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Travel Time Model of a Storage/Retrieval Machine with Acceleration and Deceleration Rate[1]

- 가 · 감속을 고려한 저장/불출 기계의 운행시간 모델 -

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요지

자동창고의 설계단계에서 생산율(throughput)을 추정하는 것은 필수적이다. 생산률은 저장/불출 기계의 운행시간에 역비례하므로, 생산율을 추정하기 위해서는 저장/불출 기계의 운행시간을 추정하여야 한다. 이러한 운행시간은 저장랙의 크기, 저장/불출 기계의 가 · 감속율, 수직/수평 방향의 최대속도, 그리고 저장정책에 의하여 결정된다. 본 논문에서는 Bozer와 White, Hwang과 Lee의 연구를 기초로 하여, 임의저장 정책하에 저장/불출 기계의 운행시간을 구하는 간단한 모델을 제시한다. 이러한 운행시간 모델은 자동창고의 설계단계에서 유용하게 사용될 수 있다.

1. Introduction

Automated storage/retrieval systems (AS/RS), appeared in early 50's in Europe, are widely used in production and distribution systems due to the immense benefits such as lower land cost (higher floor-space utilization), improved throughput level, savings in labor cost, reduced inventory cost through better material control, and improved safety considerations. Based on the application area, AS/RS can be classified as unit load AS/RS, miniload AS/RS, deep lane AS/RS, man-on-board AS/RS, and microload AS/RS. Storage and/or retrieval requests in the first three types of AS/RSs are usually performed through the single command and/or the dual command by a storage/retrieval (S/R) machine. In this paper, we study the travel time model of the single and dual commands considering acceleration and deceleration (A/D) rates of the S/R machine.

Many researchers have studied the design and operational issues of AS/RS. Since an AS/RS requires great monetary investment and is inflexible to future changes, a careful initial design is critical to the success of the system. In the initial design phase, we should determine the number of S/R machines, A/D rate and maximum horizontal and vertical velocities of an S/R machine, and physical size of the storage rack to minimize the total cost and to satisfy the required throughput level. In the operational phase, subjects on the assignment of storage location, assignment of multiple items to the same pallet, sequencing the storage/retrieval requests, and determination of the dwell point should be studied in order to maximize utilization of AS/RS. In such problems, travel time of an S/R machine is a crucial factor, since the performance of AS/RS is significantly affected by the role of the S/R machine.

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2. Literature review

The Material Handling Institute, Inc. (MHI) [6] suggests expected travel time models for both the single and dual commands. To compute the single command travel time, storage or retrieval point is assumed to be located at the center of the storage rack. To compute the dual command travel time, it is assumed that the storage point is located at the center of the rack and the retrieval point is located three-fourths of the horizontal and vertical distance from the input/output (I/O) point.

Hausman, Schwarz, and Graves [3] obtain the expected single command travel time under both the randomized and dedicated storage policies. The rack is assumed to be "square in time", i.e., time to reach the most distant row from the I/O point equals the time to reach the most distant column, given that the I/O point is located at the lower left-hand corner of the rack. It is shown that the expected one way travel time is 2/3 time units for a square rack with unit travel time in the horizontal and vertical directions, under continuous approximation of the rack face and randomized storage. They also show empirically that the continuous assumption of the rack works satisfactorily.

Graves, Hausman, and Schwarz [2] present analytical and empirical results for various combinations of alternative storage assignment rules and scheduling policies. It is again assumed that the rack is square in time. They show that the dedicated storage results in less travel time than the randomized storage, i.e., maximizes the throughput level.

Bozer and White [1] develop simple but useful analytical travel time models for both the single and dual commands under the randomized storage assignment policy. In developing the models, the square in time assumption employed in [2] and [3] is relaxed, which is a very restrictive assumption. They also study the effect of the location of the I/O point and that of the dwell point strategies. To compute the single command travel time, they first find the cumulative distribution function of the travel time and then find the probability density function, noting that the storage location is uniformly distributed in the horizontal and vertical directions. The interleave time, required to compute the dual command travel time, is computed using the order statistics.

