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ON PROJECTIVE REPRESENTATIONS OF A FINITE GROUP AND ITS SUBGROUPS I

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1. Introduction

Let G be a finite group and F be a field of characteristic $p \geq 0$. Let $\Gamma = F^fG$ be a twisted group algebra corresponding to a 2-cocycle $f \in Z^2(G, F^*)$, where $F^* = F - \{0\}$ is the multiplicative subgroup of F. By employing the concept of D_Γ-regularity where D_Γ-regular class is a certain F-class and goes back to Reynolds [6], it has been shown that the number of projective representations of G over F is related to the number of D_Γ-regular classes of G; indeed proved that the number of irreducible projective representations of G over F is equal to the number of D_Γ-regular classes of p'-elements in G. Over the complex field C, this property was originally proved by Schur [7] using "f-regular class".

In representation theory, connections between the representations of a group and those of its normal subgroups and factor groups have long been the object of study. These connections are developed by Clifford [2] for linear representations and extended by Mackey [5] to projective representations. The subject consists of three basic operations: restriction, extension and induction, and gives a motivation of this paper.

The purpose of this paper is to investigate how the number of projective representations of a group is related to the corresponding number of its subgroups. Due to the relationship between the number of representations and that of D_Γ-regular classes, the main purpose can be transfered to a question that how the condition that all F-classes of

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G are D_Γ-regular is related to corresponding condition on subgroups of G.

In what follows we denote the p-part of $t \in Z$ by t_p [resp. p'-part by $t_{p'}$], and a primitive t-th root of unity in an algebraic closure E of F by ζ_t. We refer $\mathcal{G} = \mathrm{Gal}(E/F)$ to the Galois group of E over F. As is convension, $\exp(G)$ denotes the exponent of G and $o(g)$ the order of $g \in G$.

2. Regularity Condition

Let G be a finite group and let F be a field of characteristic $p \geq 0$. The group G acts trivially on F. Let $\{a_g \mid g \in G\}$ be an F-basis of $\Gamma = F^f G$ such that $a_g a_x = f(g, x) a_{gx}$, $a_1 = 1_\Gamma$ for any $g, x \in G$.

(A) For each $\sigma \in \mathcal{G}$ and $g \in G$, choose the following for G:

(i) a positive integer n which is divisible by $\exp(G)$. Write $n = n_p n_{p'}$.

(ii) a positive integer $m(\sigma)$ such that $\zeta_{n_{p'}}^{\sigma} = \zeta_{n_{p'}}^{m(\sigma)}$, while $m(\sigma) \equiv 1 \pmod{n_p}$.

(iii) $v(g) \in E^*$ which is any n-th root of $u(g) \in F^*$ such that $a_g^n = u(g) a_1$.

Indeed $v(g)^n = \prod_{i=1}^{n-1} f(g^i, g)$ with respect to f. Upon the choices as above, for each (σ, x) in $\mathcal{G} \times G$, the mappings $D_\Gamma(\sigma, x)$ of Γ^E to Γ^E and $d_G(\sigma, x)$ of G to G are defined by

$$a_g D_\Gamma(\sigma, x) = v(g)^{\sigma^{-1}} v(g)^{-m(\sigma^{-1})} a_x^{-1} a_g^{m(\sigma^{-1})} a_x,$$

$$g d_G(\sigma, x) = x^{-1} g^{m(\sigma^{-1})} x.$$

The mappings D_Γ and d_G refer to permutation mappings as well as to conjugate actions; if $E = F$ then the maps are conjugate actions on Γ^E and G, respectively. The choices of n and $m(\sigma)$ make no use of f, and D_Γ and d_G do not depend on the particular choices of n, $m(\sigma)$, $v(g)$ and a_g (refer to [6] or [1]).

Two elements g, $x \in G$ are said to be *F-conjugate* if there exists $z \in G$ such that $x = z^{-1} g^{m(\sigma^{-1})} z$ for all $\sigma \in \mathcal{G}$. Clearly F-conjugacy is an equivalent relation so that G is a union of *F-(conjugate) classes.*

An element $g \in G$ is D_Γ*-regular* provided that $a_g D_\Gamma(\sigma, x) = a_g$ for any $(\sigma, x) \in \mathcal{G} \times G$ such that $x^{-1} g^{m(\sigma^{-1})} x = g$. If $F = E$ then the D_Γ-regularity is reduced to the well known concept of f-regularity, back to Schur (1904); g is f-regular if and only if $a_g a_x = a_x a_g$ for all $x \in G$ such that $gx = xg$. Without using any base elements, the f-regularity can be illustrated as $f(g, x) = f(x, g)$ for all $x \in G$ such that $gx = xg$. Analogously, we can derive a formula of D_Γ-regularity as follow:

