ON L–FUZZY ALMOST PRECONTINUOUS FUNCTIONS

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1. Introduction

In 1981, R. Badard introduced the notion of fuzzy pretopological spaces and their representation[1]. And in 1992, R. Badard, et al. introduced the L-fuzzy pretopological spaces and studied properties of continuity, open map, closed map, and homeomorphism in L-fuzzy pretopological spaces. In this paper we introduce and study the concepts of almost continuous functions and weakly pre-continuous functions on L-fpts’s. The symbol L denote a complete lattice, with infimum o and supremum 1, that L is equipped with an order reversing involution. For a lattice, the De Morgan laws hold for arbitrary indexed suprema and infima. Given such a lattice L and a non-empty set X, the L-fuzzy sets of X[2] are just the elements of $L^X$, i.e., the functions from X to L. 0 is the L-fuzzy set defined by 0: $X \rightarrow L$, $0(x) = o$ for each $x \in X$. 1 is the L-fuzzy set defined by 1: $X \rightarrow L$, $1(x) = 1$ for each $x \in X$. For $u, v \in L^X$, the intersection $u \land v$ and the union $u \lor v$, respectively, are defined by: $(u \land v)(x) = u(x) \land v(x), x \in X$, $(u \lor v)(x) = u(x) \lor v(x), x \in X$. Let $u, v \in L^X$. $u$ is included in $v (u \leq v)$ provided that $u(x) \leq v(x)$ holds for every $x \in X$. For any L-fuzzy set $u$, $u'$ will stand for the complement of $u$.

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Definition 1.1[2]. An L-fuzzy pretopology on a set $X$ is a function $a: L^X \to L^X$ such that

1. $a(0) = 0$,
2. $a(u) \geq u$

are satisfied for every $u \in L^X$.

The pair $(X, a)$ is said to be an L-fuzzy pretopological space (for short, L-fpts).

An L-fpts is said to be of:

1. Type I if for every $u, v \in L^X$ such that $u \leq v$ we have $a(u) \leq a(v)$.
2. Type D if for every $u, v \in L^X$ we have $a(u \vee v) = a(u) \vee a(v)$.
3. Type S if for every $u \in L^X$ we have $a^2(u) = a(u)$.

It is clear that (2) implies (1).

Definition 1.2[1]. Let $(X, a)$ and $(Y, b)$ be fpts's. A function $f: (X, a) \to (Y, b)$ is said to be precontinuous if $f(a(u)) \leq b(f(u))$, for every $u \in I^X$.

Definition 1.3[2]. Let $(X, a)$ be an L-fpts and $u \in L^X$. We define the L-fuzzy interior operator $i_a: L^X \to L^X$ by $i_a(u) = (a(u'))'$.

Then it is clear that the properties (1) to (5) become, for the operator $i_a$ (see [1]):

1. $i_a(0) = 0$.
2. $i_a(u) \leq u$ for each $u \in L^X$.
3. If $(X, a)$ is of type I, then $u \leq v$ implies $i_a(u) \leq i_a(v)$.
4. If $(X, a)$ is of type D, then $i_a(u \wedge v) = i_a(u) \wedge i_a(v)$ for each $u, v \in L^X$.
5. If $(X, a)$ is of type S, then $(i_a)^2(u) = i_a(u)$ for $u \in L^X$.

A more successful denomination would be:

1. $u$ is L-preclosed iff $a(u) = u$,
2. $u$ is L-preopen iff $i_a(u) = u$. 
It is clear that \( u \in L^X \) is preclosed if and only if \( u' \) is preopen.

**Definition 1.4[2].** Let \((X, a)\) and \((Y, b)\) be L-fpts’s. A function \( f : (X, a) \to (Y, b) \) is to be preopen (resp., preclosed) if for every \( u \in L^X \) we have \( f(i_a(u)) \leq i_b(f(u)) \) (resp., \( f(a(u)) \geq b(f(u)) \)).

Throughout this paper, we assume that every L-fuzzy pretopological space is Type I and S.

