pseudocontractive operators.

It is well known (see for example, [4]) that many physically significant problems can be modelled in terms of an initial value problem of the form

$$
\begin{cases}
\frac{dx}{dt} &= -Ux \\
x(0) &= x_0
\end{cases}
$$

(2)

where $U$ is either accretive or strongly accretive. Typical examples of how such evolution equations arise are found in models involving either the heat, the wave or the Schrödinger equation. Let $N(U)$ denote the kernel of $U$. We observe that members of $N(U)$ are, in fact, the equilibrium points of the system (2). Consequently, considerable effort has been devoted to developing constructive techniques for the determination of the kernels of accretive operators (see e.g., [5], [6], [7], [8-12], [13], [14], [22], [23-25], [27], [28], [29], [30], [32], [33], [35], [36], [37]). Moreover, since a continuous accretive operator can be approximated well by a sequence of strongly accretive ones, particular attention has been devoted to constructive techniques for the kernels of strongly accretive operators. In this connection, but in Hilbert space, Vainberg [35] and Zarantonello [39] introduced the steepest descent method:

$$x_{n+1} = x_n - c_nUx_n, \quad x_0 \in H, \quad n = 0, 1, 2, \ldots$$

(3)

and proved that if $U = I + T$ where $T$ is a monotone Lipschitz map and $c_n = \lambda, n = 0, 1, 2, \ldots; \lambda$ a constant, then the sequence $\{x_n\}$ defined by (3) converges strongly to an element of $N(U)$. This result has been generalized and extended to more general Banach spaces (see e.g., [5], [8-12], [2], [23-26], [28], [29], [32], [33], [37]). Recently, the author proved the following theorem:
Theorem 1 ([8]). Suppose \( K \) is a nonempty closed bounded and convex subset of \( L_p, p \geq 2 \), and \( T : K \rightarrow K \) is a Lipschitz strongly pseudocontractive mapping of \( K \) into itself. Let \( \{c_n\} \) be a real sequence satisfying:

1. \( 0 < c_n < 1 \) for all \( n \geq 1 \),
2. \( \sum_{n=1}^{\infty} c_n = \infty \); and
3. \( \sum_{n=0}^{\infty} c_n^2 < \infty \).

Then the sequence \( \{x_n\}_{n=0}^{\infty} \) generated by \( x_1 \in K \),

\[
x_{n+1} = x_n - c_n \ Ax_n, \quad n \geq 1
\]

converges strongly to a solution of the equation \( Ax = 0 \) where \( A = I - T \).

Several authors have generalized and extended Theorem 1 in various directions. In [32], Schu extended the theorem to the class of continuous strongly pseudocontractive maps in real Banach spaces with property \((U, \alpha, m+1, m)\) (see e.g., [32] for definition). These Banach spaces include the \( L_p \) spaces, \( p \geq 2 \); and in [33] he extended the theorem to the class of uniformly continuous maps in smooth Banach spaces. Bethlike [1] obtained a slight generalization of the theorem still in \( L_p \) spaces, \( p \geq 2 \); the author [10] and also Osilike [22] extended the theorem to the class of continuous strongly pseudocontractive maps on real uniformly smooth Banach spaces. Other generalizations can be found in Xu, Zhang and Roach [30]. The most general result for the global convergence of (4) for strongly accretive maps seems to be the main result of Xu and Roach [28] (see also a result of the author, [12]).

A natural problem of interest (see e.g., [14], [37]) is to prove convergence theorems for approximating solutions of \( Ax = 0 \) when \( A \) is locally accretive and a solution is known to exist.

