⊠ 연구논문

와이블성능분포인 경우 가속퇴화시험의 최적설계*

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Optimum Design of Accelerated Degradation Tests for Weibull Distribution

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Abstract

For highly reliable devices it is often defined to "fail" when its performance degrades below a specified value. In this paper we consider a method for optimally designing accelerated degradation tests(ADTs) in which the performance over exposure time and stress has Weibull distribution. For the product whose performance has Weibull distribution, the optimum plan - low stress level and sample proportions allocated to each test condition - is obtained, which minimize the asymptotic variance of maximum likelihood estimator of a stated quantile at design stress. We also present compromise ADTs plan that can be used for the practical purpose.

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1. Introduction

Accelerated degradation tests(ADTs) have some advantages over accelerated life tests. Performance degradation data can be analyzed earlier before any specimens fail. This is done by extrapolating performance degradation to estimate a lifetime when performance reaches a prespecified failure level. ADTs can give a good insight into the degradation process and how to improve it. ADT models are presented by several authors. Nelson(1981) provided an Arrhenius model and analysis for the breakdown strength data of electrical insulation. ADT models for some applications are treated in detail by Ballado-Perez(1986), Carey and Koenig(1991), Lu and Meeker(1993). Boulanger and Escobar(1993) provided optimum design of ADTs under the assumption of sigmodal growth curve having random measurement error. Optimum designs of ADTs having three experimental pointswere developed by Park(1993) and Lee(1995) using numerical searches.

In this paper we consider the optimum design of ADTs in which performance value of a specimen has Weibull distribution, which is widely used as the lifetime distribution for many products. Performance of a specimen is assumed to be measured only once at one of three test conditions including the measurements at the beginning of tests in an ADTs, which is called 3-point test plan, and at one of four test conditions in another ADTs which is called compromise test plan. Maximum likelihood(ML) method is used to estimate model parameters. The optimality criterion is to minimize asymptotic variance of ML estimator of a stated quantile of lifetime distribution at design stress.

The ADT model is presented in Section 2 and the procedure of estimating model parameters from degradation data is described in Section 3. In Section 4 optimum designs for the 3-point and compromise test plan are presented and an illustrative example is also given.

2. The Model

In this section we consider the ADT model in which the degradation process is characterized. The assumptions of the model are

1. At any stress s and exposure time t, the distribution of performance $U_i\ (s, t)$, I=1, 2, $\cdots$, n, is Weibull and U_i's are independently distributed. Thus log performance $Y_i = \ln U_i$ has the smallest extreme

value distribution.

2. The scale parameter δ of log performance is constant, i.e., independent of stress and exposure time.
3. The location parameter λ is a characteristic parameter and dependent on stress and exposure time. The relationship among the location parameter λ, s and t is

$$\lambda(t, s) = \alpha - \beta \, t \exp(-\gamma/s) \;, \; t \geq 0, \; \alpha > 0, \; \beta > 0, \; \gamma > 0. \qquad (1)$$

For designing test plan it is also assumed that specimens are tested at only two accelerated stresses and high stress is specified as the highest possible stress for which the assumed model is expected to hold and the longest possible exposure time t^* is pre-specified.

The following 3-point ADT plan for total test specimens n is considered:

1. Performance of $n\pi_0$ specimens randomly chosen from population are measured at the beginning of the test and design stress s_0.
2. Performance of $n\pi_1$ specimens randomly chosen from population are measured at exposure time t $(0 \leq t \leq t^*)$ and low stress s_1.
3. Performance of $n\pi_2$ specimens randomly chosen from population are measured at exposure time t^* and high stress s_2.

The following 4-point ADT plan, which is called compromise test plan, is also considered:

1. Performance of np_1 specimens randomly chosen from population are measured at a half exposure time $t = t^*/2$ and low stress $s_1 = (s_2 - s_0)/2$.
2. Performance of np_2 specimens randomly chosen from population are measured at exposure time t^* and low stress $s_1 = (s_2 - s_0)/2$.
3. Performance of np_3 specimens randomly chosen from population are measured at a half exposure time $t = t^*/2$ and high stress s_2.
4. Performance of np_4 specimens randomly chosen from population are measured at exposure time t^* and high stress s_2.

The object of an ADT for highly reliable products is to obtain performance data in a limited time. In particular, the above 3-point test plan is useful for the experimenter who wants to carry out the ADTs as simple as possible. The performance data is extrapolated to estimate the lifetime distribution at design stress. The optimum 3-point test plan specifies the optimum low stress, exposure time and proportions π_0, π_1 and π_2 $(=1-\pi_0-\pi_1)$ allocated to each test condition.

The optimum plan of compromise ADTs specifies the optimum proportions $p_1, p_2, p_3,$ and $p_4(=1-p_1-p_2-p_3)$ allocated to four test condition, respectively.

