Fast Voronoi Divider for VQ Codebook Designs

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ABSTRACT

In this paper, a new fast voronoi divider for vector quantization (VQ) is introduced, which results from Theorem that the nearest vectors in the sense of minimum mean square error (MMSE) have almost the same mean values of their elements. An improved splitting method for a VQ codebook design using the fast voronoi divider is also presented. Experimental results show that the new method reduces the complexity of training a VQ codebook several times with a high signal to noise ratio (SNR) using an appropriate extensive parameter of codebook.

KEYWORDS : Vector quantization, fast voronoi divider, full-search, partial-search

I. INTRODUCTION

VQ is one distinctive coding method which holds unusual promise. It is viewed as the extension of scalar quantization to a high dimensional space[1, 2], and used to code speech signal and vidéo sequence[3-5]. It is very important for a VQ system how to design a suitable codebook in order to get high quality of quantization. The most widely known design technique is the generalized Llyod algorithm or the LBG algorithm[6, 7], which uses an iterative method that attempts to satisfy the nearest neighbor rule and centroid condition simultaneously. Its performance is generally quite good, however, it will home in on the local optimum nearest to the initial codebook. The optimum may or may not be the global one, depending very much on how an initial codebook was set up[5, 6]. Therefore, it is necessary to research how to form the initial codebook. Three main methods had been given, i. e., selecting at random, splitting, and product codes. Some results have shown that the least quantization distortion can be obtained by the splitting method, however, its computational expense becomes a problem not to be ignored because of very high computational complexity of dividing voronois.

In this paper, a mean variable of the VQ codebook is introduced, and its properties are discussed. A new structure of VQ codebook is proposed, which consists of rearranged codewords according to the mean variable. An idea of fast-search voronoi divider is proposed based on the orderliness of the reconstructed codebook, and is used for the splitting method of the VQ codebook design. Experimental results show that the new design method not only reduces computational complexity of training the VQ codebook but also obtains the VQ codebooks with high SNR with a suitable codebook extensive parameter.

II. THEORY AND METHOD

2.1 The original splitting method

Let X be a K-dimensional training vector of size U such that $X = \{X_j : X_j = (x_{j1}, x_{j2}, \ldots, x_{jK}), X_j \in R^K, j \in J = \{1, 2, \ldots, U\}\}$, where R^K is the K-dimensional real set. Then the K-dimensional VQ codebook of size N can be generated by the splitting method as shown in Fig. 1 with the following steps :

Step_1 Let C_0^f be the center of the training vectors X and let $k = 0$.

Step_2 Set $k = k+1$ and $u = 0$. And define the k-th initial codebook $C_k^{(0)}$ as a union of two code books of C_{k-1}^f and multiplied C_{k-1}^f by r, where the variable r, called the extensive parameter, is a positive constant less than 1.

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Step_3 Obtain C_k^t by training $C_k^{(u)}$ by LBG algorithm. For this, it requires u times full search operations as shown in the dashed block of LBG algorithm in Fig. 1, where D′ is relative distortion defined as

$$D' = \frac{|D^{(u-1)} - D^{(u)}|}{D^{(u)}} \quad (1)$$

where $D^{(u)} = E(d(X_j, C_k^{(u)}))$ and ε is constant.

Step_4 Go to Step_2 until C_T^t is obtained ($T = \log_2 N$). The final codebook C_T^t with N codewords is the desired codebook by the splitting method.

To get the VQ codebook with N codewords in the above splitting method, the LBG algorithm is repeatedly used T times and at every iteration of the LBG algorithm, the corresponding full-search operations are need until $D' \le \varepsilon$. As we have seen, the LBG algorithm requires very large computational expense due to dividing voronois with the full-search method, while the VQ codebook with very high quality can be obtained by the splitting method.

2.2 Mean variable and fast-search voronoi divider

Let C be a K-dimensional VQ codebook of size N such that $C = \{C_i : C_i = (c_{i1}, c_{i2}, \ldots, c_{iK}), C_i \in R^K, i \in I = \{1, 2, \ldots, N\}\}$ and X be a K-dimensional training vector of size U as defined in section 2.1. Now we define the mean values of the codeword C_i and the training vector X_j, denoted S_{C_i} and S_{X_j}, respectively, as

$$S_{C_i} = \frac{1}{K} \sum_{u=1}^{K} c_{iu} \quad (2)$$

$$S_{X_j} = \frac{1}{K} \sum_{u=1}^{K} x_{ju} \quad (3)$$

Let C_i be the quantized vector of X_j and $d(X_j, C_i)$ be the MSE between X_j and C_i : $d(X_j, C_i) = \frac{1}{K} \sum_{k=1}^{K} (x_{ju} - c_{ik})^2$. Then SNR of a VQ codebook, denoted SNR_C, is defined as

$$SNR_C = 10 \times \log_{10}(D_0/D) \quad (4)$$

where $D = E(d(X_j, C_i))$ and $D_0 = E(\sum_{t=1}^{K} x_{jt}^2)$.

