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미지입력 비례적분 관측기 설계와 완전 LTR의 실현

김 환 성*, 川路 茂保**, 김 상 봉*

Design of Unknown-Input PI Observer and Realization of Exact LTR

Hwan-Seong KIM*, Shigeyasu KAWAJI**, and Sang-Bong KIM*

ABSTRACT

전형적인 상태 관측기에서는, 외란이 시스템 입력에 가해지는 경우 시스템의 상태 추정이 불가능하다. 이러한 상태관측 문제에 대한 한가지 대책법으로서 비례적분(PI) 관측기가 제안되어 스텝외란의 소거에 대한 유효성이 밝혀져 로바스트 제어기 설계에 대한 응용으로서 널리 연구가 행해져 왔다. 그러나, 미지입력에 대한 PI 관측기 설계는 여전히 문제로 남아 있다.

본 논문에서는 미지입력 PI 관측기의 설계법을 제안하고, 이에 대한 응용으로서 완전 LTR을 실현할 수 있는 결과를 보인다. 먼저, 입력의 정보없이 시스템의 상태를 추정할 수 있는 미지입력 PI 관측기의 충분조건을 제안하고, PI 관측기의 설계에 요구되는 필요충분조건을 보인다. 이러한 조건은 완전 LTR의 실현을 위한 직접적인 요구조건임을 보인다. 따라서, 완전 LTR을 달성하면서 지정한 관측기의 극을 지니는 PI 관측기 설계가 가능하다.

Key Words : Unknown-Input Observer(미지입력 관측기), PI(Proportional Integral)Observer[비례적분 관측기], Exact LTR (Loop Transfer Recovery, 완전 LTR)

1. Introduction

In modern control theory, the linear quadratic regulator (LQR) theory has been widely studied since early 1960's and it was used for designing the robust control system. In that approach the robust properties of control system guarantee the gain margins of -6dB to $+\infty$dB and the phase margins of $+60°$ in all channels[1]. Also, it is now well understood that the problem of constructing the observer-based-control system is not a trivial extension of the LQR theory. In particular, it has been shown by Doyle[2] that linear quadratic Gaussian (LQG) regulators have no intrinsic robustness properties and can exhibit poor stability margins contrary to the LQR theory. The attemption of recoverying those stability margins in the observer-based-control system induced to loop transfer recovery (LTR) problem. Doyle and Stein[3] have proposed the LQG/LTR method

* 부산수산대학교 공과대학 기계공학과
** 일본 구마모토대학 전기정보공학과

which asymptotically achieves the LTR by adding a fictitious noise at the input plant before designing the observer. This method drives some observer poles toward stable plant zeros and the rest toward infinity. Futhermore, for achieving the LTR exactly (Exact LTR), observer gains should be selected sufficiently large (infinite).

Generally, in the observer design we assume that all the system inputs are known and there are no parameter variations and disturbances. However in practical system, if some inputs are completely unknown, then the observed states of system can not be converged to the real states. As one of these defects, Laura[4] has shown that it is not generally feasible to use LQG/LTR method in LTR schemes with small parameter variations or disturbances.

In order to overcome these problems, robust observer is strongly required against unknown--inputs, parameter variations, and disturbances etc..

In implementation of control system, if sampling time is very short and if all disturbances are considered as step states during a sampling period, then several step disturbances can be cancelled by an observer with integrator.

As an observer with integrator, a proportional integral (PI) observer was proposed by Wojciechowski[5] with the aim of desensitizing the observer by asymptotic regulation of observer error in the case of parameter variations and step disturbances.

Recently, Kawaji and Sawada[6] showed that the PI observer has the equivalent relation to disturbance observer. Also, the PI observer is applied to the design of robust control system, and some useful results were reported.[7~8] Umeno and Hori[7] showed a design method of robust control system for DC servomotors. However, PI observer with unknown inputs is not treated until now.

In this paper, we discuss the design method of an unknown-input PI observer and show its applicability to the Exact LTR problem. The sufficient condition for the unknown-input PI observer is given, and a simple existence condition of the observer is presented. Under the condition, the Exact LTR is perfectly achieved by the unknown-input PI observer without any necessary condition except for left invertible system with stable invariant zeros.

