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An improved Version of Minty's Algorithm to solve TSP with Penalty Function

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ABSTRACT

The traveling salesman problem has been studied for many years since the model can be used for various applications such as vehicle routing, job sequencing, clustering a data array, and so on. In this paper one of the typical exact algorithms for TSP, Minty's, will be modified to improve the performance of the algorithm on the applications without losing simplicity. The Little's algorithm gives good results, however, the simple and plain Minty's algorithm for solving shortest-route problems has the most intuitive appeal. The suggested Minty's modification is based on the creation of penalty-values on the matrix of a TSP. Computer experiments are made to verify the effectiveness of the modification.

1. Introduction

The traveling salesman problem(TSP) can be stated as follows. A salesman, starting from his home city, is to visit exactly once each city on a given list and then return home. The problem is to determine the order in which he should visit the cities to minimize the total distance traveled, assuming that the direct distances between all city pairs are known. This problem takes no mathematical background to understand it and no great talent to find solutions. Thus it is fun to work on, continuously inviting recreational problem solvers. On the other hand, the TSP has resisted all efforts to find a great optimization algorithm or even an approximation algorithm that is guaranteed to be effective. Thus, the TSP contains both of the elements that have attracted mathematicians to particular problems for centuries. They are simplicity of statement and difficulty of solution[5].

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The importance of the TSP comes not from the wealth of applications, but from the fact that it is typical of other problems of its genre: combinatorial optimization. There are not many salesmen clamoring for an algorithm, and the number of other cases where the mathematical model of the TSP precisely fits an engineering or scientific situation have not to date been numerous. We are trying to minimize total distance, so the problem is one of optimization; but we cannot immediately employ the methods of differential calculus by setting derivatives to zero, because we are in a combinatorial situation: our choice is not over a continuum but over the set of all tours[6].

Despite the fact that the traveling salesman model applies directly to a very useful-sounding situation, namely that of a salesman wishing to minimize his travel distance, most of the reported applications are quite different. The possible applications of TSP are vehicle routing, computer wiring, cutting wallpaper, job sequencing, clustering a data array. There are also several other more-or-less standard combinatorial optimization problems. They are the assignment problem, integer linear programming, the quadratic assignment problem, the longest path problem, minimum spanning trees, and matroid intersection[5].

This TSP has been studied for many years, with limited success[9]. Basically, the algorithms developed so far can be classified into two categories. One is an exact algorithm, the other is a heuristic method. Exact algorithms may require inordinate running times while heuristic methods produce good answers for somewhat larger problems in reasonable times, but provide no guarantee that the optimum answer will appear. In this paper, one of the typical exact algorithms, Minty's, will be reviewed and a modification will be made to improve the performance of the Porte-Manteau version of Minty's for TSP without losing simplicity. Either the requirement of visiting all cities or and then coming back to the origin city can be used to avoid the unnecessarily required repeats in the algorithm[1] for solving the traveling salesman problems. The modification will be based on the creation of penalty-values on the matrix of a TSP.

2. Literature Review

The first use of the term traveling salesman problem in mathematical circles may have been in 1931-32. But in 1832, a book was printed in Germany entitled 'The traveling salesman, how he should be and what he should do to get commissions and to be successful in his business. By a veteran traveling salesman'. Although devoted for the most part to other issues, the book reaches the essence of the TSP in its last chapter: 'By a proper choice and scheduling of the tour, one can often gain so much time that we have to make some suggestions. The most important aspect is to cover as many locations as possible without visiting a location twice...'[6].

The NP-hardness of the traveling salesman problem makes finding optimal tours not trivial when the number of cities is large. The exact algorithms are designed to find an optimum solution in a resonable amount of time with a relatively small-sized TSPs. However, efficient heuristic algorithms that, while not guaranteed to find optimal tours, do find what one hopes are near-optimal tours are necessary for the practical and large-sized problems. The BB(branch and bound) by Little et. al. shows a good performance[12], however, Minty[10] will be reviewed in this study due to the most intuitional appeal as Conway et al.[3] indicated.