THEOREM 1. *For each* $\sigma \in \mathcal{G}$, *choose* n *and* $m(\sigma)$ *satisfying the conditions (A). An element* $g \in G$ *is* D_Γ*-regular if and only if*

$$v(g)^{\sigma^{-1}} v(g)^{-m(\sigma^{-1})} \prod_{i=1}^{m(\sigma^{-1})-1} f(g^i, g) f(g^{m(\sigma^{-1})}, x) = f(x, g)$$

for any $(\sigma, x) \in \mathcal{G} \times G$ *such that* $x^{-1} g^{m(\sigma^{-1})} x = g$.

Proof. We shall write m for $m(\sigma^{-1})$ only for convenience. For $(\sigma, x) \in \mathcal{G} \times G$ such that $x^{-1} g^m x = g$, we have

$$\begin{aligned} a_g^m a_x &= \prod_{i=1}^{m-1} f(g^i, g) a_{g^m} a_x = \prod_{i=1}^{m-1} f(g^i, g) f(g^m, x) a_{g^m x} \\ &= \prod_{i=1}^{m-1} f(g^i, g) f(g^m, x) a_{xg} = \prod_{i=1}^{m-1} f(g^i, g) f(g^m, x) f^{-1}(x, g) a_x a_g. \end{aligned}$$

If g is D_Γ-regular then $v(g)^{\sigma^{-1}} v(g)^{-m} a_x^{-1} a_g^m a_x = a_g$, thus

$$\begin{aligned} a_x a_g &= v(g)^{\sigma^{-1}} v(g)^{-m} a_g^m a_x \\ &= v(g)^{\sigma^{-1}} v(g)^{-m} \prod_{i=1}^{m-1} f(g^i, g) f(g^m, x) f^{-1}(x, g) a_x a_g, \end{aligned}$$

hence $v(g)^{\sigma^{-1}} v(g)^{-m} \prod_{i=1}^{m-1} f(g^i, g) f(g^m, x) = f(x, g)$. The converse direction is also clear.

The D_Γ-regularity form in Theorem 1 does not depend on the basis of Γ, rather it depends on f explicitly. To stress its dependence on only f, we shall refer to D_Γ-regular as (F, f)-*regular*. Suppose that f and α in $Z^2(G, F^*)$ are cohomologous over an algebraically closed field F. Then $g \in G$ is f-regular if and only if g is α-regular (refer to [4, (3.6.1)]). This can be extended to the D_Γ-regularity.

LEMMA 2. *Suppose that f, $\alpha \in Z^2(G, F^*)$ are cohomologous over F. Then for any $g \in G$, g is D_Γ-regular if and only if g is D_Ω-regular where $\Gamma = F^f G$ and $\Omega = F^\alpha G$.*

Proof. Since f is cohomologous to α, there is $c : G \to F^*$ such that $\alpha(x, y) = c(x)c(y)\cdot c^{-1}(xy)f(x, y)$ for $x, y \in G$. Let $\{a_g | g \in G\}$ be an F-basis of Γ. Let $b_g = c(g)a_g$ for all $g \in G$. Then $\{b_g | g \in G\}$ forms an F-basis of Ω. But since $b_g \in \Gamma$, Γ and Ω are equal. Hence $D_\Gamma(\sigma, x) = D_\Omega(\sigma, x)$ for all $(\sigma, x) \in \mathcal{G} \times G$.

3. Restriction, Inflation and Corestriction Maps

Let H be a subgroup of G with $|G : H| = u > 0$, and M be a left G-module. Given a 2-cocycle $f \in Z^2(G, M)$, the restriction $f_H : H \times H \to M$ of f to H is a 2-cocycle in $Z^2(H, M)$. A map that sends f to f_H is the *restriction map on cocycle groups*

$$\mathrm{Res}_{G,H} : Z^2(G, M) \to Z^2(H, M),$$

and its induced map $\mathrm{Res}^*_{G,H} : H^2(G, M) \to H^2(H, M)$ is the *restriction map on cohomology groups*. These can be generalized to any order k, and $\mathrm{Res}^*_{G,H}(fB^k(G, M)) = (\mathrm{Res}_{G,H} f)B^k(H, M)$ for $f \in Z^k(G, M)$. If Γ_H is the twisted group algebra of H over F with basis $\{a_h | h \in H\}$, then $\Gamma_H \subseteq \Gamma$ and Γ_H equals the twisted group algebra $F^{f_H} H$.

