Journal of KSNVE, Vol. 6, No. 1, pp. 35~44, 1996.

⟨Original⟩

Vibration Control of a Single-Link Flexible Manipulator Using Fuzzy-Sliding Modes

퍼지-슬라이딩 모드를 이용한 단일링크 유연 매니퓰레이터의 진동제어

Seung-Bok Choi* and Jung-Sik Kim**

최 승 복·김 정 식

⟨Received July 21, 1995 ; Accepted December 19, 1995⟩

Key Words : Fuzzy Logic(퍼지로직), Sliding Mode Control(슬라이딩모드제어), Robust Control(강건제어), Flexible Manipulator(유연매니퓰레이터)

ABSTRACT

This paper presents a new type of fuzzy-sliding mode controller for robust tip position control of a single-link flexible manipulator subjected to parameter variations. A sliding mode controller is formulated with an assumption that imposed parameter variations are bounded so that certain deterministic performance can be guaranteed. In the design of the sliding mode controller, so called moving sliding surface is adopted to minimize the reaching phase and thus mitigate system sensitivity to the variations. The sliding mode controller is then incorporated with a fuzzy technique to reduce inherently ever-existing chattering which is impediment in position control of flexible manipulators. A set of fuzzy parameters and control rules are obtained from a relation between predetermined sliding surface and representative points in the state space. Computer simulations are undertaken in order to demonstrate superior control performance of the proposed methodology.

요 약

본 논문에서는 단일 링크 유연 암 선단위치의 강건제어를 수행하기 위하여 새로운 형태의 퍼지-슬라이딩모드 제어기를 제안하였다. 우선 시스템 불확실성의 경계치를 알고 있다는 가정하에 슬라이딩모드 제어기를 먼저 설계하였다. 슬라이딩모드 제어기 설계시 주어진 시스템의 슬라이딩모드 운동 중 안정성을 보장하는 슬라이딩 평면을 최적제어기법으로 설계하였으며, 리칭상태를 최소화 시킴으로써 빠른 응답과 불확실성에 대하여 더욱 강건함을 얻도록 하기 위해서 주어진 초기조건을 고려하는 이동 슬라이딩 평면을 적용하였다. 또한, 직접 측정이 어려운 속도 상태변수들의 예측값을 구하기 위하여 비연계 저차 관측기를 설계하였다. 이와같이 설계된 슬라이딩모드 제어기는 시스템의 불확실성, 외란 등에 대해서 안정성과 강건성을 보장하는 특징을 갖고 있다. 그러나 슬라이딩모드 제어기 적용시 시스템에 존재하는 떨림현상으로 인하여 실제적인 진동제어 시스템에 적용하는데 어려운 점을 갖고 있다. 따라서 이러한 떨림현상을 감소시키기 위하여 슬라이딩 모드 제어기와 퍼지제어기를 연계시킨 퍼지-슬라이딩모드 제어기를 구성하였다. 퍼지제어기 설계시 미리 규정된 슬라이딩 평면식과 오차 공간상의 상태점과의 관계로부터 퍼지제어규칙을 선정해 제어기를 설계하였다. 이와같이 구성된 제어기에 대한 컴퓨터 시뮬레이션을 수행하여 떨림현상 감소 효과와 불확실성에 대한 강건성 유지를 입증하였다.

*정회원, 인하대학교 기계공학과
**한국타이어 중앙연구소

1. Introduction

Most industrial robot manipulators depend on bulky design in order to minimize structural vibration of each component. This massive structural design makes the manipulator slow and heavy; hence, large actuators and high mounting strength are required. The insatiable demand for high performance robotic systems quantified by high speed of operation, small energy consumption and lower overall cost have triggered a vigorous research thrust in various multi-disciplinary areas such as control of light-weight flexible manipulators. The flexible manipulators, although haveing some advantages over conventional rigid robots, have more strigent requirements for the control system design, such as accurate end-poing sensing and fast suppression of transient vibration during rapid arm movements. Furthermore, model parameter variations of the flexible manipulator, such as natural frequency, may easily arise in practice due to a wide spectrum of various conditions associated with the manufacturing process, dynamic modeling, operating environments and so forth.

A variety of control strategies for the flexible manipulators have been proposed in an attempt to discover a successful and practical feedback control. Most of the previously proposed controllers have been designed by treating the control system as a determinsitic nominal problem. A few investigators have attempted to provide control logics which account for the sensitivity of the control system to parameter variations. A robust control which guarantees stable performance of the system for all possible variations of the system parameters was designed by treatign the variations as uncertainties [1]. The properties of the ultimate uniform boundedness of the solution of the system equation were employed to formulate the controller. Sliding mode controllers which inherently possess invariant properties to the parameter variations during the sliding mode motion were proposed and successfull applied to the flexible manipulators[2~5]. However, it is generally known that the sliding mode controller (SMC) requires instantaneous change (chattering) of control input in order to compensate the parameter variations and operate effectively in sliding surfaces. This chattering may also occur due to unmodelled dynamics, time step in computer simulation or sampling period in real-time control implementation and delay of switching devices[6]. In practical engineering systems, the chattering causes severe damages to system hardware such as actuator. Thus, it has to be eliminated or alleviated as possible. One way to achieve this objective is to employ so called continuation method[6~8]. However, in this case, the trade-off between the smothing of the control input history and the control error must be considered.

More recently several researchers are trying to eliminate or attenuate the chattering by applying the fuzzy controller (FC) by adopting predetermined sliding surface and its sign as fuzzy variables. The perturbations measured from the imposed estimator were used as fuzzy input variables to handle different types of the chattering. Meystel et al.[11] used two fuzzy variables; the absolute radial distance from the origin and the absolute angle deviation from the sliding surface to alleviate the chattering of the SMC. None of these fuzzy controllers associated with the SMC has been applied to the control of flexible manipulators.