Lin and Kernighan[9] show a good example of heuristics to solve large-sized TSPs. Their method is based on a substantial generalization of the interchange transformation. This interchange strategy is applied to the traveling salesman problem by Croes, with k fixed at 2, and by Lin, with k = 3, with considerable success. Accelerated branch exchange heuristic for symmetric traveling salesman problems are given by Stewart[13]. The improvement is obtained by considering only exchanges that have give a good chance of producing a better solution. A heuristic for solving TSP with extended the planning period traveling salesman problem is suggested by Chao et al. [2]. Another heuristic search given by Knox[8] is a tabu search for the symmetric TSP.

Currently, no efficient algorithm has been developed which always finds the optimal solution for an NP-hard or NP-complete problem. Furthermore, it is generally believed that no efficient algorithm can be found for the TSP. A common approach for dealing with these problems is to use heuristic search method producing optimum tending solutions.

The traveling salesman problem has the following simple description: given a complete digraph on n nodes with an $n \cdot n$ matrix $\|d(i,j)\| \geq 0$ giving the lengths of arcs (i,j) find a minimum length circuit or tour which goes through each node exactly once. The length d(T) of a tour T is given by $d(T) = \sum(i,j)T \ \varepsilon \ d(i,j)$ [4]. The problem has been studied extensively for the past few decades and many algorithms have been proposed for its exact solution. None however have worst-case time bounds which are polynomial in n. A complexity theory (NP-completeness) initiated by Cook and Karp and extensively covered in Garey and Johnson indicates that an exact algorithm for this problem with a polynomial time bound seems unlikely to exist. Among these algorithms, the branch and bound by Little et. al. gives good results[12]. However, the simple and plain Minty's algorithm[4] for solving shortest-route problems has the most intuitive appeal as Conway et. al. [3] indicated. Also, this algorithm had been proved by Arnold[1] that it can be used to solve the traveling salesman problems. For this reason, this algorithm will be considered here and a modified version for a better performance will be developed.

3. Minty and the Porte-Manteau version

Minty[10] suggested a simple and plain algorithm to solve the shortest-route problem where the distance-matrix is symmetrical. The algorithm can be stated as below.

Two functions defined on the set of nodes are used as working variables in the algorithm:

① a(x) During the course of the algorithm this will be a lower bound on the time required to get from s to x

② b(x) will represent the node before x on the route of smallest total time from s to x.

At any phase of the algorithm the set of nodes, {N}, is partitioned into two sets:

① {U} at set of unpassed nodes. If $x \in \{U\}$, then the shortest route from s to x has not been determined.

② {P} a set of passed nodes. If $x \in \{P\}$, then the shortest route from s to x has been determined.

Initially, $a(s) = 0$, $a(x) = \infty$ for $x \neq s$, and {U} contain all the nodes.

General steps are:

① Step 1. Let x be a node in {U} such that

$$a(x) = \min_{y \in \{U\}} a(y)$$

② Step 2. If x equals t, then a(t) is the shortest time from s to t and the shortest route is (s,....., b(b(t)), b(t), t). Stop.

③ Step 3. If x is not t, then remove x from {U} and place x in {P}. For all

$y \in \{\{A(x)\} \cap \{U\}\}$,

if $a(y) > a(x) + p(x,y)$, then let $a(y) = a(x) + p(x,y)$ and $b(y) = x$. Return to step 1.

To find the shortest route from s to all other nodes, step 2 is omitted and steps 1 and 3 are cycled until {U} is empty or until $a(x) = \infty$ at step 1. If a(x) equals ∞ at termination, then there is no route from s to x.

However this algorithm has an unfortunate feature such as going far out along one path and then being forced to return some intermediate point or origin to start out in a new direction again and again as pointed out by Arnold[1]. This tedious task will be exponentially grown if the algorithm is applied to solve the traveling salesman problems. The Porte-Manteau version of Minty's algorithm to solve the traveling salesman problems can be stated simply by changing the stopping rule of the Minty's algorithm. The stopping rules of Minty's algorithm to be applicable to TSP are changed to :

① Traveling salesman problem with the N cities appears exactly once - Repeat the steps in Minty until a route which contains all cities on it and has minimum total distance or cost among all candidates is found.

② Traveling salesman problem with coming back to the origin city - Repeat the steps in Minty until a route which contains all cities on it, visit the origin finally and minimum total distance or cost among all candidates is found.