In the case that H is a normal subgroup of G, we may consider $M^H = \{m \in M \mid hm = m,\ h \in H\}$ as a G/H-module. Then the *inflation map on cocycle groups*

$$\mathrm{Inf}_{G/H} : Z^2(G/H, M^H) \to Z^2(G, M)$$

is defined by $(\mathrm{Inf}_{G/H} f)(g, x) = f(gH, xH)$ for $f \in Z^2(G/H, M^H)$. Its induced homomorphism is $H^2(G/H, M^H) \to H^2(G, M)$ the *inflation map* $\mathrm{Inf}^*_{G/H}$ *on cohomology groups*.

Let H be any subgroup of G and let $S = \{s_1, \dots, s_\mu\}$, $s_1 = 1$ be a right transversal of H in G. Then $G = \cup_{i=1}^{\mu} Hs_i$ and for any $g \in G$ there exists a unique s_i such that $g \in Hs_i$. We shall write $\bar{g}$ for s_i, hence $g\bar{g}^{-1} \in H$. For $f \in Z^2(H, M)$, a map $Tf : G \times G \to M$ is defined by

$$Tf(g, x) = \prod_{i=1}^{\mu} s_i^{-1} \cdot f\left(s_i g \overline{(s_i g)}^{-1}, \overline{(s_i g)} x \overline{(s_i g x)}^{-1}\right),$$

for $g, x \in G$. Then Tf is in $Z^2(G, M)$ and the *corestriction map* $\mathrm{Cor}_{H,G}$ *on cocycle groups* relative to S is the homomorphism

$$\mathrm{Cor}_{H,G} : Z^2(H, M) \to Z^2(G, M)$$

that maps f to Tf. Further, $\mathrm{Cor}^*_{H,G} : H^2(H, M) \to H^2(G, M)$ is the *corestriction map on cohomology groups*. For any positive k, $\mathrm{Cor}^*_{H,G}(fB^k(H, M)) = (\mathrm{Cor}_{H,G} f) \cdot B^k(G, M)$ for $f \in Z^k(H, M)$. If $M = F^*$ with trivial G-action then $(\mathrm{Cor}_{H,G} f)(g_1, \cdots, g_k)$ is defined by

$$\prod_{i=1}^{\mu} f\left(s_i g_1 \overline{(s_i g_1)}^{-1}, \cdots, \overline{(s_i g_1 \cdots g_{k-1})} g_k \overline{(s_i g_1 \cdots g_k)}^{-1}\right).$$

If $k = 1$, the corestriction map on cohomology groups is the group theoretical transfer map. A known theorem due to Gaschütz is:

LEMMA 3. ([4, (2.3.23)]) *For $f \in Z^k(G, F^*)$, $k \geq 0$, the composition map $Cor^*_{H,G}Res^*_{G,H}$ on $H^k(G, F^*)$ sends $fB^k(G, F^*)$ to $f^\mu B^k(G, F^*)$. Hence $(Cor_{H,G}Res_{G,H})f$ is cohomologous to f^μ.*

LEMMA 4. *If $f \in Z^2(G, F^*)$ is a noncoboundary then for some Sylow q-subgroup Y_q of G, $(Res_{G,Y_q} f)$ is a noncoboundary.*

Proof. Let $|G| = q_1^{b_1} \cdots q_t^{b_t}$ where all q_i are distinct prime divisors of $|G|$ and $b_i \geq 1$, $1 \leq i \leq t$. For Sylow q_i-subgroup Y_{q_i}, $\mu_i = |G : Y_{q_i}| = q_1^{b_1} \cdots q_{i-1}^{b_{i-1}} \cdot q_{i+1}^{b_{i+1}} \cdots q_t^{b_t}$. By Lemma 3,

$$\left(\mathrm{Cor}^*_{Y_{q_i}, G}\mathrm{Res}^*_{G, Y_{q_i}}\right)(fB^2(G, F^*)) = f^{\mu_i} B^2(G, F^*)$$

so that $o\left(f^{\mu_i}B^2(G,F^*)\right)$ divides $o\left(f_{Y_{q_i}}B^2(Y_{q_i},F^*)\right)$. Suppose that every $f_{Y_{q_i}}$ is a coboundary. Then since

$$1 = o\left(f_{Y_{q_i}}B^2(Y_{q_i},F^*)\right) = o\left(f^{\mu_i}B^2(G,F^*)\right) = o\left(fB^2(G,F^*)^{\mu_i}\right),$$

every μ_i is divisible by $o\left(fB^2(G,F^*)\right)$. But every μ_i are relative primes, hence $o\left(fB^2(G,F^*)\right) = 1$ and f is a coboundary.