In this paper, we propose a new type of fuzzy-slding mode controller (FSMC) to attenuate the chattering, and apply it to a robust position control of a single-link flexible manipulator subjected to parameter variations. We first formulate a SMC by assuming that imposed parameter variations are bounded so that certain determinstic performance can be guaranteed. In the design of the controller, so called moving sliding surface (MSS)[12] is adopted to minimize the reaching phase and hence mitigate system sensitivity to the variation. In addition, a decoupled reduced-order observer is formulated to estimate velocity state variables, while the position state variables are obtained directly from output sensor measurements. The SMC is then incorporated with a fuzzy logic. We choose the position of representative points (RP) of the sliding surface and its gradient as fuzzy input variables. In this case, we can determine fuzzy control rules more systematically compared with other design methods. Computer simulations are performed in order to demonstrate the effectiveness and robustness of the proposed fuzzy-sliding mode controller.

2. System Modeling

Consider the horizontal motion of a single-link flexible manipulator as shown in Fig. 1. The uniform beam that has a total length of l and width of b is attached to the rotating hub that has a moment of inertia I_H. The axis ov' is the fixed reference line and ov is the tangential line to the beam's neutral axia at the hub. Upon assuming Euler-Bernoulli beam theory, small elastic deflections, small angular velocities and neglecting axial deflections, the governing equation of motion and the associated boundary conditions are obtained as follows[13].

$$EI\frac{\partial^4 w(v,t)}{\partial v^4}+p\frac{\partial^2 w(v,t)}{\partial t^2}=0 \tag{1}$$

$$EI\frac{\partial^2 w(v,t)}{\partial v^2}\bigg| + T(t) - I_H\ddot{\theta}(t)=0$$

$$w(0,t)=0$$

$$EI\frac{\partial^2 w(v,t)}{\partial v^2}\bigg|_{v=l}=0$$

$$EI\frac{\partial^3 w(v,t)}{\partial v^3}\bigg|_{v=l}=0$$

where, EI is the bending stiffness of the beam, ρ is the mass per unit lenght of the beam, $T(t)$ is the input torque and $w(v,t)$ is the total displacement give by

$$w(v,t)=\tilde{w}(v,t)+v\theta(t) \tag{3}$$

By using the assumed-mode method the system

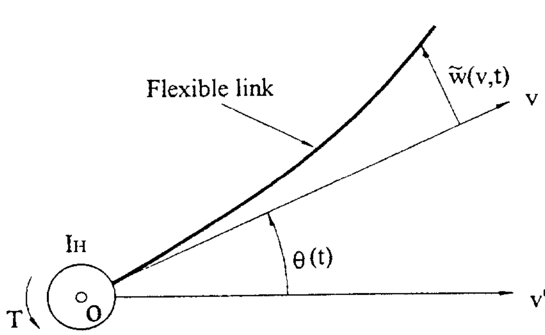

Fig. 1 A single-link flexible manipulator

variables $w(v,t)$ in a partial differential equation (1) can be expressed as

$$w(v,t)=\sum_{i=0}^{\infty}\phi_i(v)\,q_i(t) \tag{4}$$

Here, the space-dependent function $\phi_i(v)$ is the eigen function of the i-th mode and the time-dependent function $q_i(t)$ is the generalized coordinate of the system. After applying Lagrange's equation, an infinite set of ordinary differential equations which are decomposed from each other may be obtained. Upon retaining fintie number of control modes, a reduced dynamic model can be obtained in the state space representation as follows:

$$\dot{x}(t)=\boldsymbol{A}x+\boldsymbol{B}T,\ \boldsymbol{y}=\boldsymbol{C}x \tag{5}$$

where,

$$\boldsymbol{x}=[q_0\,\dot{q}_0 q_1\,\dot{q}_1\cdots q_n\,\dot{q}_n]^T=[x_1 x_2\cdots x_{2n+2}]^T\in\boldsymbol{R}^{2n+2}$$

$$\boldsymbol{A}=\begin{bmatrix} 0 & 1 & & & & \\ 0 & 0 & & & 0 & \\ & & 0 & 1 & & \\ & & -\omega_1^2 & -2\zeta_1\omega_1 & & \\ & & & & & \\ & & 0 & & 0 & 1 \\ & & & & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix}$$

$$\boldsymbol{B}=\frac{1}{I_t}[0\ 1\ 0\ \phi_1'(0)\cdots 0\ \phi_n'(0)]^T$$

$$\boldsymbol{y}=[y_t\ y_\theta\ y_s]^T$$

$$\boldsymbol{C}=\begin{bmatrix} l & 0 & \phi_1(l) & 0 & \cdots & \phi_n(l) & 0 \\ 1 & 0 & \phi_1'(0) & 0 & \cdots & \phi_n'(0) & 0 \\ 0 & 0 & \frac{h}{2}\phi_1''(0) & 0 & \cdots & \frac{h}{2}\phi_n''(0) & 0 \end{bmatrix} \tag{6}$$

Viscous damping terms have been added to each flexible mode in an heuristic fashion. In equations (5) and (6), ω_i and ζ_i are the natural frequency (pinned-mode) and the damping ratio of the i-th mode, respectively. I_t is the total moment of inertia (hub and beam), h is the thickness of the arm, $\phi'(\cdot)$ is the modal slope coefficient, the subscript n is the number of primary flexible modes to be controlled. It is noted that the output matrix $\boldsymbol{C}$ is related to non-colocated tip position sensor (y_t) and two colocated sensors: hub angle sensor (y_θ) and strain gage sensor at the root (y_s).