It is our purpose in this paper to prove that in real \( q \)-uniformly smooth Banach spaces (defined below) the steepest descent approximation method (4) converges strongly to a solution of the equation \( Ax = 0 \) (when one exists) for locally strongly accretive operators, \( A \). In particular, our result (Theorem 2) will extend Theorem 1 to real \( q \) uniformly smooth Banach spaces (which include the \( L_p \) spaces, \( 1 < p < \infty \)) and to the class of locally strongly pseudocontractive maps (see our Remarks 1 and 2). Furthermore, since Banach spaces with property
$(U, \alpha, m + 1, m)$ are $q$-uniformly smooth, Theorem 2 also extends the
result of Schu (Theorem 1 of [32]) to these more general Banach spaces
and to operators which are continuous and locally strongly pseudocon-
tractive, while Theorem 4 extends Theorem 2 of [32] to the class of
locally Lipschitz continuous and strongly pseudocontractive maps. In
addition, we shall prove a theorem (Theorem 3) on the convergence of
the iteration process (4) to a solution of the equation $x + Ux = f$ where
$U$ is a continuous locally accretive map on a real $q$-uniformly smooth
Banach space. This result is related to the results of Bruck [5], the
author [9] and Carbone [6].

2. Preliminaries

Let $E$ be a Banach space. We shall denote by $J$ the normalized
duality mapping from $E$ to $2^{E^*}$ given by

$$Jx = \{f^* \in E^* : \|f^*\|^2 = \|x\|^2 = \langle x, f^* \rangle\}$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. If $E$ is uniformly
convex then $J$ is single-valued, and is uniformly continuous on bounded
sets. In the sequel we shall denote single-valued normalized duality map
by $j$.

Now, with $p > 1$, following [38], we shall associate the generalized
duality map $J_p$ from $E$ to $E^*$ defined by

$$J_p(x) = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^p, \quad \text{and} \quad \|f^*\| = \|x\|^{p-1}\}.$$ 

In particular, $J_2$ is the usual normalized duality map on $E$. It is known
(see e.g., [38]) that

$$J_p(x) = \|x\|^{p-2} J(x) \quad \text{for} \quad x \neq 0. \quad (5)$$

Let $E$ be a Banach space with $\dim E \geq 2$. The modulus of smooth-
ness $\rho_E(\tau), \tau > 0$, of $E$ is defined by

$$\rho_E(\tau) = \sup\{ (\|x + y\| + \|x - y\|)^2 - 1 : x, y \in E, \quad \|x\| = 1, \|y\| = \tau\}.$$
The Banach space $E$ is uniformly smooth (see e.g., [34]) if
\[ \lim_{\tau \to 0} \rho_E(\tau)/\tau = 0, \] and $E$ is called $q$-uniformly smooth (see e.g., [38]) if there exists a constant $c > 0$ such that
\[ \rho_E(\tau) \leq c \tau^q, \quad 0 < \tau < \infty. \]

It is known (see e.g., [38], [34]) that
\[ L_p \text{ is } \begin{cases} p - \text{uniformly smooth if } & 1 < p \leq 2 \\ 2 - \text{uniformly smooth if } & p \geq 2. \end{cases} \]

A Banach space $E$ is called smooth (see e.g., [34], p.60) if, for every $x \in E$ with $\|x\| = 1$, there exists a unique $f^* \in E^*$ such that $\|f^*\| = f^*(x) = 1$. In [38], the following result which will be needed in the sequel is proved.

**Lemma 1 ([38]).** Let $q > 1$ be a real number and $E$ be a smooth Banach space. Then the following are equivalent:

(i) $E$ is $q$-uniformly smooth;

(ii) There is a constant $c > 0$ such that for every $x, y \in E$, the following inequality holds:

\[ \|x + y\|^q \leq \|x\|^q + q(y, J_q(x)) + c\|y\|^q \]

A mapping $U$ is called locally strongly accretive if each point in the domain of $U$ has a neighbourhood $N$ for which there exist a constant $k > 0$ and $j(x - y) \in J(x - y)$ such that

\[ \langle Ux - Uy, j(x - y) \rangle \geq k\|x - y\|^2. \]

holds for $x, y \in N$.