4. Modified Porte-Manteau version

The above changes in Porte-Manteau version to find an optimal solution will increase the number of repeating steps exponentially to reach an optimal solution in the traveling salesman problems. This could be happen due to the additional strong condition visit all the cities to be an optimal solution. But if we use the requirement visit all cities or and then come back to origin rather will help us to reduce the number of repeating steps. To use the requirements to reduce the repeats, we need to set up penalties to apply to a route which has less number of visited city on it than the other on the selection step for future expansions. The procedures to set up penalties are as follows:

① STEP 1

Find a minimum value of each row on the distance or cost matrix. Then we will have N elements with N cities case. Delete the largest one among these. Calculate the sum of the N-1 elements and name it SUMA.

② STEP 2

Find a minimum value of each column on the distance or cost matrix. Then we will have N values. Delete the largest one among these. Calculate the sum of the N-1 elements and name it SUMB.

③ STEP 3

Compare SUMA with SUMB and find bigger one. If SUMA is greater than or equal to SUMB, use the N-1 elements found on STEP 1 as penalties. Otherwise, use the other elements found on STEP 2.

④ STEP 4

Sort the N-1 elements on the STEP 3 from the smallest one to the largest one. Make penalties such as

PENALTY(1) = The smallest one

〃 (2) = PENALTY(1) + the second smallest one

〃 (3) = PENALTY(2) + the third smallest one

.

.

PENALTY(N-1)= PENALTY(N-2) + the N-1 th smallest one

⑤ STEP 5

Apply these penalties to the route which has less number of visited cities than the other on the comparison stage and find the minimum one for further expansion. For example, if we have a route which includes 4 cities on it and the other, 7 cities, so far. Then add PENALTY(3) (because 7-4=3) to the total distance of the route which includes 4 cities on it and then compare the two routes to find minimum one. Same process to all candidates should be followed. Take the minimum one for expansion. Do not link a route with once visited city except going back to the origin case for trip type solution.

⑥ STEP 6

If the minimum route has N cities for TRIP version or N+1 cities for TOUR version, repeat the STEP 5 to see it is global. If it is true, stop the iteration and report the results as a solution. Otherwise, go back to STEP 5.

In the Porte-Manteau version of Minty's algorithm[1], we may have to go back to a route which contains less cities on it than the other even though it will have a value larger than the other when it is extended up to the same number of visited cities appearing on it as the other. To avoid this possibility as much as we can, we need to use the above penalties on comparisons. This simple procedure will show us the ideally smallest value of a route when it is extended up to the same number of visited cities as the other which is comparing with. Therefore we could avoid unnecessary going backs to some intermediate point to start over in a new directions.

5. Computer experiments and results

The distance or cost matrix to be used in this experiment is same as in Table 1. For example, the distance between city 1 and city 2 is 3 and city 2 and 3 is 5 as shown in the table. Detailed calculations based on the procedures are follows. See Figure 2 and Figure 3 for complete calculation results based on this version and the Minty's, respectively.

Table 1. Distance matrix

city	1	2	3	4
1	*	3	7	4
2	2	*	5	3
3	6	6	*	4
4	4	7	5	*

① STEP 1

The minimum value on the row 1 = 3

〃 〃 〃 2 = 2

〃 〃 〃 3 = 4

〃 〃 〃 4 = 4

Delete one of the 4's and then we have 3, 2, and 4.

SUMA = 3 + 2 + 4 = 9

② STEP 2

The minimum value on the column 1 = 2

〃 〃 〃 2 = 3

〃 〃 〃 3 = 5

〃 〃 〃 4 = 3

Delete 5 and then we have 2,3, and 3

SUMB= 2 + 3 + 3 = 8

③ STEP 3

SUMA is greater than SUMB. We are going to use 3, 2, and 4 as our penalties.

④ STEP 4n

3, 2, 4 --- sorting --- 2, 3, 4

PENALTY(1) = 2

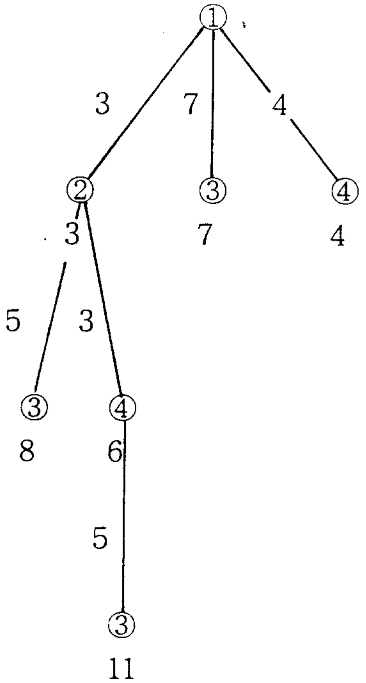
PENALTY(2) = PENALTY(1) + 3 = 5

PENALTY(3) = PENALTY(2) + 4 = 9

⑤ STEP 5

Necessary calculation procedures are given in Figure 1. Please refer the figure.