Let b_o and SNR_O be the computational expense and the SNR of the full-search dividing voronoi, respectively, and in the same way let b_f and SNR_f be those of the fast-search voronoi divider as we will see, respectively. Note that b_o equals U×K×N. Then we can use the following β and ΔSNR as the computational measures of search algorithms :

Computation ratio	$\beta = b_o/b_f$	(5)
SNR loss	$\Delta SNR = SNR_o - SNR_f$	(6)

Therefore we see that the most desirable fast-searching algorithm takes the highest β for a given small ΔSNR.

Theorem : If $C_i (i \in I)$ is the quantized vector of $X_j (j \in J)$, then the mean values of C_i and X_j have the following relationship :

$$S_{C_i} \cong S_{X_j}. \quad (7)$$

Proof : If C_i is the real quantized vector of X_j, then $d(X_j, C_i) \le d(X_j, C_t)$, $\forall t \in I$. And thus we may assume that $d(X_j, C_i)$ is very small and tends to zero. Therefore it can be said that $x_{jk} \cong c_{ik}$, $k = 1, 2, \ldots, K$, and thus the mean value of C_i in Eg. (1), S_{C_i}, is approximately equal to that of X_j in Eq. (3). Q.E.D.

Let $Z = \{Z_i : i \in I, Z_i \in C\}$ be a rearranged VQ codebook C according to the of mean values of codewords such that $S_{Z_1} \le S_{Z_2} \le \ldots \le S_{Z_N}$ or $S_{Z_1} \ge S_{Z_2} \ge \ldots \ge S_{Z_N}$. We call Z a reconstructed codebook.

Using Theorem and the reconstructed codebook Z, we introduce a fast-search algorithm as shown in Fig. 2. The key point of the algorithm is use of a limited subset Z^S as a search area instead of Z. The procedure for the fast algorithm takes the following steps :

Step_1 Compute the mean value of the input $X_j : S_{X_j}$.

Step_2 Search an index i of codebook Z, whose mean value is the closest to S_{X_j}, i.e. $i = \{k \mid \min_k (|S_{Z_k} - S_{X_j}|)\}$.

Step_3 Search a codeword Z_m in a subset $Z^S = \{Z_k : \max(1, i-M) \le k \le \min(i+M, N)\}$ minimizing MSE such that $Z_m = \{Z_k \mid \min_k (d(Z_k, X_j)), Z_k \in Z^S\}$.

Finally Z_m is the codeword of the input vector X_j.

2.3 The improved splitting method

As noted in section 2.1, the original splitting method requires much time for dividing voronois with full-search. We propose an improved splitting method using partial-search which is mainly due to the property of the mean variable in section 2.2.

The improved method is the same as the original one as shown in Fig.1 except for the partial-search in the block of voronoi divider.

The partial-search voronoi divider is achieved using the following three steps, which is faster than the full-search one because in the former all the voronois can be easily determined by only searching in a subset in the sense of the nearest mean as Fig.2.

Step_1 Calculate each mean value of $C_k^{(u)}$.

Step_2 Rearrange $C_k^{(u)}$ according to their mean values and then obtain a reconstructed codebook.

Step_3 Divide fast each voronoi only searching in a small subset of $C_k^{(u)}$ instead of $C_k^{(u)}$ as in Fig. 2.

Ⅲ. RESULTS AND ANALYSES

The improved splitting method to design the VQ codebook is tested for speech signals. In the process of training VQ codebook, each current codebook $C_k^{(u)}$ is reconstructed in order to get the orderliness of the