The following lemma has been proved:

**Lemma 2 ([37]).** Let $E$ be a real Banach space, $K$ a subset of $E$ and $U : K \to E$. Then $U$ is locally strongly pseudocontractive if and only if $(I - U)$ is a locally strongly accretive.
3. Main results

In the sequel, $c$ will denote the constant appearing in inequality (6). We prove the following theorems.

**Theorem 2.** Let $E$ be a real $q$-uniformly smooth Banach space. Suppose $T$ is a continuous locally strongly accretive map with open domain $D(T)$ in $E$ and that $Tx = 0$ has a solution $x^*$ in $D(T)$. Then there exist a neighbourhood $B$ in $D(T)$ of $x^*$ and a real number $r_1 > 0$ such that for any $r > r_1$ and some real sequence $\{c_n\}$, any initial guess $x_1 \in B$, the sequence $\{x_n\}$ generated from $x_1$ by

$$x_{n+1} = x_n - c_n \quad T \quad x_n, \quad n \geq 1,$$

remains in $D(T)$ and converges strongly to $x^*$ with

$$\|x_n - x^*\| = O(n^{-(q-1)/q}).$$

**Proof.** Since $T$ is locally strongly accretive, there exists a neighbourhood $U$ of $x^*$ such that for each $x \in U$,

$$(Tx - Tx^*, j(x - x^*)) \geq k\|x - x^*\|^2.$$

Accretiveness of $T$ on $U$ implies $T$ is locally bounded at each interior point of $U$ (see e.g., Rockafellar [31], Reich [26]). So, we can choose $B = B_d(x^*)$, the closed ball of radius $d > 0$, $B \subseteq U$ so that $T(B)$ is bounded and $T$ is strongly accretive on $B$. Let $D$ be a constant such that $2d + \text{diam}(T(B)) \leq D$. Let $r_1 = [c^1/qD]^{q/(q-1)} (dk)^{-q/(q-1)}$. Then $r_1 > 0$ and for $r \geq r_1$,

$$D \leq r^{(q-1)/q} dk \ c^{-q-1}.$$

Let $c_n = \frac{1}{k(n+r)}$, $d_n = \frac{1}{k(n+r-1)(q-1)/q}$. Observe that $(1 - k\ c_n)^q d_n^q + c_n^q = d_{n+1}^q$.

Starting with an initial guess $x_1 \in B$, define the sequence $\{x_n\}_{n=1}^{\infty}$ inductively by (8).

**Claim** For all $n \geq 1$, $x_n$ is well defined and

$$\|x_n - x^*\| \leq d_n \ d \ r^{(q-1)/q}. k.$$
The proof of this claim is by induction. For \( n = 1 \), \( x_n \) is clearly in \( B \). Suppose now that the claim has been proved for a particular choice of \( n \). Then,

\[
\|x_n - x^*\| \leq d_1 \, d \, r^{(q-1)/q} k = d, \quad \text{so} \quad x_n \in B.
\]

Thus, \( x_n \) is well defined by (8). Using (5), (6), (7) and the induction hypothesis, we obtain:

\[
\|x_{n+1} - x^*\|^q = \|(1 - c_n)(x_n - x^*) + c_n(Sx_n - Sx^*)\|^q,
\]

where \( Sx := x - Tx \) for each \( x \in B \). Observe that \( x^* \) is a solution of \( Tx = 0 \) and only if it is a fixed point of \( S \). Moreover,

\[
\langle Sx_n - Sx^*, J_q(x_n - x^*) \rangle = \langle x_n - x^* - (Tx_n - Tx^*), J_q(x_n - x^*) \rangle \\
= \|x_n - x^*\|^q - \langle Tx_n - Tx^*, J_q(x_n - x^*) \rangle \\
\leq (1 - k)\|x_n - x^*\|^q.
\]