It will not be hard to set up penalties since we usually have the costs or distances between two cities or nodes in matrix. Picking up the smallest one in each row like setting up the lower bound in branch and bound[6] and adding them up will not take time much. This simple procedure will make the performance of Porte−Manteau version of Minty's algorithm better without hurting the attractive simplicity of Minty's algorithm.

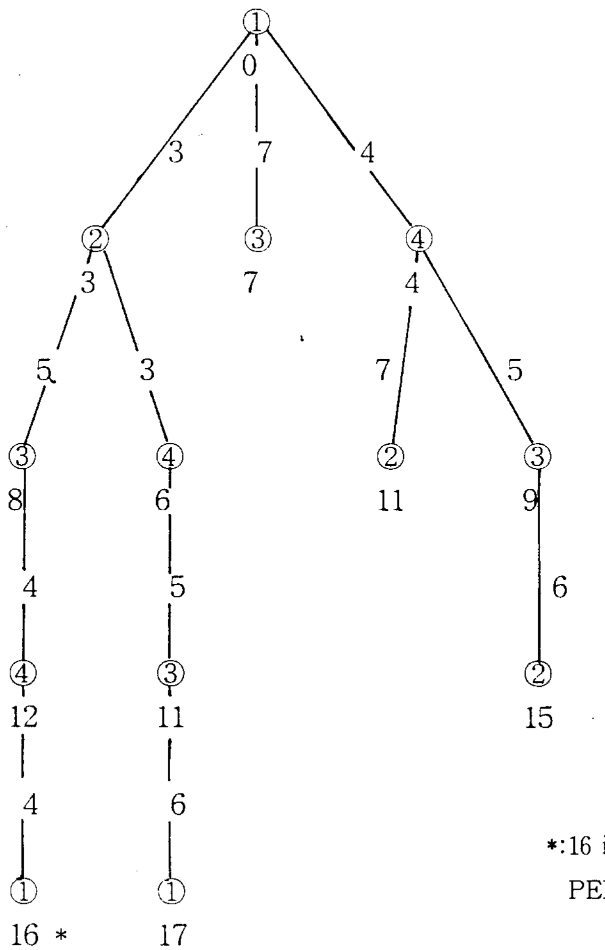


Take route 1-2 since 3 is the min.

Take route 1-2-4 since
route 1-2-3 : 8
route 1-2-4 : 6
route 1-3 :7 + PENALTY(1) = 9
route 1-4 :4 + PENALTY(1) = 6

Figure 1. Step 5

The reduced number of repeating steps and the number of created candidates to find an optimal route were investigated with example problems using computer. The reduced number of repeats were various problem by problem, but about 40-70% of the repeating steps and candidate calculations by the Porte-Manteau version were necessary on N = 6 case[3]. Please refer Table 2 for details. If N = 9, the necessary rate with this version drops to 10-20% of the Porte-Manteau version does. The worst case to this version can happen with the arc matrix which contains a lot of zero's. Even though it is not possible, all penalties will be equal to zero if we have a cost matrix which contains at least one zero in each row and this version will perform same as the Porte-Manteau version.