Single and dual travel time models considering the A/D rates are developed by Hwang and Lee [4]. The randomized storage policy and the continuous rack assumptions are also employed in developing the model. The models estimate the travel time very accurately. However, the travel time models are so complicated that application of the models may be limited. To build the model, they assume that the maximum horizontal velocity is greater or equal to the maximum vertical velocity and the storage rack is long and high enough for the S/R machine to reach the maximum velocity from the I/O point. It is also assumed that the vertical A/D rates and the horizontal A/D rates are the same, which may not be true in the field.

As discussed above, the travel time models developed by Bozer and White [1] are simple and can be used to estimate the throughput rate of a proposed or existing AS/RS. However, those models ignore the A/D rate which has very significant impact on the throughput, especially when the A/D rate is low and/or the physical size of the storage rack is relatively small, i.e., most of the storage locations can be reached below the maximum velocity. (Bozer and White assume constant velocity in both directions. However, they do not discuss how to obtain the constant velocity.) Hwang and Lee consider the A/D rate in developing the travel time model. However, this model is complicated and A/D rates in the horizontal and vertical directions are assumed to be the same. However, both models estimate the travel time very accurately under the given assumptions.

3. The Model

In this section, we develop a simple heuristic travel time model in which the A/D rates in both directions are different and the storage rack is small so that the maximum velocity may not be reached. The model is developed under the following assumptions.

1. The randomized storage assignment policy is used.
2. The Tchebychev travel is used. That is, the S/R machine travels simultaneously in the horizontal and vertical directions.
3. The rack length and height, as well as the specification of the S/R machine such as horizontal/vertical velocity and the A/D rate, are known.
4. The absolute values of the A/D rates in a given direction are the same.
5. The rack face is considered to be a continuous rectangle and the I/O point is located at the lower left-hand corner of the face.

Notation used is shown below:

V_h = maximum velocity of an S/R machine in the horizontal direction,
V_v = maximum velocity of an S/R machine in the vertical direction,
a_h = A/D rate of an S/R machine in the horizontal direction,
a_v = A/D rate of an S/R machine in the vertical direction,
L_h = horizontal length of the rack,
L_v = vertical length of the rack.

Since the S/R machine travels independently in the horizontal and vertical directions, we first examine the horizontal movement. The rack face can be divided into two regions as shown in Fig. 1, in which the minimum distance required for the S/R machine to reach V_h is denoted by d. To reach a storage/retrieval point in region I, the S/R machine cannot reach to its maximum velocity, V_h. However, to reach a point in region II, the S/R machine can reach to V_h. In this case, the distance travelled with V_h is $x-d$, where x is the storage/retrieval location in the horizontal direction.

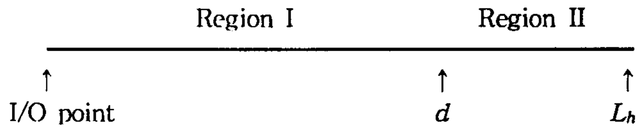


Figure 1. Division of storage rack in horizontal direction.

Hwang and Lee [4] show that the travel time to point x in the horizontal direction from the I/O point, t_x, is given by

$$t_x = \begin{cases} \sqrt{\dfrac{4x}{a_h}} & \text{for} \quad 0 \le x \le d \\ \dfrac{x}{V_h} + \dfrac{V_h}{a_h} & \text{for} \quad d \le x \le L_h \end{cases} .$$

They also show that d is obtained as

$$d = \frac{V_h^2}{a_h} .$$

Suppose that $L_h > d$. In this case, the expected travel time in the horizontal direction, i.e., T_h, can be obtained by conditioning on the location of the storage/retrieval point, i.e., Z. That is,

$$T_h = E[t_r \mid Z<d]P[Z<d] + E[t_r \mid Z\geq d]P[Z\geq d]. \qquad \cdots\cdots(1)$$

The probability that the horizontal distance to the point is less than d, i.e., $P[Z<d]$, is given by

$$P[Z<d] = \frac{d}{L_h} = \frac{V_h^2}{a_h L_h}, \qquad \cdots\cdots(2)$$

since the storage/retrieval location is uniformly distributed over the interval $(0,L_h)$. Similarly, the probability that the point is located $d \leq x \leq L_h$, i.e., $P[Z\geq d]$, is given by

$$P[Z\geq d] = \frac{L_h - d}{L_h} = \frac{a_h L_h - V_h^2}{a_h L_h}. \qquad \cdots\cdots(3)$$

When the storage/retrieval location is located between [0,d], the expected travel time is obtained as follows.