Hence, from (10), using (6):

\[
\|x_{n+1} - x^*\|^q \leq (1 - c_n)^q \|x_n - x^*\|^q \\
+ q \, c_n(1 - c_n)^{q-1} \langle Sx_n - Sx^*, J_q(x_n - x^*) \rangle + c \, c_n^q \|Sx_n - Sx^*\|^q \\
\leq [(1 - c_n)^q + q(1 - k)c_n(1 - c_n)^{q-1}] \|x_n - x^*\|^q \\
+ c \, c_n^q \|Sx_n - Sx^*\|^q,
\]

For \( x \in (0, 1) \), consider the function

\[
f(x) = (1 + x)^q, \quad q > 1
\]

Then, there exists \( \xi \in (0, x) \) such that

\[
f(x) = f(0) + xf'(0) + x^2 \frac{f''(\xi)}{2} = 1 + xq + \frac{x^2}{2} f''(\xi).
\] (i)

Observe that \( f''(\xi) \geq 0 \). Set \( x = (1 - k)c_n(1 - c_n)^{-1} \) in (i) to get,

\[
\left[1 + \frac{(1 - k)c_n}{1 - c_n}\right]^q = 1 + \frac{q(1 - k)c_n}{(1 - c_n)} + \frac{(1 - k)^2 c_n^2 f''(\xi)}{2}
\]
which simplifies to
\[
[1 - c_n + (1 - k)c_n]^q
= (1 - c_n)^q + q(1 - k)c_n(1 - c_n)^{q-1} + \frac{1}{2}(1 - k)^2c_n^2(1 - c_n)^{q-2}f''(\xi)
\]
and implies (since \(f''(\xi) \geq 0\)):
\[
(1 - c_n)^q + q(1 - k)c_n(1 - c_n)^{q-1} \leq [1 - c_n + (1 - k)c_n]^q = (1 - kc_n)^q
\]
Hence, using this inequality, (11) yields:
\[
\|x_{n+1} - x^*\|^q \leq (1 - kc_n)^q\|x_n - x^*\|^q + c c_n^q\|Sx_n - Sx^*\|^q.
\]
Observe that \(\|Sx_n - Sx^*\| \leq D\) so that
\[
\|x_{n+1} - x^*\|^q \leq (1 - kc_n)^q\|x_n - x^*\|^q + c c_n^qD^q
\]
which implies, by induction hypothesis
\[
\|x_{n+1} - x^*\|^q \leq [(1 - kc_n)^qd_n^q + c_n^q]d^q r^{q-1} k^q = d_{n+1}^q r^{q-1} k^q d^q
\]
so that
\[
\|x_{n+1} - x^*\| \leq d_{n+1}dk r^{(q-1)/q},
\]
completing the induction process. Since \(d_n = O(n^{-(q-1)/q})\), the error estimate of the theorem has also been established. This completes the proof.

**Corollary 1.** Let \(E\) be a real \(q\)-uniformly smooth Banach space. Suppose \(U\) is a continuous locally strongly pseudointractive map with open domain \(D(U)\) in \(E\) and that \(U\) has a fixed point in \(D(U)\). Then there exist a neighbourhood \(B\) in \(D(U)\) of \(x^*\) and a real number \(r_1 > 0\) such that for any \(r > r_1\) and some real sequence \(\{c_n\}\), any initial guess \(x_1 \in B\), the sequence \(\{x_n\}\) generated from \(x_1\) by
\[
x_{n+1} = x_n - c_n(I - U)x_n \quad n \geq 1,
\]
remains in \(D(U)\) and converges strongly to \(x^*\) with
\[
\|x_n - x^*\| = O(n^{-(q-1)/q}).
\]

**Proof.** Follows immediately from Lemma 2 and Theorem 1.
REMARK 1. In [14], the author claimed to have generalized Theorem 1 to \textit{locally} Lipschitzian and strongly pseudocontractive operators in $L_p$ spaces, $p \geq 2$. He stated that if the mapping $U : D(U) \to E(E = L_p, p \geq 2)$ is locally Lipschitzian and strongly pseudocontractive, then there exists a closed region $B(x^*)$ containing a solution $x^*$ of the equation $Tx = y$ such that, for arbitrary $x_0 \in B(x^*)$, the process $x_{n+1} = x_n + \lambda(y - Tx_n)$ for a suitable $\lambda$ converges strongly to the solution $x^*$. However, as has already rightly been observed (MR. 92h:47090) the author fails to prove the existence of the region $B(x^*)$ where the iteration process is well defined. Moreover, there are several other inconsistencies in this result (see e.g., MR. 92h:47090).