The accuracy of the reduced model (5) is very important in the sense of spillover problems. The accuracy depends heavily upon the geometrical and material properties of the flexible arm as well as on the dynamic characteristics of actuators and sensors. It may be estimated by comparing transfer function between exact and reduced representations or investigating dynamic response before and after employing the controller[11]. Several works on this point confirm that a sufficient finite number of modes can provide an effectively exact model of the link dynamics. In this study, the number of control modes is determined from the investigations of the open-loop responses and closed-loop responses of the system by considering the participation factor of each vibrational mode.

From the earlier discussion on the lack of knowledge of model parameters such as natural frequencies, a possible variation of the parameters can be expressed as follows:

$$\omega_i = \omega_{0,i} + \Delta\omega_i \quad \zeta_i = \zeta_{0,i} + \Delta\zeta_i, \quad i = 1,2,3,\cdots,n \tag{7}$$

where $\omega_{0,i}$ and $\zeta_{0,i}$ are nominal natural frequency and damping ratio, respectively, under the conditions of no payload and all system parameters knwon. The $\Delta\omega_i$ and $\Delta\zeta_i$ are corresponding possible deviation (uncertain in practice) due to, for instance, the varying payload. Now, substituting the equation (7) into equation (5) yields following dynamic model

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_0 + \Delta\boldsymbol{A})\boldsymbol{x}(t) + \boldsymbol{B}T(t), \quad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{8}$$

where $\Delta\boldsymbol{A} = \boldsymbol{A} - \boldsymbol{A}_0$. It is noted that the variation limits of the uncertainties of $\Delta\omega_i$ and $\Delta\zeta_i$ need to be known in the synthesis of the controller.

3. Sliding Surface Design

The control objective is to enforce the tip position of the flexible arm to the desired set point; regulating control problem. Thus, we may set a sliding surface in the state space as followings:

$$\sigma = \sum_{i=1}^{2n+2} c_i x_i = [c_1 \ c_2 \cdots c_{2n+2}][x_1 \ x_1 \cdots x_{2n+2}]^T = \boldsymbol{G}\boldsymbol{x} \tag{9}$$

where $\boldsymbol{G}$ is surface gradient vector and $det(\boldsymbol{GB}) \neq 0$. Then the state of the system during the sliding mode motion is constrained to the subspace $\boldsymbol{E}^{2n+1}$ defined by the equation

$$\boldsymbol{G}\boldsymbol{x} = 0 \tag{10}$$

Differentiating (10) with respect to time and substituting from (8) yields following equivalent control T_{eq} in an unique manner.

$$T_{eq} = -(\boldsymbol{GB})^{-1}\boldsymbol{G}(\boldsymbol{A}_0 + \Delta\boldsymbol{A})\boldsymbol{x} \tag{11}$$

Thus, the resultant sliding mode equations are obtained as follows.

$$\begin{aligned} \dot{\boldsymbol{x}} &= [\boldsymbol{I} - \boldsymbol{B}(\boldsymbol{GB})^{-1}\boldsymbol{G}](\boldsymbol{A}_0 + \Delta\boldsymbol{A})\boldsymbol{x} \\ &= \boldsymbol{A}_e\boldsymbol{x}, \ \boldsymbol{G}\boldsymbol{x} = 0 \end{aligned} \tag{12}$$

It is seen from above equations that $\Delta\boldsymbol{A}$ could have an intrinsic influence on the system behavior during the sliding mode motion. The invariance condition for the sliding mode system (12) to be completely insensitivie for the parameter variation $\Delta\boldsymbol{A}$ is given by[14]

$$rank[\boldsymbol{B} \vdots \Delta\boldsymbol{A} \ \boldsymbol{M}] = rank[\boldsymbol{B}] \tag{13}$$

where $\boldsymbol{M}$ is the state transformation matrix whose columns are the basis of the subspace $\boldsymbol{E}^{2n+1}$. From the dynamic model (5), we know that the system $(\boldsymbol{A},\boldsymbol{B})$ is controllable. Thus, the system(8) can be transformed into the controllable canonical form which in nature fulfills the invariance condition (13). This implies that the $\Delta\boldsymbol{A}$ will act as the equation in the reaching phase. However, the $\Delta\boldsymbol{A}$ will have no influence on the system motion in the sliding phase. From the fulfillment of the invariance condition (13) we can design the surface parameter c_i so that all eigenvaluse of $\boldsymbol{A}_e$(in the absence of $\Delta\boldsymbol{A}$) in equation (12) have negative real parts. This can be easily achieved by pole assignment technique[15].

However, we know that the surface (9) is designed without consideration of given initial conditions. Therefore, the SMC associated with the surface (9) may be sensitive to parameter variations during the reaching phase. In order to reduce the reaching phase, and hence to improve the system robustness we modify the surface (9) to adapt arbitrarily given initial conditions. We first introduce new axis coordinates as follows:

$$e_{m1}=\frac{1}{C_1}\sum_{i=1}^{n+1} c_{2i-1}x_{2i-1},\ e_{m2}=\sum_{i=1}^{n+1} c_{2i}x_{2i} \qquad (14)$$

Note that the e_{m1} axis is the function of the position state variables, while the e_{m2} axis is the function of the velocity state variables of the flexible vibration modes. Using these variables, we construct a moving sliding surface(MSS) in the e_{m1}-e_{m2} coordinates as follows:

$$\sigma=\begin{cases} c(t_0)e_{m1}(t_0)+e_{m2}(t_0)+a(t_0), & t=t_0 \\ c(t)e_{m1}(t)+e_{m2}(t)+a(t), & t_0<t\le t_s \\ c_p e_{m1}(t)+e_{m2}(t), & t>t_s \end{cases} \qquad (15)$$