Notation

I_n n – square matrix with 1's on the diagonal and 0's elsewhere

$I_{n\times m}$ $n \times m$ dimension matrix with 1's on the diagonal of min(n, m) and 0's elsewhere

O_n n – square matrix with 0's

$O_{n\times m}$ $n \times m$ dimension matrix with 0's

A^g Generalized inverse matrix of A

2. Design of Unknown-Input PI Observer

Consider a linear time-invariant system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the unknown input, and $y(t) \in \Re^p$ is the output. A, B and C are known constant matrices of appropriate dimensions, and rank $B = m$ and rank $C = p$. It is assumed that $p \geq m$ and the pair (C, A) is observable. Further, assume without loss of generality the matrix C has the form

$$C = [C_m \;\; O_{p\times(n-p)}]$$

where C_m is nonsingular.

Consider a related system represented by

$$\begin{cases} \dot{z}(t) = \hat{A}z(t) + \hat{B}y(t) + \hat{J}u(t) + \hat{H}\omega(t) \\ \hat{x}(t) = \hat{C}z(t) + \hat{D}y(t) \\ \dot{\omega}(t) = y(t) - C\hat{x}(t) \end{cases} \tag{2}$$

where $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$, $\hat{H}$ and $\hat{J}$ are unknown matrices of appropriate dimensions.

Definition 1 : The system (2) is said to be a proportional integral observer (PI observer) for the system (1) if and only if

$$\lim_{t\to\infty} e(t) = 0,\ \forall x(0\_),\ z(0\_),\ u(\cdot) \tag{3}$$

$$\lim_{t\to\infty} \omega(t) = 0,\ \forall \omega(0\_) \tag{4}$$

where $e(t) = \hat{x}(t) - x(t)$ represents the observer error.

We can have the following relations between the system and the observer.

Theorem 1 : *The system (2) is an unknown-input PI observer for the system (1) if*

$$\mathrm{Re}\ \lambda_i \begin{bmatrix} \hat{A} & \hat{H} \\ -C\hat{C} & 0 \end{bmatrix} < 0,\ \ i = 1,\dots,n+p \tag{5}$$

and if there exists a matrix $U \in \Re^{n\times n}$ *such that*

$$\hat{A}U + \hat{B}C = UA \tag{6}$$

$$\hat{J} = UB = 0 \tag{7}$$

$$\hat{C}U + \hat{D}C = I_n \tag{8}$$

where $\mathrm{Re}\ \lambda_i[\cdot]$ *denotes the real part of the i-th eigenvalue.*

Proof : Define the estimation error by

$$\xi(t) = z(t) - Ux(t)$$

From (1) and (2), the dynamics of this error obeys

$$\dot{\xi}(t) = \hat{A}\xi(t) + \left(\hat{A}U + \hat{B}C - UA\right)x(t) + \left(\hat{J} - UB\right)u(t) + \hat{H}\omega(t) \tag{9}$$

And eq. (2) leads to

$$\hat{x}(t) = \hat{C}\xi(t) + \left(\hat{C}U + \hat{D}C\right)x(t) \tag{10}$$

$$\dot{\omega}(t) = C\left(x(t) - \hat{x}(t)\right) \tag{11}$$

By substituting (6)-(8) into (9)-(11), we have

$$\dot{\xi}(t) = \hat{A}\xi(t) + \hat{H}\omega(t)$$
$$\hat{x}(t) = \hat{C}\xi(t) + x(t)$$
$$\dot{\omega}(t) = -C\hat{C}\xi(t)$$

or

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{H} \\ -C\hat{C} & 0 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \omega(t) \end{bmatrix}$$
$$\hat{x}(t) = \hat{C}\xi(t) + x(t)$$

Thus, under the condition (5), $e(t) \to 0$ and $\omega(t) \to 0$ $(t \to \infty)$.

Remark 1 : For designing an unknown-input PI observer, the matrix U is selected such that (7) is satisfied. This is directly related with designing the Exact LTR and shown in Section 3

In the following, we let $\hat{C} = I_n$ for simplicity. Then, from (8)

$$U = I_n - \hat{D}C \tag{12}$$

By substitution of (12) into (6), we have

$$\hat{A} = UA - KC \tag{13}$$

$$\hat{B} = \hat{A}\hat{D} + K \tag{14}$$

where

$$K = \hat{B} - \hat{A}\hat{D}$$

Also eq. (5) is rewritten as

$$\mathrm{Re}\ \lambda_i[R] < 0,\ i = 1,\dots,n+p$$
$$;\ R = \begin{bmatrix} UA - KC & H \\ -C & 0 \end{bmatrix} \tag{15}$$

The remained problem is how to find the matrices K and $\hat{H}$ with the designed matrix U. This is the standard problem for designing the PI observer, and was solved systematically by Kawaji and Kim[11]. The procedure of solving the problem is summarized in Appendix.