REMARK 2. In [37], the author claimed to have extended Theorem 1 to general uniformly smooth Banach spaces $E$ and to the class of \textit{local} strongly pseudocontractive operators. He published the following theorem:

THEOREM XW ([37]). Let $K$ be a subset of a uniformly smooth Banach space $E$ and $U : K \to E$ be a local pseudocontractive mapping. If $F(U) = \{x \in K : Ux = x\} \neq \emptyset$ and the range of $U$ is bounded, then $\{x_n\} \subseteq K$ generated by $x_1 \in K$,

$$x_{n+1} = x_n - c_n(I - U)x_n$$

with $\{c_n\} \subseteq (0, 1]$, satisfying: $\sum_{n=1}^{\infty} c_n = \infty$, $c_n \to 0$, converges strongly to $x^* \in F(U)$ and $F(U)$ is a singleton set.

We remark immediately that the sequence $\{x_n\}$ in Theorem XW is not even well defined, as can be seen from the following easy example.

COUNTER-EXAMPLE TO THEOREM XW. Take $E = \ell_2, K = \{x \in \ell_2 : \|x\| \leq 1\}$. Define $U : K \to E$ by

$$U(x_1, x_2, x_3, \ldots) = (-4x_1, -4x_2, -4x_3, \ldots)$$

for arbitrary $(x_1, x_2, x_3, \ldots) \in K$ Then,

(i) $E$ is clearly uniformly smooth;
(ii) $Ux = x$ if and only if $x = 0$. Hence $F(U) \neq \emptyset$.
(iii) $\|Ux\| \leq 4$ for each $x \in K$. Hence, the range of $U$ is bounded
(iv) $\langle (I - U)x - (I - U)y, j(x - y) \rangle = 5\|x - y\|^2$ for each $x, y \in K$. 


Now, choose \( c_n = \frac{1}{n+1}, n = 1, 2, \ldots \) and \( x_1 = (1, 0, 0, \ldots) \in K \). Then \( x_2 = (-\frac{3}{2}, 0, 0, \ldots) \notin K \), and so \( x_3 \) is not defined. In fact, the above choice of \( x_1 \) is not crucial. For example, for any \( \lambda \in (\frac{\theta}{2}, 1), x_1 = (\lambda, 0, 0, \ldots) \in K \) and \( x_2 = (-\frac{3}{2} \lambda, 0, 0, \ldots) \notin K \). Again \( x_3 \) is not defined. Other choices are obviously possible. This completes the counter-example.

We now prove the following theorem on the convergence of the steepest-descent method to a solution of the equation \( x + T x = f \) for a locally accretive operator \( T \) in \( q \)-uniformly smooth Banach spaces.

**Theorem 3.** Let \( E \) be a real \( q \)-uniformly smooth Banach space. Suppose \( T \) is a continuous locally accretive map with open domain \( D(T) \) in \( E \) and that \( f \in R(I + T) \). Suppose the equation \( x + T x = f \) has a solution \( x^* \in D(T) \). Then there exist a neighborhood \( B \subseteq D(T) \) of \( x^* \) and a real number \( r_1 > 0 \) such that for any \( r > r_1 \), any initial guess \( x_1 \in B \), the sequence \( \{x_n\}_{n=1}^{\infty} \) generated from \( x_1 \) by

\[
(12) \quad x_{n+1} = x_n - c_n (I - f + T)x_n, \quad n = 1, 2, \ldots
\]

for some real sequence \( \{c_n\}_{n=1}^{\infty} \) remains in \( D(T) \) and converges strongly to \( x^* \) with

\[
\|x_n - x^*\| = O(n^{-(q-1)/q}).
\]