Here t_s is the time at which the sliding mode of the system begins, $c(t)$ is the time-vaying slope and $a(t)$ is the time-varying imtercept of the e_{m2} axis. It is obvious that the surface (15) goes through arbitrarily given initial conditions $e_{mi}(t_0)$ with corresponding slope $c(t_0)$ and intercept $a(t_0)$. If the intial states are located in the stable zone, the intercept $a(t)=0$ for $t\in[t_0,\infty]$. If the initial states are located in the unstable zone, the intercept $a(t)$ varies in a shifting manner until the states arrive to the stable zone. The surface moves until the slope $c(t)$ becomes equal to $c_p(=c_1)$. The detailed moving algorithms are well described in reference[12].

4. Controller Design

4.1 SMC Design

As a first step towards developing a sliding mode controller, we take a time derivative of the sliding surface defined by (15).

$$\begin{aligned}\dot\sigma_m(t)&=\dot c(t)e_{m1}(t)+c(t)\dot e_{m1}(t)+\dot e_{m2}(t)+\dot a(t)\\ &=\sum_{i=1}^{n+1}[(r_{2i-1}+d_{2i-1})x_{2i-1}+(r_{2i}+d_{2i})x_{2i}]\\ &\quad+DT+\frac{\dot c(t)}{C_1}\sum_{i=1}^{n+1}c_{2i-1}x_{2i-1}+\dot a(t) \end{aligned}\qquad (16)$$

where,

$$\begin{aligned} r_{2i-1}&=-c_{2i}\omega_{0i-1}^2,\ \omega_{0,0}=0\\ r_{2i}&=\frac{c(t)}{C_1}c_{2i-1}-2\zeta_{0,i-1},\ \zeta_{0,0}=0\\ d_{2i-1}&=-c_{2i}(2\omega_{0,i-1}\delta\omega_{i-1}+\delta\omega_{i-1}^2),\ \delta\omega_0=0\\ d_{2i}&=-2c_{2i}(\delta\zeta_{i-1}\omega_{0,i-1}+\zeta_{0,i-1}\delta\omega_{i-1}),\ \delta\zeta_0=0\\ D&=\frac{1}{I_t}\sum_{i=1}^{n+1}c_{2i}\phi_{i-1}'(0),\ \phi_0'(0)=1 \end{aligned}\qquad (17)$$

Since $c(t)$, $a(t)$ are chosen to be step functions, there is partition, $p=\{\tilde\nu_1,\ \tilde\nu_2,\cdots,\tilde\nu_n\}$ such that $t_0=\tilde\nu_1<\tilde\nu_2<\cdots<\tilde\nu_n=t_s$ and $c(t)$ and $a(t)$ become constant in the open interval $(\tilde\nu_{k-1},\ \tilde\nu_k)$. Certainly we can prove p measurable and $m(p)=m_e(p)=0$[12]. Here m is Lebesque measure and m_e is Lebesque exterior measure. Thus we know that $\dot c(t)=\dot a(t)=0$ in the time interval $t\in[t_0,t_s]-p$. From this fact, we can easily construct the controller T which satisfies the following sliding condition.

$$\sigma_m\dot\sigma_m<0 \qquad (18)$$

To formulate such a control law, we assume that the parameter variations are bounded as follows.

$$|\delta\omega_i|\le\varepsilon_i\omega_{0,i},\ |\delta\zeta_i|\le\gamma_i\zeta_{0,i},\ i=1,2,3,\cdots,n \qquad (19)$$

Here, ε_i and γ_i are weighting factors representing the limits of the parameter variations.

Now we propose following sliding mode controller:

$$\begin{aligned}T=-\frac{1}{D}\{&\sum_{i=1}^{n+1}(r_{2i-1}x_{2i-1}+r_{2i}x_{2i})\\ &+[k+\left|\sum_{i=1}^{n+1}(g_{2i-1}x_{2i-1}+g_{2i}x_{2i})\right|]\mathrm{sgn}(\sigma_m)\},\\ &k>0\end{aligned}\qquad (20)$$

where,

$$\begin{aligned} g_{2i-1}&=-c_{2i}(2\omega_{0,i-1}\varepsilon_{i-1}\omega_{0,i-1}+\varepsilon_{i-1}^2\omega_{0,i-1}^2),\ \varepsilon_0=0\\ g_{2i}&=-2c_{2i}(\gamma_{i-1}\zeta_{0,i-1}\omega_{0,i-1}+\zeta_{0,i-1}\varepsilon_{i-1}\omega_{0,i-1}\\ &\quad+\gamma_{i-1}\zeta_{0,i-1}\varepsilon_{i-1}\omega_{0,i-1}),\ \gamma_0=0 \end{aligned}$$

Then we can show that the system (8) with the proposed controller (20) satisfies the sliding condition (18) as follows:

$$\begin{aligned}\sigma_m\dot\sigma_m&=\sigma_m\{\sum_{i=1}^{n+1}[r_{2i-1}+d_{2i-1})x_{2i-1}\\ &\quad+(r_{2i}+d_{2i})x_{2i}]+DT\}\\ &=\sigma_m\{\sum_{i=1}^{n+1}[r_{2i-1}+d_{2i-1})x_{2i-1}\\ &\quad+(r_{2i}+d_{2i})x_{2i}]-\sum_{i=1}^{n+1}(r_{2i-1}x_{2i-1}+r_{2i}x_{2i})\\ &\quad-[k+\left|\sum_{i=1}^{n+1}(g_{2i-1}x_{2i-1}+g_{2i}x_{2i})\right|]\mathrm{sgn}(\sigma_m)\\ &=\sum_{i=1}^{n+1}(d_{2i-1}x_{2i-1}+d_{2i}x_{2i})\sigma_m\\ &\quad-\left|\sum_{i=1}^{n+1}(g_{2i-1}x_{2i-1}+g_{2i}x_{2i}\right||\sigma_m|-k|\sigma_m|<0\end{aligned}\qquad (21)$$