$$\begin{aligned} E[t_r \mid Z<d] &= \int_0^d \frac{1}{d}\sqrt{\frac{4x}{a_h}}\,dx \\ &= \frac{4}{3}\frac{V_h}{a_h}. \end{aligned} \qquad \cdots\cdots(4)$$

When the storage/retrieval location is located between $[d, L_h]$, the expected travel time is computed as follows.

$$\begin{aligned} E[t_r \mid Z\geq d] &= \int_d^{L_h} \frac{1}{L_h - d}\left[\frac{x}{V_h} + \frac{V_h}{a_h}\right]dx \\ &= \frac{a_h}{a_h L_h - V_h^2}\left[\frac{L_h^2}{2V_h} + \frac{V_h L_h}{a_h} - \frac{3V_h^3}{2a_h^2}\right]. \end{aligned} \qquad \cdots\cdots(5)$$

Therefore, substituting equations (2)-(5) into equation (1), we obtain

$$T_h = \frac{L_h}{2V_h} + \frac{V_h}{a_h} - \frac{V_h^3}{6L_h a_h^2}. \qquad \cdots\cdots(6)$$

In the randomized storage rack, the expected travel distance in the horizontal direction, D_h, is given by

$$D_h = \frac{L_h}{2}. \qquad \cdots\cdots(7)$$

Noting that the expected velocity of an S/R machine is obtained by dividing the expected travel distance by the expected travel time, the average velocity, V_h', is obtained as follows from equations (6) and (7).

$$\begin{aligned} V_h' &= \frac{D_h}{T_h} \\ &= \frac{L_h}{\frac{L_h}{V_h} + \frac{2V_h}{a_h} - \frac{V_h^3}{3a_h^2 L_h}}. \end{aligned} \qquad \cdots\cdots(8)$$

Now consider the case where $L_h < d$. In this case, the expected travel time is computed as follows.

$$T_h = \int_0^{L_h} \frac{1}{L_h}\sqrt{\frac{4x}{a_h}}\,dx = \frac{4}{3}\sqrt{\frac{L_h}{a_h}}\,. \qquad \cdots\cdots (9)$$

Hence, V'_h is obtained as follows, since the expected travel distance is $L_h/2$.

$$V'_h = \frac{D_h}{T_h} = \frac{3}{8}\sqrt{a_h L_h}\,. \qquad \cdots\cdots (10)$$

So far, we derive the expected travel velocity in the horizontal direction. The expected travel velocity in the vertical direction, V'_v, is also obtained by a similar method as follows.

$$V'_v = \begin{cases} \dfrac{L_v}{\dfrac{L_v}{V_v} + \dfrac{2V_v}{a_v} - \dfrac{V_v^3}{3\,a_v^2 L_v}} & \text{for } L_v \geq \dfrac{V_v^2}{a_v} \\[2ex] \dfrac{3}{8}\sqrt{a_v L_v} & \text{for } L_v \leq \dfrac{V_v^2}{a_v} \end{cases} . \qquad \cdots\cdots (11)$$

Since we find the expected travel velocities in the horizontal and vertical directions, these velocities, shown in equations (8), (10), and (11), can be used to compute the single and dual command travel time using the travel time models developed by Bozer and White [1], that is

$$T = Max\left(\frac{L_h}{V'_h},\ \frac{L_v}{V'_v}\right),$$

$$Q = \frac{1}{T}\,Min\left(\frac{L_h}{V'_h},\ \frac{L_v}{V'_v}\right),$$

$$E[SC] = T\left(\frac{Q^2}{3}+1\right) + 2T_{p/d},$$

$$E[DC] = T\left(\frac{4}{3} + \frac{Q^2}{2} - \frac{Q^3}{30}\right) + 4T_{p/d},$$

where E[SC] is the expected single command travel time, E[DC] is the expected dual command travel time, and $T_{p/d}$ is the pickup/deposit time.

4. Numerical results

Travel time of a single command consists of travel time from the I/O point to the storage (retrieval) point, travel time from the storage (retrieval) point to the I/O point, and two pickup/deposit times. Travel time of a dual command consists of travel time from the I/O point to the storage point, travel time from the storage point to the retrieval point, travel time from the retrieval point to the I/O point, and four pickup/deposit times. To evaluate the performance of the proposed model, we ignore the pickup/deposit time. However, these time factors can be easily included in the model.