**Proof.** Let \( x^* \) denote a solution of the equation \( x + T x = f \). So, as in the proof of Theorem 2, we can choose \( B = B_d(x^*) \), the closed unit ball of radius \( d > 0 \), \( B \subseteq D(T) \) so that \( T(B) \) is bounded and \( T \) is accretive on \( B \). Let

\[
r_1 = \left[ \frac{C^1}{q} \text{diam } T(B) \right]^{q/(q-1)} d^{1/(q-1)}.
\]

Then \( r > 0 \) and \( \text{diam } T(B) \leq r^{(q-1)/q} d^{-q} \) for \( r \geq r_1 \). Let \( c_n = \frac{1}{n+r}, d_n = \frac{1}{(n+r-1)^{1/(q-1)}} \) so that \((1 - c_n)^q d_n^q + c_n^q = d_{n+1}^q \). Starting with an initial guess \( x_1 \in B \), define the sequence \( \{x\}_{n=1}^{\infty} \) inductively by (12). As in the proof of Theorem 2, \( \{x_n\} \) is well defined by (12). We now prove

\[
\|x_n - x^*\| \leq d_n d^{r^{(q-1)/q}}.
\]
Now, using an induction argument as in the proof of Theorem 2, we have,

\[ \|x_{n+1} - x^*\|^q \leq (1 - c_n)^q \|x_n - x^*\|^q - q \ c_n (1 - c_n)^{q-1} \left\langle T x_n - T x^*, J_q(x_n - x^*) \right\rangle + c \ c_n^q \|T x^* - T x_n\|^q. \]

Since \(c_n(1 - c_n) \geq 0\) and \(T\) is accretive, it follows that

\[ \|x_{n+1} - x^*\|^q \leq (1 - c_n)^q \|x_n - x^*\|^q + c \ c_n^q \|T x^* - T x_n\|^q. \]

Using the induction hypothesis and the fact that \(T x_n\) and \(T x^*\) belong to \(T(B)\), the last inequality yields:

\[ \|x_{n+1} - x^*\|^q \leq ((1 - c_n)^q d_n^q + c_n^q) d_n^q r^{(q-1)} = d_{n+1}^q \ r^{(q-1)} \]

so that \(\|x_{n+1} - x^*\| \leq d_{n+1} d \ r^{(q-1)/q}\), completing the induction argument and completing the proof of the theorem.

**Corollary 2.** Let \(E\) be a real \(q\)-uniformly smooth Banach space. Suppose \(U\) is continuous locally pseudocontractive map with open domain \(D(U)\) in \(E\) and that \(U\) has a fixed point \(x^*\) in \(D(U)\). Then there exist a neighbourhood \(B\) in \(D(U)\) of \(x^*\) and a real number \(r_1 > 0\) such that for any \(r > r_1\) and for some real sequence \(\{c_n\}_{n=1}^\infty\), any initial guess \(x_1 \in B\), the sequence \(\{x_n\}_{n=1}^\infty\) generated from \(x_1\) by

\[ x_{n+1} = x_n - c_n(\overline{i} - U)x_n, \quad n \geq 1, \]

remains in \(D(U)\) and converges strongly to \(x^*\) with

\[ \|x_n - x^*\| = O(n^{-(q-1)/q}). \]

**Proof.** Obvious, from Lemma 2 and Theorem 3.

**Acknowledgments.** The author is grateful to Professor Abdus Salam, the International Atomic Energy Agency, UNESCO and the International Centre for Theoretical Physics, Trieste, for support.
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International Centre for Theoretical Physics
Trieste, Italy