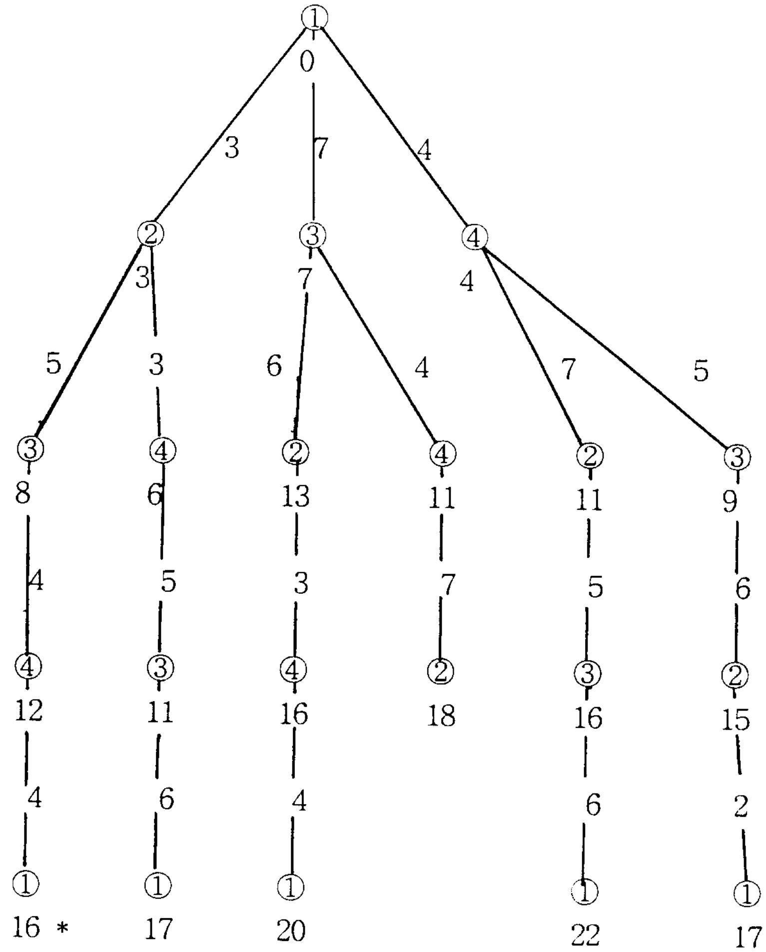


*:16 is the minimum one if we apply PENALTY to all candidates.

Figure 2. Modified Porte-Manteau version

6. Conclusions

The possible applications of TSP are various and TSP has been studied for many years with limited success. In this paper, a modified version of Minty's algorithm based on the creation of penalty-values is introduced and examined. The suggested version performed well without losing simplicity, one of the most important characteristics of the Minty. The computer experiments said that the reduced number of repeating steps and the number of created candidates to find an optimal route were about 40-70% of the repeating steps and candidate calculations by the Porte-Manteau version with N = 6 case and 10-20% with N = 9.



*: 16 is the minimum one.

Figure 3. Porte-Manteau version of Minty

Table 2. Computer Experiments

[NOTE] NR : Number of repeats
NC : Number of candidate-calculation and comparisons
NECESSARY(%)=NR or NC of P-M vsn. /PLNT vsn.×100
* : Unsolvable within 3000 candidates calculations

SOURCE		COME BACK TO THE ORIGIN						VISIT EXACTLY ONCE					
ORIGIN		P-M	vsn	PNLT	vsn	NECESSARY		P-M	vsn	PNLT	vsn	NECESSARY	
		NR	NC	NR	NC	NR	NC	NR	NC	NR	NC	NR	NC
C	1	87	167	58	113	67	68	30	76	18	46	60	61
O	2	78	157	49	104	63	66	41	96	22	55	54	57
N	3	85	164	59	110	69	67	24	64	9	26	38	41
W	4	68	142	44	96	65	68	47	109	32	77	68	71
A	5	94	174	55	108	59	62	34	83	21	52	62	63
Y	6	59	120	30	64	51	53	34	78	21	51	62	65
P	1	105	195	86	157	82	81	64	133	57	121	89	91
H	2	115	210	105	189	91	90	44	108	32	74	73	69
I	3	130	231	121	209	93	90	38	90	33	77	87	86
L	4	94	183	90	176	96	96	50	118	42	96	84	81
L I	5	99	175	83	140	84	80	39	87	32	71	82	82
P	6	111	211	96	183	86	87	53	123	50	117	94	95
	1	*	*	240	836			522	2012	79	133	15	16
R	2	*	*	205	647			678	2421	81	297	12	12
A	3	*	*	363	1250			*	*	355	1242	–	–
N	4	*	*	223	764			687	2525	140	504	20	20
D	5	*	*	314	1066			*	*	192	700	–	–
O	6	*	*	275	909			393	1587	51	214	13	14
N	7	*	*	409	1273			549	2105	75	281	14	13
	8	*	*	367	1173			704	2580	95	373	13	14
	9	*	*	224	784			667	2455	149	550	22	22

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