Since we assume that the position state variables are readily obtainable through the measurement equation, the subatrix relating position state variables and output measurements is nonsingular. Therefore the position variables obtained can be viewed as new system's output as follows;

$$\bar{y}=\begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ & & & \vdots & & & \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n+2} \end{bmatrix}=\bar{C}x \quad (22)$$

where $\bar{C}$ is an $(n+1)\times(2n+2)$ transformed output matrix. The state-space description of the nominal system (A_0, B) is decomposed, and thus we can construct a reduced-order observer to estimate the velocity state variables[16]. The resulting observer for the velocity state variables takes the form

$$\dot{z}_i=L_{i1}z_i+L_{i2}x_{2i-1}+L_{i3}T,\ \hat{x}_{2i}=\lambda_i x_{2i-1}+z_i,$$
$$i=1,\cdots,n \quad (23)$$

Here,

$$L_{i1}=-(\lambda_i+2\zeta_{0,i-1}\omega_{0,i-1})$$
$$L_{i2}=-[(\omega_{0,i}^2+\lambda_i(\lambda_i+2\zeta_{0,i-1}\omega_{0,i-1})]$$
$$L_{i3}=\frac{\phi'_{i-1}(0)}{I_t}$$

In the above equation, z_i is an estimated value of transformed state variable, λ_i is a desired eigenvalue of the observer and $\hat{x}_{2i}$ is an estimate value of the velocity state variable x_{2i}.

4.2 FSMC Formulation

The feedback gain k in the control law (20) should be chosen according to the magnitude of the parameter variations, measurement and process noise. However, it is difficult to favorably select a proper k, because these are in general numeasurable. Hence, if we choose sufficiently large k, we may achieve fast and robust tracking control effect. But this may cause the chattering of discontinuous control input to be increased.

When we design a fuzzy controller(FC), how to select appropriate fuzzy control rules is the first problem to be resolved. For this, we consider Fig. 2 that shows the motion of the RP neighboring moving sliding surface in the state space. RP_{ri} and RP_{si} denote RP in the reaching phase and the sliding phase, respectively. When the RP is far from the sliding surface$(\sigma_m(\cdot)=0)$, the feedback gain k must be chosen relatively large in order to drive the system states to the surface as soon as possible, and *vice versa* in order to reduce the magnitude of chattering in the vicinity of the sliding surface. Therefore, our final control is represented as following conditional statement.

If *RP is far from the sliding surface,*
then *feedback gain is large and vice versa.* (24)

For the construction of the FC that has linguistic rule(24), we choose two fuzzy variables as follows:

$$\alpha(t)=c(t)e_{m1}(t)+e_{m2}(t) \quad (25)$$
$$\beta(t+\Delta t)=\alpha(t+\Delta t)-\alpha(t) \quad (26)$$

Here, Δt is the time step to solve the control system. It is seen that the first fuzzy variable(25) is equivalent to the sliding surface iteself (15) with $\alpha(t)=0$, and the second fuzzy variable (26) implies the gradient of this variable. Now, two linguistic input variables are defined to describe α and β as follows:

$$s=\{NB,\ NM, ZO,\ PM,\ PB\} \quad (27)$$
$$cs=\{NB,\ NM,\ ZO,\ PM,\ PB\} \quad (28)$$

where, *NB= negative big, NM= negative medium, ZO=zero, PM=positivie medium* and *PB=positive big,* respectively. Also linguistic output variable is

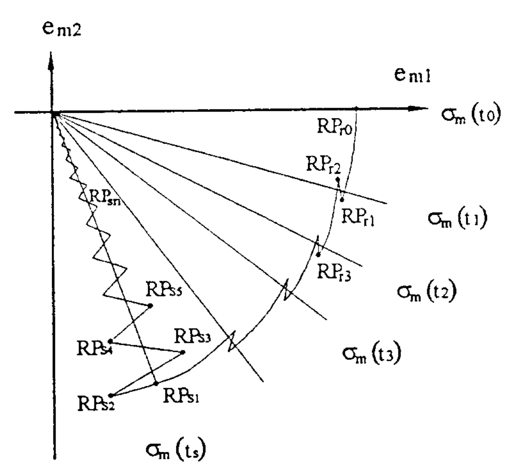

Fig. 2 Configuration of knowledge base for fuzzy input variables and rules

Table 1 Linguistic fuzzy rule base for feedback gain

s \ cs	PB	PM	ZO	NM	NB
PB	P6	P5	P5	P4	P4
PM	P5	P5	P4	P3	P3
ZO	P2	P1	ZO	N1	N2
NM	N3	N3	N4	N5	N5
NB	N4	N4	N5	N5	N6

defined to describe the discontinuous feedback gain k as follows:

$$kd = \{N6,\ N5,\ N4,\ N3,\ N2,\ N1,\ ZO,\ P1,\ P2,\ P3,\ P4,\ P5,\ P6\} \quad (29)$$