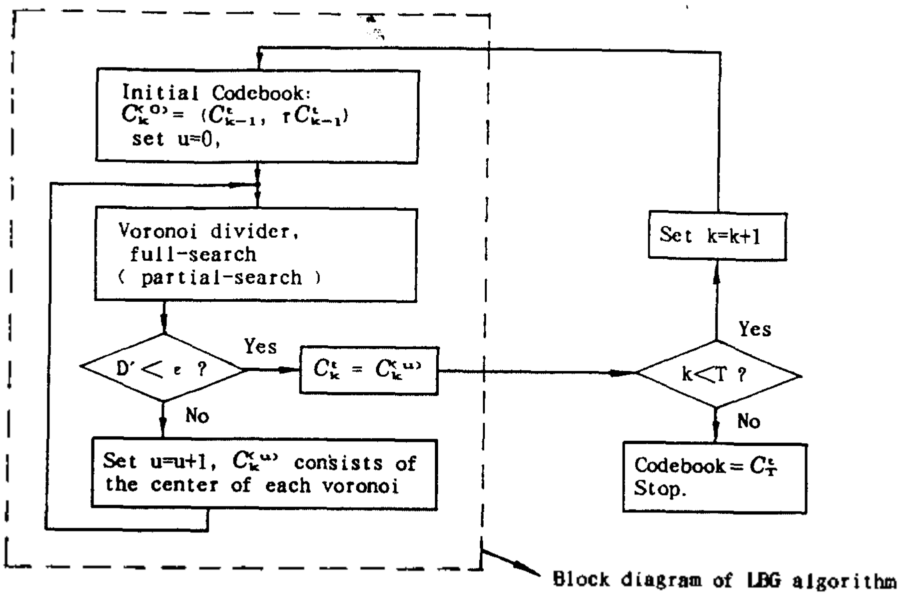


Fig 1. Block diagram of the original splitting method.
C_k^t : trained $C_k^{(u)}$ by LBG algorithm

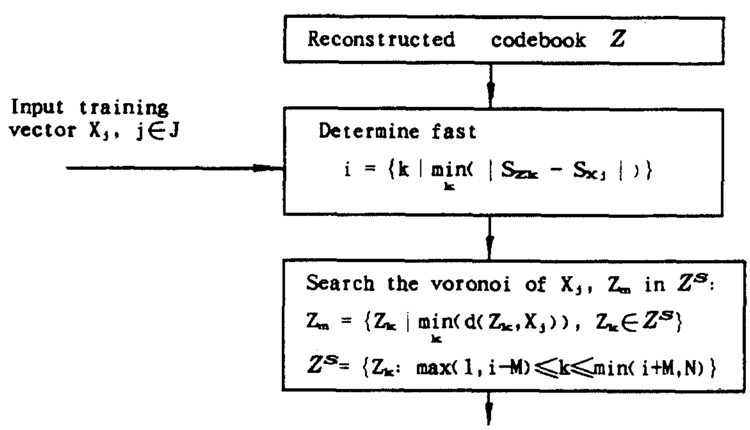
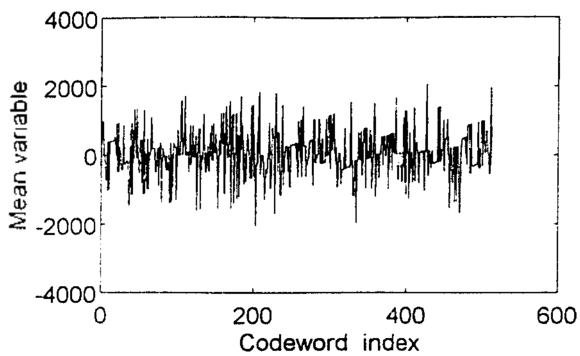


Fig 2. Partial-search voronoi divider.

codebook structure. Mean variable distribution of the current codebook and its reconstructed codebook is shown as Fig.3 (a) and (b), which have 512 codewords. It is obvious that the reconstructed codebook is orderly on its mean variable. The curves of β and ΔD are shown in Fig.4, where K of the VQ codebooks is 4. The curves (I) and (II) in Fig.4 correspond to N=128 and 256, respectively. When M is small, the computation ratio is high and on the other hand the loss of the SNR is large. β and ΔD decrease along with increment of M, while M gets to certain value ΔD becomes very small and tends to 0 but β is always greater than 1. The computational complexity of training codebook can be thus reduced by using the fast-search algorithm. Table 1 shows a comparison of the computational expense between the full-search and the fast-search algorithms, where N is the size of codebook. ΔD in Table 1 is approximately equal to 0. It is evident that β increases along with increment of N. Therefore, the new design method of the VQ codebook can reduce the computational complexity of training codebooks and maintain the same codebook SNR as the original splitting method. SNR of the VQ codebooks according to the extensive coefficient r are shown in Table 2. When N is small, it is not evident for r to influence the SNR of the VQ codebook. When N increases, SNR of the VQ codebook increases with the increment of r, but after r is larger than 0.90 empty voronois appear increasingly in the initial codebook $C_i^{(0)}$.