Among the three morphisms over cohomology groups, corestriction map is rather complicated than the other mappings, and still remains ackward. The fundamental source on corestriction map is the paper by Eckmann [3]. In [3, Theorem 7], he proved that the corestriction map on cohomology groups do not depend on the choices of transversal. However, the corestriction map on cocycle groups will be used here, and as far as we know, the map may depend on the choice made. It is therefore essential to choose suitable transversal in studying corestriction maps on cocycle groups. The next theorem will be useful for computation of $\mathrm{Cor} = \mathrm{Cor}_{H,G}$.

THEOREM 5. *Let $G = H \times K$ be a direct product with $|G : H| = \mu$, and $\alpha \in Z^2(H,F^*)$. Assume that K is the right transversal $\{s_1, \ldots, s_\mu\}$, $s_1 = 1$ of H in G with respect to which $\mathrm{Cor}\alpha$ is defined. Then for any $g, x \in G$,*

$$(\mathrm{Cor}_{H,G}\ \alpha)(g,x) = \alpha(g\bar{g}^{-1}, x\bar{x}^{-1})^{\mu},$$

where $\bar{g}$ means the unique s_i such that $g \in Hs_i$.

Proof. For each $g \in G$, $\bar{g}$ is the image of g under the natural projection of $G = H \times K$ onto K, and $g = h\bar{g}$ is the factorization of g where $h \in H$ and $\bar{g} \in K$. Thus we have

$$g\bar{g} = \bar{g}g,\ \overline{(g^i)} = \bar{g}^i, \text{ and } g^i\overline{(g^i)}^{-1} = (g\bar{g}^{-1})^i = h^i$$

for all $i \in Z$. In fact, $g\bar{g}g^{-1} = h\bar{g}\bar{g}\bar{g}^{-1}h^{-1} = h\bar{g}h^{-1} = \bar{g}$. Also $g^2 = h\bar{g}h\bar{g} = h^2\bar{g}^2$ yields $\overline{g^2} = \bar{g}^2$, hence $\overline{g^i} = \bar{g}^i$. Further, $s_ig\overline{(s_ig)}^{-1} = h = g\bar{g}^{-1}$. Indeed, $s_ig = s_i(h\bar{g}) = hs_i\bar{g}$ (the last equality is true, since $s_i \in K$ and $h \in H$), so that $\overline{s_ig} = s_i\bar{g}$, thus $s_ig\overline{(s_ig)}^{-1} = s_ig\cdot\bar{g}^{-1}s_i^{-1} = g\bar{g}^{-1} = h$. Similarly, let $x \in G$ and let $x = z\bar{x}$ where

$z \in H$, $\bar{x} \in K$. Then $\overline{(s_i g)} x \overline{(s_i g x)}^{-1} = z$. Thus Cor relative to the transversal K is

$$(\mathrm{Cor}_{H,G}\alpha)(g,x) = \prod_{i=1}^{\mu} \alpha(s_i g \overline{(s_i g)}^{-1}, \overline{(s_i g)} x \overline{(s_i g x)}^{-1})$$

$$= \prod_{i=1}^{\mu} \alpha(h,z) = \alpha(h,z)^{\mu} = \alpha(g\bar{g}^{-1}, x\bar{x}^{-1})^{\mu}.$$

4. Regularity Conditions on Groups: Global condition

This section contains several relations between D_Γ-regular F-classes of G and those of its subgroup. We ask how the condition that all F-classes of G are D_Γ-regular is related to corresponding condition on subgroups. As a global condition, we will prove in Theorem 6 that if all F-classes of G are D_Γ-regular then all F-classes of any subgroup H of G are D_{Γ_H}-regular. On the other hand, the local condition which is a converse question of Theorem 6 is somewhat troublesome. We will study the local condition in separate paper.