where, Ni, ZO, $Pi\,(i=1,\cdots,6)$ are fuzzy values of kd, Input-output relation of the FC with fuzzy variabels (27), (28) and (29) is written by

$$s,cs \rightarrow kd \quad (30)$$

Tabel 1 presents "look up" table of fuzzy control rules adopted in the present study. These fuzzy control rules can be inferred from the center-of-gravity method[17]. Fig. 3 represents the block-diagram of the proposed FSMC. The basic configuration of the FC comprises three componets; a fuzzification interface, a decision-making logic and a defuzzification interface. In the fuzzification interface, α and β are estimated, and these are modified into linguistic values s and cs. The decision-making logic is the kernel of the FC since it has the capability of both simulating decision-making based on fuzzy concepts and inferring fuzzy control actions. In the defuzzification interface, fuzzy output kd decided from the decision-making logic is changed into a numerical value for the purpose of using real control input. From the SMC (20), the proposed FSMC can be now formulated as follows:

$$T = -\frac{1}{D}\Big\{\sum_{i=1}^{n+1}(r_{2i-1}x_{2i-1}+r_{2i}x_{2i}) + \Big[kd(s,cs) + \Big|\sum_{i=1}^{n+1}(g_{2i-1}x_{2i-1}+g_{2i}x_{2i})\Big|\,\mathrm{sgn}(\sigma_m)\Big\} \quad (31)$$

It is noted that unlike conventional SMC (20), the FSMC (31) has varying feedback gain which is properly adjusted according to the commanded fuzzy rules.

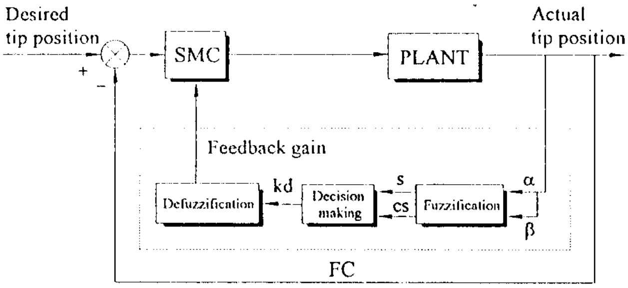

Fig. 3 A block-diagram of the proposed fuzzy-sliding mode controller

5. Simulations and Results

In order to demonstrate superior control performance characteristics of the proposed methodology, the nominal single-link flexible manipulator which has physical properties and model parameters given in Table 2, is considered. The natural frequency of the nominal system was determined from the eigen characterstic equaiton and the value of the damping ratio was adopted from the reference[13]. The rigid body mode and the first flexible mode were considered as the primary modes to be controlled.

The open-loop responses of the nominal system is shown in Fig. 4. The vibration of the first flexible mode was produced by taking the initial conditions as $x(0)=[0\ 0\ -0.17698\ 0]^T$. It is clear from the figure that the amplitude decays when the first flexible mode is assumed to be damped with the damping

Table 2 Physical properties and model parameters of the flexible arm.

Parameters	Values
Length (l)	1.0 m
Thickness (h)	0.002 m
Width (b)	0.02 m
Mass per unit length (ρ)	0.106 kg/m
Young's modulus (E)	64 Gpa
Moment of inertia of the hub (I_H)	0.023 kg/m^2
First-mode damping ratio ($\zeta_{0,1}$)	0.02
First-mode natural frequency ($\omega_{0,1}$)	2.47 Hz

ratio of 0.02.

For the closed-loop control simulations, the following numerical values were employed: $x(0)=[0.2\ 0\ 0\ 0]^T$, $\hat{z}_1(0)=-0{,}57$, $\hat{z}_2(0)=0.05$, $\lambda_1=\lambda_2=3.0$, $\varepsilon_1=0.2$, $\gamma_1=0.2$, $\delta\omega_1=0.2\sin(2t)$, $\delta\zeta_1=0.2\sin(2t)$, $G=[1.0\ 0.196\ 2.01\ 0.0773]$, $k=0.2$. On the other hand, the moving parameters for the surface (15) was chosen as 0.001sec for the dwelling time, 0.09 for the switching vicinity magnitude, 0.15 for the rotating and shifting boundary width. In addition, time-delay simulation scheme[18] was adopted to emulate real-time implementation of the controller. Time step for the Runge-Kutta integration was chosen by 0.001 sec and 0.05 sec for the sampling period.

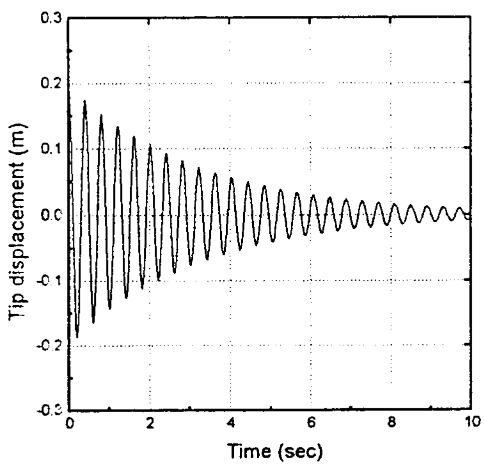

Fig. 4 Open-loop tip displacement of the nominal system

The controlled responses with the SMC (20) are presented in Fig. 5. At the beginning of the system motion, we can observe the nonminimum phase character which frequently plagues existing control methods to be successfully implemented. Despite the nonminimum phase character of the system, the system is successfully controlled by the SMC. One can clearly see that the controller associated with the moving sliding surface (15) considerably improves the system response by shortening the reaching phase without increasing the maximum magnitude of the control torque as well as the undesired chattering. This is due to the increment of the fastness for the velocity state variables of each flexible mode represented by the e_{m2}. Here, e_{m1} and