Lemma 1 : For the unknown-input PI observer (2), there exist the matrices K and $\hat{H}$ if and only if the pair (C, UA) is observable.

Proof : For satisfying eq. (15), there is necessary that rank $\hat{H} = p$. Also, the matrix R is rewritten as

$$R = \begin{bmatrix} UA & \hat{H} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} K \\ I_p \end{bmatrix} [C \ \ 0]$$

Therefore, if the matrix $\hat{H}$ is arbitrary selected such that rank $\hat{H} = p$, the existence condition of matrix R which satisfies eq. (5) is the observability of $\left([C \ \ 0], \begin{bmatrix} UA & \hat{H} \\ 0 & 0 \end{bmatrix}\right)$. And, it can be easily known the pair $\left([C \ \ 0], \begin{bmatrix} UA & \hat{H} \\ 0 & 0 \end{bmatrix}\right)$ is observable if and only if the pair (C, UA) is observable.

Next, we will consider the existence condition of unknown-input PI observer. Substituting (12) into (7), we have

$$\hat{D}CB = B \tag{16}$$

In order for the matrix $\hat{D}$ satisfying (16) to exist

$$\text{rank } CB = \text{rank } B = m \tag{17}$$

must hold. The condition (17) requires that $p \geq m$.

The general solution of (16) can be written as

$$\hat{D} = B(CB)^g + G\{ I_p - CB(CB)^g\} \tag{18}$$

where G is an arbitrary matrix.

By substituting (18) into (12), we can get

$$U = (I_n - GC)\{I_n - B(CB)^g C\}$$

From the above equation, there exists a matrix G which makes $(I_n - GC)$ nonsingular, and then the rank $U = n - m$.

Since rank $B = m$, there exists the left-inverse of matrix B, *i.e.*,

$$B^g B = I_m$$

Under the condition of rank $U = n - m$, we have Ker $U \cap$ Ker $B^g = \{0\}$, *i.e.*,

$$\text{rank} \begin{bmatrix} U \\ B^g \end{bmatrix} = n$$

Then, we have the relation

$$\begin{bmatrix} U & 0 \\ B^g & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I_n & 0 \\ -B^g(sI_n - A) & I_m \end{bmatrix}$$
$$= \begin{bmatrix} U(sI_n - A) & 0 \\ 0 & I_m \\ C & 0 \end{bmatrix}$$
$$= \begin{bmatrix} I_n & 0 & -s\hat{D} \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} \begin{bmatrix} sI_n - UA & 0 \\ 0 & I_m \\ C & 0 \end{bmatrix}$$

It follows that

$$\text{rank} \begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} = m + \text{rank} \begin{bmatrix} sI_n - UA \\ C \end{bmatrix}$$

Consequently, for $\forall s \in C$, $\text{Re}(s) \geq 0$

$$\text{rank} \begin{bmatrix} sI - A & B \\ C & 0 \end{bmatrix} = n + m \tag{19}$$

which means that the invariant zeros of the system (1) must be stable.

From the above statements, we summary the following theorem.

Theorem 2 : *For the system (1), the unknown-input PI observer (2) exists if*

(i) rank CB = rank $B = m$

(ii) rank $\begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} = n + m, \ \forall s \in C, \ \mathrm{Re}(s) \geq 0$

Remark 2 : In Theorem 2, the condition (ii) is equivalent to following condition

(i) rank $\begin{bmatrix} sI_n - UA \\ C \end{bmatrix} = n, \forall s \in C (\forall s \in C, \mathrm{Re}(s) \geq 0)$

(ii) pair (C, UA) is observable (detectable)

If $I_p - C\hat{D} = 0$, then the pair (C, UA) is unobservable. This case is obtained for example $p = m$, and CB is nonsingular[12].

3. Exact LTR by Unknown-Input PI Observer

In this section, we will show that the Exact LTR can be achieved by the unknown-input PI observer. Let the system (1) be controlled by an observer based controller.

$$u(t) = -F\hat{x}(t)$$

where F is the state feedback gain and $\hat{x}(t)$ the estimated state vector. The PI observer based control system is illustrated by Fig. 1.