THEOREM 6. *Suppose that every F-class of G is (F,f)-regular. Then every F-class of H is (F,f_H)-regular, where $f \in Z^2(G,F^*)$ and $f_H = Res_{G,H} f$.*

The proof is clear since $f_H(h,k) = f(h,k)$ for any $h,k \in H$. Theorem 6 says that if the number of irreducible representations of G over F equals that of irreducible f-representations of G over F then the corresponding numbers with respect to H over F are same. Further this implies a simple fact that if every class of G is f-regular then every class of H is f_H-regular. In the case that f cobounds, every elements of G is of course (F,f)-regular. To avoid this uninteresting case, Lemma 4 and Theorem 1 together give the following corollary.

COROLLARY 7. *Suppose that G has all F-classes (F,f)-regular for $f \in Z^2(G,F^*)$ while f does not cobound. Then for some prime divisor q of $|G|$, a Sylow q-subgroup Y_q of G has all F-classes (F,f_{Y_q})-regular, while f_{Y_q} does not cobound.*

For two finite groups G_1 and G_2 with 2-cocycles $\alpha \in Z^2(G_1,F^*)$ and $\beta \in Z^2(G_2,F^*)$, define $\alpha\beta$ on the direct product $G_1 \times G_2$

by $\alpha\beta(hk, zu) = \alpha(h, z)\beta(k, u)$ for any $h, z \in G_1$ and $k, u \in G_2$. Certainly, $\alpha\beta \in Z^2(G_1 \times G_2, F^*)$.

THEOREM 8. *Let* $\alpha \in Z^2(G_1, F^*)$ *and* $\beta \in Z^2(G_2, F^*)$. *If every* F*-class of* G_1 *is* (F, α)*-regular and every* F*-class of* G_2 *is* (F, β)*-regular, then every* F*-class of* $G_1 \times G_2$ *is* $(F, \alpha\beta)$*-regular, and every* F*-class of* $G_1 \cap G_2$ *is* $(F, \alpha\beta_{G_1 \cap G_2})$*-regular. Conversely, if every* F*-class of* $G_1 \times G_2$ *is* (F, f)*-regular for* $f \in Z^2(G_1 \times G_2, F^*)$ *then every* F*-class of* G_i *is* (F, f_i)*-regular, where* f_i *is a restriction of* f *to* G_i, $(i = 1, 2)$.

Proof. We shall prove the first statement and the others follow from Theorem 6. Choose integers n and $m(\sigma)$ for $G_1 \times G_2$ as in (A). The integers work for both G_1 and G_2. For any $g \in G_1 \times G_2$, consider $(\sigma, x) \in \mathcal{G} \times (G_1 \times G_2)$ which satisfies $x^{-1}g^{m(\sigma^{-1})}x = g$. For convenience, write $m = m(\sigma^{-1})$, and let $g = hk$ and $x = zu$ for $h, z \in G_1$ and $k, u \in G_2$. Then $z^{-1}h^m z = h$ and $u^{-1}k^m u = k$, thus we have

$$v_{G_1}(h)^{\sigma^{-1}} v_{G_1}(h)^{-m} \prod_{i=1}^{m-1} \alpha(h^i, h)\alpha(h^m, z) = \alpha(z, h),$$

and

$$v_{G_2}(k)^{\sigma^{-1}} v_{G_2}(k)^{-m} \cdot \prod_{i=1}^{m-1} \beta(k^i, k)\beta(k^m, u) = \beta(u, k),$$

where $v_{G_1}(h)$ and $v_{G_2}(k)$ are as in (A) with respect to α and β, respectively. Since

$$\left(v_{G_1}(h)v_{G_2}(k)\right)^n = \prod_{i=1}^{n-1} \alpha(h^i, h)\beta(k^i, k) = \prod_{i=1}^{n-1} \alpha\beta(g^i, g),$$

we choose an n-th root $v(g)$ of $\prod_{i=1}^{n-1} \alpha\beta(g^i, g)$ as $v_{G_1}(h)v_{G_2}(k)$; this satisfies the conditions (A) with respect to $\alpha\beta$, therefore

$$\begin{aligned}
&v(g)^{\sigma^{-1}} v(g)^{-m} \prod_{i=1}^{m-1} \alpha\beta(g^i, g)\alpha\beta(g^m, x) \\
&= \left(v_{G_1}(h)v_{G_2}(k)\right)^{\sigma^{-1}} \left(v_{G_1}(h)v_{G_2}(k)\right)^{-m} \\
&\qquad \prod_{i=1}^{m-1} \alpha(h^i, h)\alpha(h^m, z)\beta(k^i, k)\beta(k^m, u) \\
&= \alpha(z, h)\beta(u, k) = \alpha\beta(x, g).
\end{aligned}$$

The next theorem is for the induction from a subgroup.