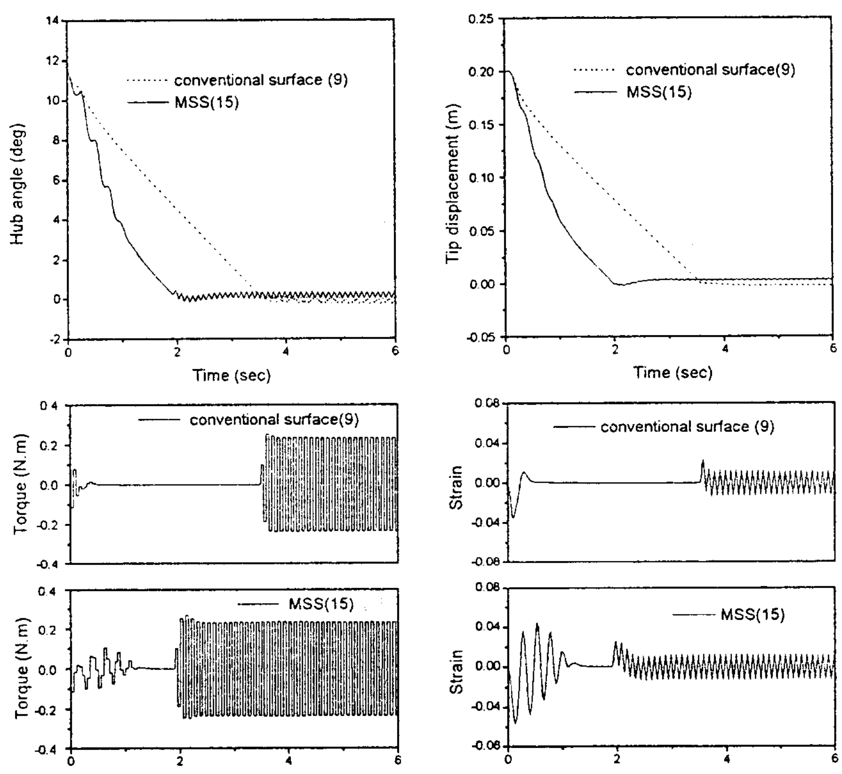

Fig. 5 System responses with the SMC

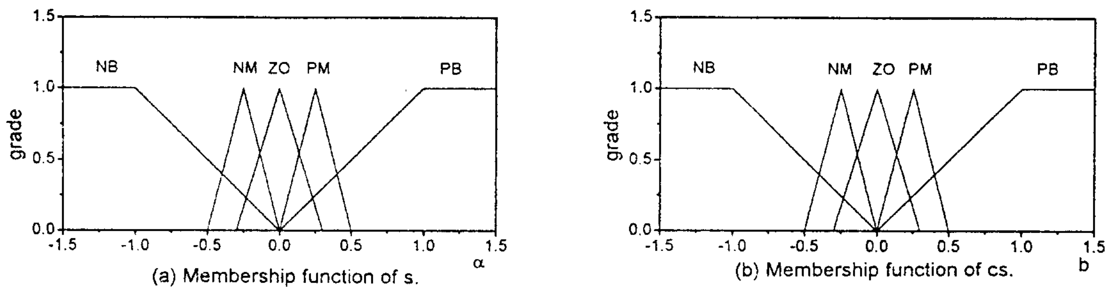

Fig. 6 Membership function of input variables (a) Membership function of s. (b) Membership function of cs.