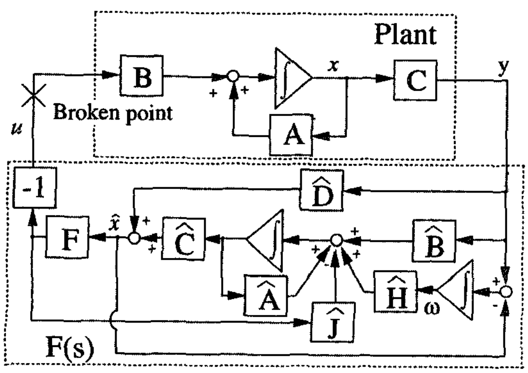

Fig. 1 PI observer based feedback control system

Assuming that the broken point is located in plant input, the loop transfer function is given by

$$L_{obs}(s) = F(s)G(s) \tag{20}$$

where

$$G(s) = C\Phi(s)B$$
$$: \ \Phi(s) = (sI - A)^{-1}$$

is the transfer function of plant, and

$$F(s) = -F\{\phi(s) + \hat{J}K + s^{-1}\hat{H}C\}^{-1}(\phi(s)\hat{D} + \hat{B} + s^{-1}\hat{H})$$
$$: \ \phi(s) = (sI - \hat{A})$$

is the transfer function of the PI observer based controller. The corresponding loop transfer function with state feedback law is given by

$$L_{sf}(s) = -F\Phi(s)B \tag{21}$$

The difference between the two loop transfer functions $L_{obs}(s)$ and $L_{sf}(s)$ is defined as the loop recovery error at the input loop braken point

$$E_I(s) = L_{obs}(s) - L_{sf}(s)$$
$$= M_I(s) \ \{I + M_I(s)\}^{-1}\{I + F\Phi(s)B\} \tag{22}$$

where

$$M_I(s) = F(sI - \hat{A} + s^{-1}\hat{H}C)^{-1}\hat{J} \tag{23}$$

So, it is obvious that the Exact LTR is achieved if $M_I(s) = 0$. Eq. (23). called the recovery matrix for the Exact LTR, is rewritten as

$$M_I(s) = F(sI - UA + KC + s^{-1}HC)^{-1}UB \tag{24}$$

It follows that if $UB = 0$, the recovery matrix $M_I(s)$ equals zero exactly.

Theorem 3 : *The Exact LTR for the system (1) is achieved, if unknown-input PI observer is constructed.*

Remark 3 : The condition of Exact LTR in Theorem 3 is equivalent to

$$\langle \hat{A}, \ \mathrm{Im}\hat{J} \rangle \subset \mathrm{Ker} \ \hat{C}$$

where, $\langle \hat{A}, \mathrm{Im}\hat{J} \rangle$ denotes the controllable subspace for the system $(\hat{A}, \hat{J})$. This result was shown in Niemann et al[10]. But, method for designing the observer was not shown.

4. Numerical example

We consider the following system which was used as an example in Kawaji and Kim[11].

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ -1.6 & -2.3 & -1.2 \end{bmatrix}, \; B = \begin{bmatrix} 2.7 \\ 0.5 \\ -1.2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix}$$

The state feedback gain matrix F is designed by the conventional LQR method as

$$F = [\, 0.9678 \quad 0.2692 \quad 0.0409 \,]$$

In this system, the conditions of Theorem 2 are satisfied, so that there exists an arbitrary matrix G such that PI observer is stable. Thus, the unknown-input PI observer can be designed.

The matrix $\hat{D}$ is calculated from (18)

$$\hat{D} = \begin{bmatrix} 0.821 & 0.967 \\ 0.603 & -2.259 \\ -0.218 & -1.225 \end{bmatrix}$$

with

$$G = \begin{bmatrix} 1.000 & 1.000 \\ 2.000 & -2.000 \\ 1.000 & -1.000 \end{bmatrix}$$

Then, the matrix U is iobtained from (12) as

$$U = \begin{bmatrix} 0.179 & -0.967 & 0.000 \\ -0.603 & 3.259 & 0.000 \\ 0.218 & 1.225 & 1.000 \end{bmatrix}$$

Let the eigenvalues of PI observer be {-1, -2, -3, -4, -5}. Then, the PI observer gains K and $\hat{H}$ are obtained from Appendix A as

$$K = \begin{bmatrix} -6.011 & -0.544 \\ 0.090 & 8.411 \\ -1.768 & 186.685 \end{bmatrix}, \; \hat{H} = \begin{bmatrix} 7.872 & 178.529 \\ 0.296 & -591.894 \\ 0.000 & 0.000 \end{bmatrix}$$

The other unknown parameters are calculated from (13) and (14).

$$\hat{A} = \begin{bmatrix} -6.011 & 0.723 & -0.967 \\ 0.090 & -9.014 & 3.259 \\ 0.168 & -188.767 & 0.025 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} 1.723 & -6.805 \\ -6.165 & 24.865 \\ -115.547 & 613.169 \end{bmatrix}$$

The result shown in Fig. 2 is the loop transfer recovery by unknown-input PI observer, which is equal to that of state feedback control system perfectly. In this case, the compared conventional PI observer was designed by LQG/LTR method (observation noise intensity $\rho = 0.5$)[11].