The results obtained from the proposed model are compared with those obtained from empirical experiments. To obtain the empirical results, SIMAN developed by Pegden *et al.* [5] is used. The warmup period is determined by the MOVAVERAGE command and the results obtained in the initial bias period are not included for output analysis. To analyze the steady state results, the batch means methodology is used with 10 batches. The length of each batch is determined by CORRELOGRAM command.

For the experiments, we consider two data sets. The first data set is identical to the data used in Hwang and Lee [4]. The storage rack is 60m long and 20m high, where each storage location is 1m long and 1m high. That is, there are 1,200 storage locations which are square. The maximum velocities are 5m/sec and 2m/sec in the horizontal and vertical directions, respectively. Five cases are tested for the A/D rate, i.e., 0.5, 0.6, 0.7, 0.8, 0.9 m/sec^2. (When $a_h=a_v=0.5\text{m/sec}^2$, the minimum distances to reach the maximum horizontal and vertical velocity are 50m and 8m, respectively.)

In the second data set, three different rack sizes are tested, i.e., 30m(L)×10m(H), 30m×30m, and 10m×30m. The dimension of each storage location is 1m by 1m. The maximum velocities are 2.666m/sec and 0.666m/sec in the horizontal and vertical directions, respectively. The A/D rates are 0.3m/sec^2 and 0.1m/sec^2 in the horizontal and vertical directions, respectively. (The minimum distances to reach the maximum horizontal and vertical velocity are 23.76m and 4.49m, respectively.)

Table 1 summarizes results obtained from the first data set. The simulation result indicates that the Bozer and White (B&W) model with maximum velocities in both directions severly underestimates the travel time. When the A/D rate is 0.5 m/sec^2, the estimated single command travel time from the B&W model is about 48% of the travel time obtained from the simulation, in which acceleration and deceleration are considered. In addition, the estimated dual command travel time from the B&W model is about 46% of that from simulation. That is, if the B&W model is used in the initial design phase with only the maximum velocities, one may overestimate the throughput level of the AS/RS, and as a result the installed AS/RS cannot meet the required throughput. We can also note that the discrepancy between results from the simulation and the B&W model decreases as the A/D rate increases. (It is empirically shown that the travel time obtained from simulation approaches to the value obtained from the B&W model as the A/D rate increases.)

The travel time obtained from simulation is very close to the travel time obtained from the Hwang and Lee (H&L) model, regardless of the A/D rate and the types of the travel commands. That confirms the accuracy of the H&L model. However, recall that the H&L model is complicated and based on the assumption of $a_h=a_v$.

The second and third columns in Table 1 shows the expected travel velocities in the horizontal and vertical directions, which are function of the A/D rate, maximum velocity, and the physical size of the storage rack. Applying these expected velocities to the B&W model, we obtain travel times listed in the seventh and twelfth columns for single and dual commands, respectively. The maximum difference between the travel times from simulation and the estimated travel times from the proposed model is less than 7%. However, when the dual command is used, the maximum difference is less than 1.4%. That is, the proposed model estimates the expected single and dual command travel times relatively accurately. The single and dual command travel times obtained from the B&W model, simulation, and the proposed model are graphically shown in Fig. 2. Similar results are obtained for the different rack sizes, which are presented in Table 2 and Table 3. (The only difference among Tables 1-3 is the dimension of the rack.) Note that the difference, presented in Table 2, between the values obtained from the B&W model and from the simulation is smaller than the difference presented in Table 1. The rack size in Table 2 is larger than the rack size in Table 1. Therefore the effect of A/D rate in Table 2 becomes smaller than that in Table 1.

From Table 4 which summarizes the results obtained from the second data set, we are able to find the similar pattern observed from the first data set, except the case with

10mx30m rack where the proposed model underestimates the travel time. Note that, in the second data set, the A/D rates in the horizontal and vertical directions are different. (H&L model cannot be applied, since the model assumes the same A/D rates in both directions.) The single and dual command travel times obtained from the B&W model, simulation, and the proposed model are graphically shown in Fig. 3.

As a summary, our proposed model tends to overestimate the expected single and dual command travel times. Note that the travel times shown in Tables 1-4 are computed without considering the pickup/deposit times. Hence, if the pickup/deposit time is considered, the difference between the results obtained from simulation and the results obtained from the proposed model is expected to become even smaller.