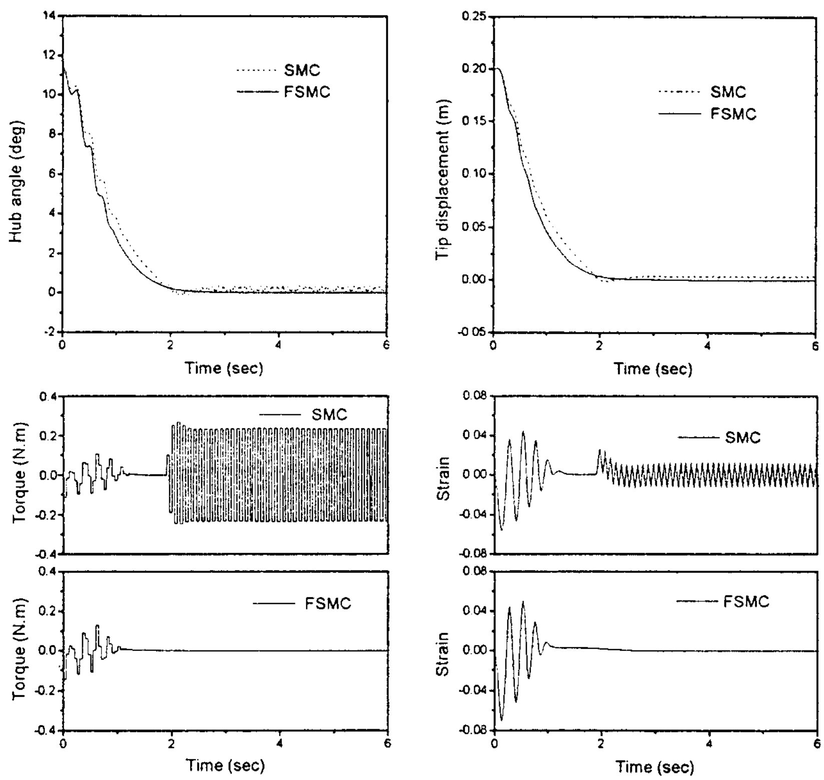

Fig. 7 Comparison of system responses between the SMC and the FSMC

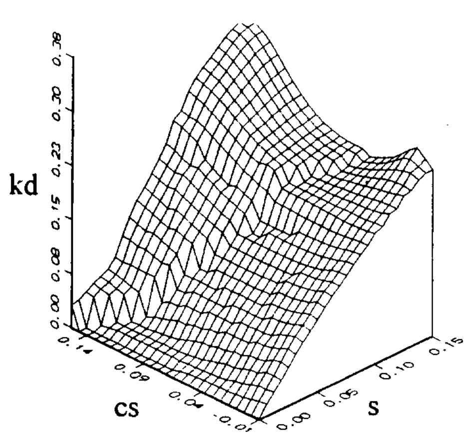

Fig. 8 Surface of feedback gain for the proposed FSMC

e_{m2} differ from the tip displacement and tip velocity so that there is steady state error due to the time delay (sampling period) and the discontinuous control input. Fig. 6 shows the membership functions of the fuzzy input variables of s and cs. It is noted that the membership functions of s and cs are selected to be same. Fig. 7 compares controlled responses between the SMC and the proposed FSMC. The control accuracy is also enhanced maintaining the system robustness. Fig. 8 presents surface of the kd (s, cs) obtained by the center-of-gravity method. We can easily see that this represents in nature the fuzzy control rules given by Table 1. The results presented in this study clearly justify that the proposed FSMC favorably eliminates the chattering of control input and hence improves the control performance of the flexible manipulator subjected parameter variations.

6. Conclusions

A fuzzy-sliding mode controller was formulated to attenuate the chattering and successfully applied to the position control of a single-link flexible manipulator. A robust sliding mode controller associated with the moving sliding surface was synthesized to account for the parameter variations such as natural frequency. This controller was then incorporated with a fuzzy technique which features two fuzzy variables obtained from the sliding surface. It has been demonstrated that the proposed controller furnishes favorable control responses such as significant reduction of the chattering and enhancement of control accuracy.

REFERENCES

(1) Choi, S. B., Cheong, C. C., Thompson, B. S. and Gandhi, M. V., 1993, "Vibration Control Robust to Parameter Variations in a Very Flexible Manipulator Fabricated from Composite Laminates," J. Sound and Vibration, Vol. 162, No. 2, pp. 195~207.

(2) Yeung, K. S. and Chen, Y. P., 1989, "Regulation of a One-Link Flexible Robot Arm Using Sliding Mode Technique," Int. J. Control, Vol. 49, No. 6, pp. 1965~1978.

(3) Yeung, K. S, and Chen, Y. P., 1992, "Sliding Mode Control of Multi Flexible Manipulator," Int. J. Control, Vol. 54, No. 2, pp. 257~278.

(4) Nathan, P. J. and Singh, S. N., 1991, "Sliding Mode Control and Elastic Mode Stabilization of a Robotic Arm with Flexible Links," ASME J. Dynamic Systems, Measurement and Control, Vol. 113, pp. 669~676.

(5) Qian, W. T. and Ma, C. C. H., 1992, "A New Controller Design for Flexible One-Link Manipulator," IEEE Trans. on Automatic Control, Vol. 37, No. 1, pp. 132~137.

(6) Slotine, J. J. E. and Sastry, S. S., 1983, "Tracking Control of Nonlinear Systems Using Sliding Surfaces with Application to Robot Manipulators," Int. J. Control, Vol. 38, No. 2, pp. 465~492.

(7) Utkin, V. I., 1977, "Variable Structure System with Sliding Modes," IEEE Trans. on Automatic Control, Vol. AC-22, No. 2, pp. 212~222.

(8) Choi, S. B., Park, D. W. and Jayasuriya, S., 1994, "A Time-Varying Sliding Surface for Fast and Robust Tracking Control of Second-Order Uncertain Systems," Automatica, Vol. 30, No. 5, pp. 899 ~904.

(9) Lin, S. C. and Kung, C. C., 1992, "A Linguistic Fuzzy-Sliding Mode Controller," Proc, of ACC, Vol. 3, pp. 1904~1905

(10) Hung, J. C., 1993, "Chattering Handling for Variable Structure Control System," Proc. of IECON'93, Vol. 3, pp. 1968~1972.

(11) Meystel, A., Nisenzon, Y. and Nawathe, R., 1993, "Moving Switching Surface for Robust Control of Second-Order Varable Structure Systems," Int. J. Control, Vol. 58, No. 1, pp. 229~245.

(12) Choi, S. B., Cheong, C. C, and Park, D. W., 1993, "Moving Switching Surface for Robust Control of Second-Order Variable Structure System," Int. J. Control, Vol, 58, No. 1, pp. 229~245.

(13) Schmitz, E., 1985, "Experiments on the End-point Position Control of a Very Flexible One-Link Manipulator," Ph. D. Thesis, Department of Aeronautics and Astronautics, Stanford University.

(14) Drazenovic, B., 1969, "The Invariance Conditions in Variable Structure Systems," Automatica, No. 5, pp. 287~295.

(15) Utkin, V. I., 1978, Sliding Modes and Their Application in Variable Structure Systems, MIR Publishers, Moscow.

(16) Chen, C. T., 1984, Linear System Theory and Design. CBS College Publishing: New York.

(17) Li, Y. F. and Lau, C. C., 1989, "Development of Fuzzy Algorithms for Servo Systems," IEEE, Control Syst, Mag. Vol. 9, No. 3, pp. 65~72.

(18) Lewis, F. L., Abdallah. C. T. and Dawson, D. W., 1993, Control of Robot Manipulators, Macmillan Publishing Company, New York.