THEOREM 9. *Let H be a normal subgroup of G. If every F-class of G/H is (F, α)-regular for $\alpha \in Z^2(G/H, F^*)$, then every F-class of G is $(F, \mathrm{Inf}\,\alpha)$-regular, where $\mathrm{Inf} = \mathrm{Inf}_{G/H}$.*

Proof. Choose integers n and $m(\sigma)$ for G. Then these integers work for G/H. For $g \in G$, choose $(\sigma, x) \in \mathcal{G} \times G$ such that $x^{-1}g^{m(\sigma^{-1})}x = g$, and write m for $m(\sigma^{-1})$ for convenience. Since $x^{-1}g^m xH = gH$ in G/H, we have

$$v_1(gH)^{\sigma^{-1}} v_1(gH)^{-m} \prod_{i=1}^{m-1} \alpha(g^i H, gH)\alpha(g^m H, xH) = \alpha(xH, gH)$$

where $v_1(gH)$ is as in (A) with respect to α. Then

$$\begin{aligned}
&(\mathrm{Inf}\ \alpha)(x, g) \\
&= v_1(gH)^{\sigma^{-1}} v_1(gH)^{-m} \prod_{i=1}^{m-1} (\mathrm{Inf}\alpha)(g^i, g)(\mathrm{Inf}\ \alpha)(g^m, x).
\end{aligned}$$

It now suffices to show that we may choose $v(g)$ in (A) with respect to Inf α as $v_1(gH)$. For, since $v(g)^n = \prod_{i=1}^{n-1}(\mathrm{Inf}\ \alpha)(g^i, g) = \prod_{i=1}^{n-1} \alpha(g^i H, gH) = v_1(gH)^n$, there is $v'(g)$ such that $v'(g) = v_1(gH)$. Further since the choices of $v(g)$ in (A) do not make any change of D_Γ where $\Gamma = F^{\mathrm{Inf}\ \alpha}G$, we may choose without lose of generality $v(g) = v'(g) = v_1(gH)$, as is required.

It follows immediately that for $\alpha \in Z^2(G/H, F^*)$, if every F-class of G/H is (F,α)-regular then every F-class of any subgroup K of G is $(F, \mathrm{Res}_{G,K}(\mathrm{Inf}_{G/H}\alpha))$-regular. When $K = H$, it is much clear that if every F-class of G/H is (F,α)-regular then every F-class of H is $(F, (\mathrm{Inf}\ \alpha)_H)$-regular, since $(\mathrm{Inf}^*_{G/H}\alpha)_H = 0$ (refer to inflation-restriction sequence [8]).

THEOREM 10. *Let $G = H \times K$ and $\alpha \in Z^2(G/H, F^*)$. Then there exists $\gamma \in Z^2(K, F^*)$ such that every F-class of K is (F,γ)-regular if and only if every F-class of G/H is (F,α)-regular.*

Proof. For $k, u \in K$ there are unique corresponding elements gH, $xH \in G/H$ such that $g = hk$, $x = zu$ for some $h, z \in H$. Define $\gamma : K \times K \to F^*$ by $\gamma(k,u) = \alpha(gH, xH)$. Then $\gamma \in Z^2(K, F^*)$, and indeed $\gamma = \mathrm{Res}_{G,K}\mathrm{Inf}_{G/H}\alpha$. Choose integers n and $m(\sigma)$ for G. Then these work for both G/H and K. Write $m(\sigma^{-1}) = m$. Then $x^{-1}g^m xH = gH$ in G/H if and only if $u^{-1}k^m u = k$ in K, and we may choose $v_1(gH) = v_K(k)$ where $v_1(gH)$ [resp. $v_K(k)$] is an analogue of $v(g)$ in (A) with respect to α [resp. γ]. This completes the proof.

Let $G = H \times K$ and let $\alpha \in Z^2(G/H, F^*)$. Then theorems 6 and 10 guarantee that if every F-class of G is $(F, \mathrm{Inf}_{G/H}\alpha)$-regular then every F-class of G/H is (F,α)-regular.

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