Table 1. Expected single and dual command travel times.
(Data 1 : L_h = 60m, L_v = 20m, V_h = 5m/sec, V_v = 2m/sec)

	Expected velocity		Single command travel time					Dual command travel time				
a_h=a_v	V_h'	V_v'	B&W model	simulation(95% C.I.)	H&L model	Proposed model	Deviation from simulation	B&W model	simulation (95% C.I.)	H&L model	Proposed model	Deviation from simulation
0.5	2.053	1.145	14.78	30.6±0.33	30.54	32.70	6.9%	19.94	43.5±0.53	42.87	43.98	1.1%
0.6	2.244	1.227		28.1±0.31	28.04	30.05	6.9%		40.1±0.56	39.34	40.41	0.8%
0.7	2.413	1.295		26.3±0.29	26.19	28.06	6.7%		37.5±0.54	36.70	37.75	0.7%
0.8	2.562	1.352		24.9±0.28	24.76	26.53	6.5%		35.3±0.41	34.66	35.70	1.1%
0.9	2.696	1.401		23.8±0.27	23.64	25.31	6.3%		33.6±0.42	33.04	34.06	1.4%

Table 2. Expected single and dual command travel times.
(Data 1 : L_h = 60m, L_v = 60m, V_h = 5m/sec, V_v = 2m/sec)

	Expected velocity		Single command travel time				Dual command travel time			
a_h=a_v	V_h'	V_v'	B&W model	simulation(95% C.I.)	Proposed model	Deviation from simulation	B&W model	simulation (95% C.I.)	Proposed model	Deviation from simulation
0.5	2.053	1.586	31.60	43.2±0.61	45.35	5.0%	42.34	59.2±0.63	61.14	3.3%
0.6	2.244	1.642		41.3±0.48	43.06	4.3%		56.6±0.62	58.03	2.5%
0.7	2.413	1.684		40.0±0.34	41.41	3.5%		54.3±0.56	55.77	2.7%
0.8	2.562	1.718		39.0±0.39	40.16	3.0%		53.0±0.38	54.07	2.0%
0.9	2.696	1.745		38.2±0.42	39.19	2.6%		51.7±0.54	52.74	2.0%

* Results for L_h = 60m and L_v = 60m are not available from Hwang and Lee [4].

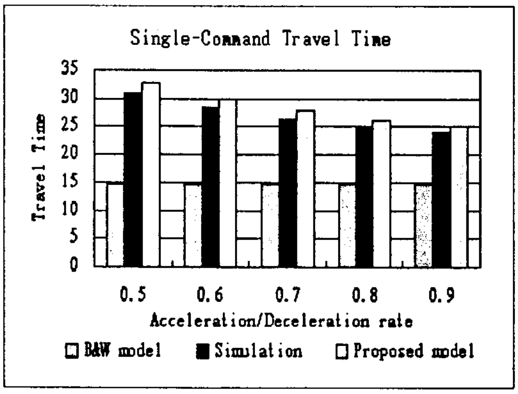
Table 3. Expected single and dual command travel times.
(Data 1 : L_h = 20m, L_v = 60m, V_h = 5m/sec, V_v = 2m/sec)

a_h=a_v	Expected velocity		Single command travel time				Dual command travel time			
	V_h'	V_v'	B&W model	simulation(95% C.I.)	Proposed model	Deviation from simulation	B&W model	simulation (95% C.I.)	Proposed model	Deviation from simulation
0.5	1.186	1.586	30.18	39.8±0.33	40.33	1.3%	40.26	53.6±0.37	54.08	0.9%
0.6	1.299	1.642		38.4±0.34	38.71	0.8%		51.6±0.46	51.88	0.5%
0.7	1.403	1.684		37.4±0.34	37.52	0.3%		50.0±0.82	50.27	0.5%
0.8	1.500	1.718		36.6±0.25	36.63	0.1%		48.8±0.66	49.05	0.5%
0.9	1.591	1.745		36.0±0.29	35.92	0.2%		47.9±0.66	48.09	0.4%

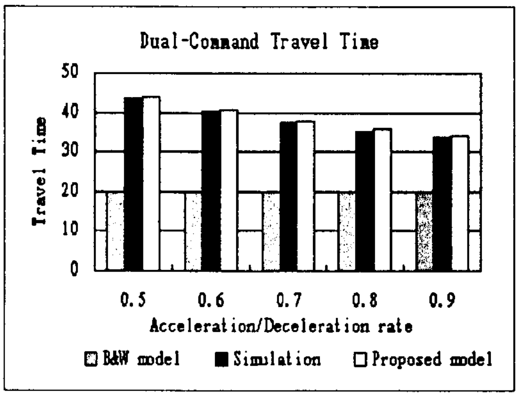
* Results for L_h = 60m and L_v = 60m are not available from Hwang and Lee [4].