### 2. Main Theorems

**Definition 2.1.** A fuzzy subset \( u \) of an L-fpts\((X, a)\) is called a regularly preopen L-fuzzy set if \( i_a(a(u)) = u \). An L-fuzzy set whose complement is a regularly preopen L-fuzzy set is called a regularly preclosed L-fuzzy set.

We obtain easily the following lemma by Definition 1.3.

**Lemma 2.2.** Let \((X, a)\) be an L-fpts and \( u \in L^X \).

1. \( a(u') = (i_a(u))' \),
2. \( i_a(u') = (a(u))' \).

**Definition 2.3.** Let \((X, a)\) and \((Y, b)\) be L-fpts’s. A fuzzy mapping \( f : (X, a) \to (Y, b) \) is called an L-fuzzy almost precontinuous mapping if for each preopen L-fuzzy set \( u \) in \((Y, b)\), \( f^{-1}(u) \leq i_a(f^{-1}(i_b(b(u)))) \).

**Theorem 2.4.** Let \((X, a)\) and \((Y, b)\) be L-fpts’s. A fuzzy mapping \( f : (X, a) \to (Y, b) \) is an L-fuzzy almost precontinuous mapping if and only if for each regular preopen L-fuzzy set \( u \) in \( Y \), \( f^{-1}(u) \) is a preopen L-fuzzy set.

**proof.** Assume that \( f : (X, a) \to (Y, b) \) is an L-fuzzy almost precontinuous mapping. And let \( u \) be a regular preopen L-fuzzy set in \( Y \). Then \( f^{-1}(u) \leq i_a(f^{-1}(i_b(b(u)))) \)
and by the definition of regular preopen L-fuzzy set, we obtain \( f^{-1}(u) \leq i_a(f^{-1}(u)) \). Thus \( f^{-1}(u) \) is a preopen L-fuzzy set.

For the converse, let \( u \) be a preopen L-fuzzy set. Then \( i_b(b(u)) \) is a regular preopen L-fuzzy set, and \( f^{-1}(i_b(b(u))) = i_a(f^{-1}(i_b(b(u)))) \). This means \( f^{-1}(u) \leq i_a(f^{-1}(i_b(b(u)))) \), since \( u = i_b(u) \leq i_b(b(u)) \). Consequently, \( f \) is an L-fuzzy almost precontinuous mapping.

**Theorem 2.5.** Let \((X, a)\) and \((Y, b)\) be L-fpts's. A fuzzy mapping \( f: (X, a) \to (Y, b) \) is an L-fuzzy almost precontinuous mapping if and only if for each preclosed L-fuzzy set \( u \) in \( Y \), \( a(f^{-1}(b(i_b(u)))) \leq f^{-1}(u) \).

**Proof.** Let \( u \) be a preclosed L-fuzzy set in \( Y \). Since \( u' \) is a preopen L-fuzzy set in \( Y \), \( f^{-1}(u') \leq i_a(f^{-1}(i_b(b(u')))) \). By the definition 1.3 and lemma 2.2, we obtain

\[
\begin{align*}
f^{-1}(u) &\geq a(f^{-1}(i_b(b(u'))))' \\
&= a(f^{-1}(b(b(u')))) \\
&= a(f^{-1}(b(i_b(u)))).
\end{align*}
\]

Thus \( a(f^{-1}(b(i_b(u)))) \leq f^{-1}(u) \).

The converse is obvious.

**Definition 2.6.** Let \((X, a)\) and \((Y, b)\) be L-fpts's. A fuzzy mapping \( f: (X, a) \to (Y, b) \) is called L-fuzzy weakly precontinuous if for each preopen L-fuzzy set \( u \) of \( Y \), \( f^{-1}(u) \leq i_a(f^{-1}(b(u))) \).