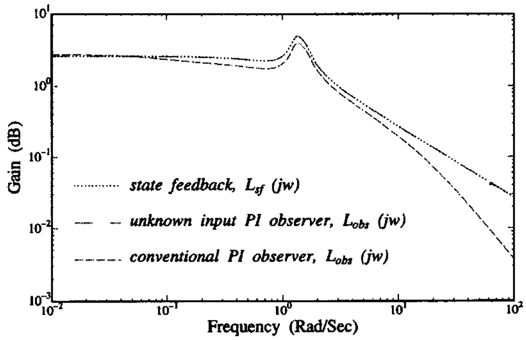

Fig. 2 Gain plots for loop transfer functions

5. Conclusions

In this paper, we have presented a simple design method of unknown-input PI observer and shown a relation between unknown-input PI observer and Exact LTR.

The sufficient condition for the unknown-input PI observer is derived. A simple existence condition is proposed to construct the PI observer and can be checked by rank conditions. Also, it was shown that the Exact LTR by unknown-input PI observer is achieved without any condition except for left invertible system with stable invariant zeros.

Finally, the effectiveness of unknown-input PI observer was shown by an example.

References

1. Safonov M. G. and Athans, M., "Gain and Phase Margin for Multiloop LQG Regulators", IEEE Trans. on Automatic Control, Vol. 22, pp. 173~179, 1979.
2. Doyle, J. C., "Guaranteed margins for LQG Regulators", IEEE Trans. on Automatic Control, Vol. 23, pp. 756~757, 1978.
3. Doyle, J. C. and Stein, G., "Multivariable feedback design : concepts for a classical/modern synthesis", IEEE Trans. on Automatic Control, Vol. 26, pp. 4~16, 1981.
4. Laura, R. R., "Stability Robustness of Uncertain LQG/LTR", IEEE Trans. on Automatic Control, Vol. 38, pp. 304~308, 1993.
5. Wojciechowski, B., "Analysis and synthesis of proportional - integral observers for single - input single - output time-invariant continuous systems", Ph. D. dissertation, Gliwice, Poland. 1978.
6. Kawaji, S. and Sawada, K., "On the relation between PI observer and disturbance observer", Trans. of IEEJ, Vol. 114-C, pp. 1326~1327, 1994. (in Japanese)
7. Umeno, T. and Hori, Y., "Robust Speed Control of DC Servomotors Using Modern Two Degrees-of-Freedom Controller Design",Trans. on IEEE Industrial Electronics, Vol. 38. pp. 363, 1991.
8. Kawaji, S. and Sawada, K., "Robust Impedance Control by Using the Environment Observer", Proc. of 2nd IEEE Int. Workshop on Advanced Motion Control, pp. 9~12, 1992.
9. Beale, S. and Shafai, B., "Robust control system design with a proportional integral observer", International Journal of Control, Vol 50, pp. 97 ~111, 1989.
10. Niemann, H. H., Sϕgaard-Andersen, P., and Stoustrup, J., "Loop transfer recovery for general observer architectures", International Journal of Control, Vol. 53, pp. 1177~1203, 1991.
11. Kawaji, S. and Kim, H. S., "Simultaneous Recovery of Loop Transfer Property and Disturbance Attenuation Property by PI observer", Proceeding of the Asian Control Conference, Vol. I, pp. 17~20, 1994.
12. Darouach, M., Zasadzinski, M., and Xu, S. J., "Full-Order Observer for Linear Systems with Unknown Inputs", IEEE Trans. on Automatic Control, Vol. 39, pp. 606~609, 1994.

Appendix A

⟨Design algorithm of PI observer⟩

Step 1 : Construct augmented matrices as

$$A_e = \begin{bmatrix} UA & I_{n\times p} \\ I_{p\times n} & 0_p \end{bmatrix},\ C_e = \begin{bmatrix} C & 0_p \end{bmatrix}$$

Step 2 : Design a matrix Le by conventional pole assignment, LQG, or etc.

$$\text{Re}\ \lambda_i[A_e - L_eC_e] < 0,\ i = 1,\ldots,n+p$$

where

$$L_e = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

Step 3 : Calculate the matrices K and

$$K = L_1$$

$$\hat{H} = I_{n\times p}(L_2 - I_{p\times n}C^g)$$