(a) Mean variable distribution of the current codebook

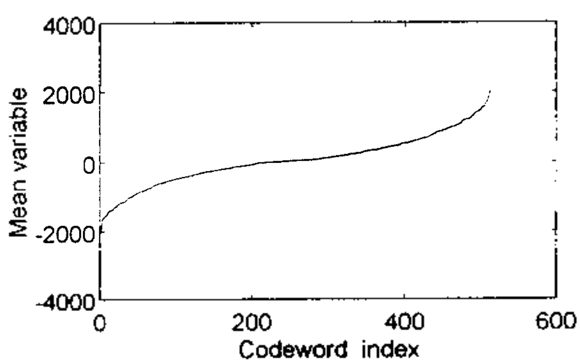
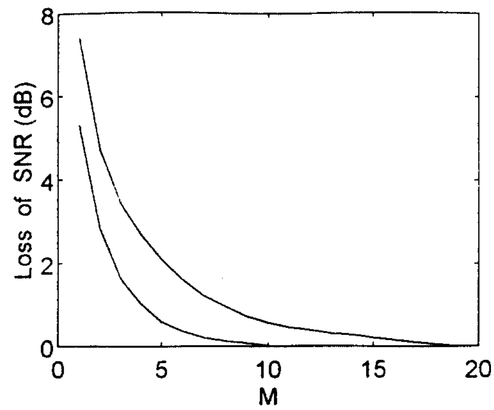


Fig 3. (b) Mean variable distribution of the reconstructed codebook



(a) SNR Loss versus M

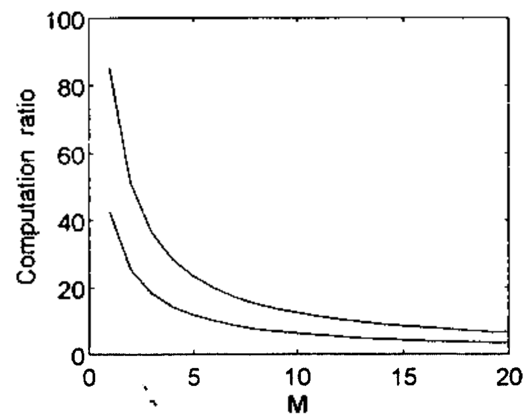


Fig 4. (b) Computation ratio versus M

Table 1. Complexity Comparison between full-search voronoi divider and fast-search voronoi divider

N	br	bf	β
4	16 × U	10.72 × U	1.49
8	32 × U	11.81 × U	2.70
16	64 × U	19.20 × U	3.36
32	128 × U	33.42 × U	3.71
64	256 × U	58.10 × U	4.40
128	512 × U	89.83 × U	5.69
256	1024 × U	159.25 × U	6.43
512	2048 × U	275.93 × U	7.42
1024	4096 × U	486.69 × U	8.41

Table 2. SNR of codebooks versus the extensive coefficient r

N \ r	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.96	0.97	0.98	0.99
4	6.48	6.48	6.45	6.48	6.50	6.50	6.50	6.50	6.50	6.50	6.50
8	9.14	9.14	9.15	9.17	9.20	9.20	9.20	9.20	9.20	9.20	9.20
16	11.57	11.62	11.16	11.59	11.62	11.62	11.61	11.61	11.61	11.61	11.70
32	13.82	13.80	13.91	13.69	13.79	13.78	13.78	13.78	13.78	13.78	13.78
64	15.78	15.72	15.95	15.95	16.08	16.07	16.13	16.14	16.14	16.14	16.14
128	17.17	17.30	17.78	18.05	18.13	18.24	18.24	18.28	18.29	18.30	18.30
256	18.11	18.55	19.22	19.46	20.05	20.42	20.47	22.48	20.50	20.53	20.56
512	18.86	19.57	20.32	20.89	21.82	22.56	22.94	22.94	22.95	22.95	22.95
1024	19.21	20.55	21.45	22.32	23.49	24.28	24.38	24.42	24.43	24.45	24.45

Ⅳ. CONCLUSIONS

An algorithm of the fast-search voronoi divider is presented according to the orderliness of the reconstructed codebook and used to improve the splitting method to design the VQ codebook. Experimental results show that the new design method can get not only the codebooks of high SNR, but also reduce computational complexity of training the VQ codebook several times.

REFERENCES

1. R. M. Gray, *Vector Quantization*, IEEE ASSP Magazine, 1, 4-29, 1984.
2. A. Gerho et al., *Vector quantization and signal compression*, Kluwer Academic Publishers, 1992.
3. J. Makhol, et al., "Vector Quantization in Speech Coding," Proceedings of IEEE, 73(11), pp. 1551-1685, 1985.
4. T. W. Parsons, *Voice and speech processing*, McGraw-Hill Book Company, 1986.
5. M. I. Sezan, R. L. Lagendijk, *Motion analysis and image sequence processing*, Kluwer Academic Publishers, 1993.
6. Y. Linde, A. Buzy, R. M. Gray, "An Algorithm for Vector Quantizer Design," IEEE Trans. on Commun., 28(1), pp. 55-66, 1980.
7. S. P. Lloyd, "Least Squares Quantization in PCM," IEEE Trans. on Inform. Theory, 28(2), pp. 129-137, 1982.

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