Table 4. Expected single and dual command travel times.
(Data 2 : V_h = 2.666m/sec, V_v = 0.666m/sec, a_h = 0.3m/sec^2, a_v = 0.1m/sec^2)

L_h/L_v	Expected velocity		Single command travel time				Dual command travel time			
	V_h'	V_v'	B&W model	simulation(95% C.I.)	Proposed model	Deviation from simulation	B&W model	simulation (95% C.I.)	Proposed model	Deviation from simulation
30m/10m	1.124	0.366	17.83	33.5±0.25	36.03	7.6%	24.03	46.6±0.39	48.64	4.4%
30m/30m	1.124	0.517	45.98	59.7±0.99	62.13	4.1%	61.44	83.2±1.43	83.33	0.2%
10m/30m	0.650	0.517	45.15	61.3±1.51	59.40	-3.1%	60.22	80.8±1.39	79.39	-1.7%



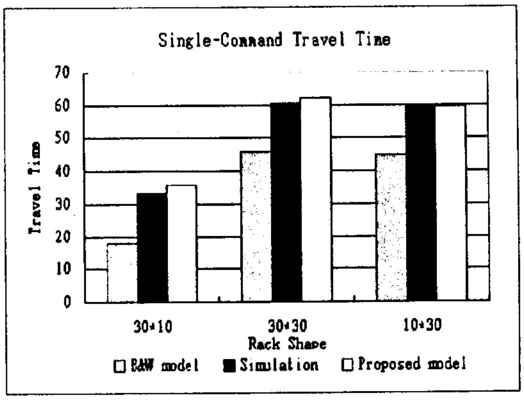
a) Single command travel time.



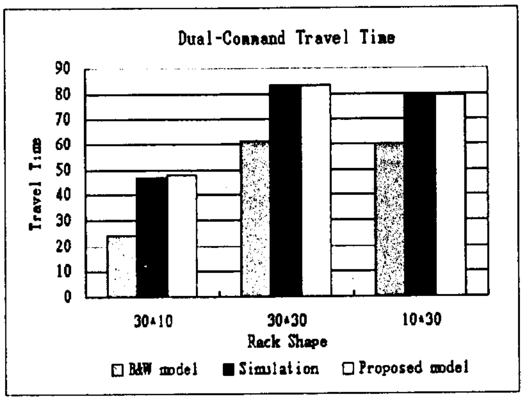
b) Dual command travel time.

Figure 2. Expected single and dual command travel time from the B&W model, simulation, and the proposed model.

(Data 1 : L_h = 60m, L_v = 20m,
V_h = 5m/sec, V_v = 2m/sec)



a) Single command travel time.



b) Dual command travel time.

Figure 3. Expected single and dual command travel time from the B&W model, simulation, and the proposed model.

(Data 2 : V_h = 2.666m/sec, V_v = 0.666m/sec,
a_h = 0.3m/sec^2, a_v = 0.1m/sec^2)

5. Conclusions

Travel time of an S/R machine plays an important role in estimating the throughput level of AS/RS in the early design phase. The B&W model is simple but tends to underestimate the travel time. That is, an AS/RS designed without considering the A/D rate would not meet the required throughput capacity. On the other hand, the H&L model estimates the travel time very accurately. However, the computation procedure is so complicated that application of the H&L model is not straightforward. Further, the H&L model assumes that the A/D rates in the horizontal and vertical directions are the same and the physical size of the storage rack is large enough for an S/R machine to reach the maximum velocity.

The model proposed in this paper overcomes the problems mentioned above. Moreover, it is very straightforward to compute the single and dual command travel times with the suggested model. Although the accuracy of the proposed model is slightly worse than that of the H&L model, if the pickup/deposit time is included for comparison, the difference between results from simulation and from the proposed model would become even smaller.

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