**Theorem 2.7.** The following properties are equivalent

1. \( f \) is L-fuzzy weakly precontinuous in L-fpts.
2. \( f^{-1}(u) \geq a(f^{-1}(i_b(u))) \) for each preclosed L-fuzzy set \( u \) in \( Y \).
3. \( a(f^{-1}(u)) \leq f^{-1}(b(u)) \) for each pre-open L-fuzzy set \( u \) in \( Y \).
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Proof. (1)⇒(2). Let \( u \) be preclosed L-fuzzy set in \( Y \). Then \( u' \) is a preopen L-fuzzy set in \( Y \) and \( f^{-1}(u') \leq i_a(f^{-1}(b(u'))). \) This implies \( f^{-1}(u) \geq a(f^{-1}(b(u')))'. \) By Lemma 2.2, \( a(f^{-1}(i_b(u))) \leq f^{-1}(u). \)

(2)⇒(3). Let \( u \) be a preopen L-fuzzy set in \( Y \). Since \( b(u) \) is a preclosed L-fuzzy set in \( Y \), then \( f^{-1}(b(u)) \geq a(f^{-1}(i_b(b(u)))) \) and \( f^{-1}(i_b(b(u))) \geq f^{-1}(u) \). Therefore \( f^{-1}(b(u)) \geq a(f^{-1}(u)). \)

(3)⇒(1). Let \( u \) be a preopen L-fuzzy set in \( Y \). Since \( b(u)' \) is a preopen L-fuzzy set, \( a(f^{-1}(b(u)')) \leq f^{-1}(b(b(u)))'. \) By Lemma 2.2, \( (i_a(f^{-1}(b(u))))' \leq f^{-1}(i_b(b(u)))'. \) This means that \( f^{-1}(i_b(b(u))) \leq i_a(f^{-1}(b(u))). \) Therefore \( f^{-1}(u) \leq i_a(f^{-1}(b(u))). \)

**Theorem 2.8.** Let \((X, a)\) and \((Y, b)\) be L-fpts's. If \( f: (X, a) \to (Y, b) \) is an L-fuzzy weakly precontinuous, onto and fuzzy preopen mapping, then \( f \) is fuzzy almost precontinuous.

**Proof.** Since \( f \) is L-fuzzy weakly precontinuous, for each preopen L-fuzzy set \( u \) in \( Y \), we have \( f^{-1}(u) \leq i_a(f^{-1}(b(u))). \) And we have \( i_a(f^{-1}(b(u))) \leq f^{-1}(i_b(b(u))), \) since \( f \) is a fuzzy preopen, onto mapping. Consequently, \( f^{-1}(u) \leq i_a(f^{-1}(b(i_b(b(u)))). \)

**Theorem 2.9.** Let \((X, a), (Y, b)\) and \((Z, c)\) be L-fpts's. If \( f: (X, a) \to (Y, b) \) is a fuzzy preopen, onto, and L-fuzzy precontinuous mapping and \( g: (Y, b) \to (Z, c) \) is an L-fuzzy mapping. Then \((g \circ f)\) is L-fuzzy almost precontinuous if and only if \( g \) is L-fuzzy almost precontinuous.

**Proof.** Assume that \((g \circ f)\) be L-fuzzy almost precontinuous and let \( u \) be a preopen L-fuzzy set in \( Z \). Since \((g \circ f)\) is L-fuzzy almost precontinuous, we have \((g \circ f)^{-1}(u) \leq i_a((g \circ f)^{-1}(i_c(c(u))). \) Since \( f \) is a fuzzy preopen onto mapping, \( g^{-1}(u) \leq i_b(g^{-1}(i_c(c(u))). \)
For the converse, let $u$ be a preopen $L$-fuzzy set in $Z$. Then by Proposition 2.4 in [2], we obtain the following implications:

$$(g \circ f)^{-1}(u) \leq f^{-1}(i_b(g^{-1}(i_c(c(u))))$$

$$\leq i_a(f^{-1}(g^{-1}(i_c(c(u))))$$

Therefore $(g \circ f)$ is $L$-fuzzy almost precontinuous.

REFERENCES


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