3. Estimation Procedure

In this section we present the method of estimating lifetime distribution and its quantiles at design stress using performance data from ADTs. Let $Y(t, s)$ be the random variable denoting the log performance at exposure time t and stress s. Let lifetime T at stress s be a random variable denoting the smallest time at which $Y(t, s)$ goes below a design value y_0 and failure of the specimen occurs. And denote the distribution of random variable T as $F_T(t, s)$.

$$\begin{aligned} F_T(t,s) &= \Pr\ [\, Y(t,s) \le y_0 \,] \\ &= \Pr\ [\, (Y(t,s)-\lambda(t,s))/\delta \le (y_0-\lambda(t,s))/\delta \,] \\ &= \Psi[\, (y_0-\alpha+\beta t \exp(-\gamma/s))/\delta \,]\ , \end{aligned} \quad (2)$$

where $\Psi(\cdot)$ is the standard extreme value distribution function. The distribution function of T is as follows.

$$\begin{aligned} F_T(t,s) &= \Psi[(t-\lambda_T)/\delta_T\,] && \text{if } t \ge 0 \\ &= 0 && \text{if } t < 0, \end{aligned} \quad (3)$$

where $\lambda_T=[\,(\alpha-y_0)/\beta\,]\exp(\gamma/s)$ and $\delta_T=(\delta/\beta)\exp(\gamma/s)$.

Let p_j be the probability that a specimen fails at the beginning of tests and design stress s_0.

The relationship between δ and p_f is

$$\alpha - y_0 = -\delta\, z(p_f)\ , \tag{4}$$

where $z(p_f)$ is the 100 p_f^{th} quantile of standard extreme value distribution.

The 100 q^{th} quantile of the lifetime distribution at design stress s_0, say t_q, is

$$\begin{aligned} t_q &= [\,(\alpha - y_0 + \delta z(q))/\beta\,]\exp(\gamma/s_0) && \text{if } q \geq p_f \\ &= 0 && \text{if } q < p_f\ . \end{aligned} \tag{5}$$

The maximum likelihood method can be used to estimate the parameters α, β, γ and δ from performance data.

The MLE of t_q is

$$\hat{t}_q = [\,(\hat{\alpha} - y_0 + \hat{\delta} z(q))/\hat{\beta}\,]\exp(\hat{\gamma}/s_0)\ , \tag{6}$$

where $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\delta}$ are MLEs of α, β, γ and δ, respectively.

It is convenient to define a transformed stress $x_j = 1/s_j$, j = 0, 1, 2 and the standardized stress $\eta = (x - x_2)/(x_0 - x_2)$. Then η=0 for high stress s_2 and η=1 for design stress s_0. We also define standardized exposure time $\tau = t/t^*(0 \leq \tau \leq 1)$. Then $\lambda(t, s)$ in formula (1) may be written in terms of τ and η as

$$\lambda(\tau, \eta) = \alpha_0 - \beta_0 \tau\ \exp(-\eta\gamma_0) \tag{7}$$

where $\alpha_0 = \alpha$, $\beta_0 = \beta t^* \exp(-x_2\gamma)$, $\gamma_0 = \gamma(x_0 - x_2)$, and $\delta_0 = \delta$. (8)

The $\hat{t}_q$ in formula (6) may be written in terms of estimates $\hat{\alpha}_0$, $\hat{\beta}_0$, $\hat{\gamma}_0$, $\hat{\delta}_0$ as

$$\hat{t}_q = t^*\ \hat{\beta}_0^{\,-1}\ [\,\hat{\alpha}_0 - y_0 + \hat{\delta}_0\ z(q)]\exp(\hat{\gamma}_0). \tag{9}$$

The log performance $Y_i(\tau, \eta)$, I=1, 2, $\cdots$, n, are independent and identically distributed under the same test condition. The p.d.f of a random variable $Y(\tau, \eta)$ for a single observation is

$$f(y) = (1/\delta_0)\ \exp((y-\lambda)/\delta_0)\ \exp[-\exp((y-\lambda)/\delta_0)]. \tag{10}$$

The log likelihood function ,$\mathcal{L}$, of an observation $y(\tau,\eta)$ at a transformed test condition (τ, η) is

$$\mathcal{L} = -\ln\delta_0 + w - \exp(w), \tag{11}$$

where $w=[y-\alpha_0+\beta_0\ \tau\ \exp(-\eta\gamma_0)\ ]/\delta_0$.

For a single observation, the first derivatives are

$$\begin{aligned}
\partial\mathcal{L}/\partial\alpha_0 &= -1/\delta_0+(1/\delta_0)\ \exp(w),\\
\partial\mathcal{L}/\partial\beta_0 &= B/\delta_0-(B/\delta_0)\ \exp(w),\\
\partial\mathcal{L}/\partial\gamma_0 &= -(\eta\beta_0B)/\delta_0+(\eta\beta_0B/\delta_0)\exp(w),\\
\partial\mathcal{L}/\partial\delta_0 &= -1/\delta_0-w/\delta_0+(w/\delta_0)\ \exp(w),
\end{aligned} \tag{12}$$

where B $= \tau\ \exp(-\eta\gamma_0)$, and the second derivatives are

$$\begin{aligned}
\partial^2\mathcal{L}/\partial\alpha_0^2 &= -(1/\delta_0^2)\ \exp(w), \qquad \partial^2\mathcal{L}/\partial\beta_0^2 = -(1/\delta_0^2)B^2\ \exp(w),\\
\partial^2\mathcal{L}/\partial\gamma_0^2 &= -(1/\delta_0^2)\eta^2\beta_0B\{(\delta_0+\beta_0B)\exp(w)-\delta_0\},\\
\partial^2\mathcal{L}/\partial\delta_0^2 &= -(1/\delta_0^2)\{w(w+2)\ \exp(w)-2w-1\},\\
\partial^2\mathcal{L}/\partial\alpha_0\partial\beta_0 &= (1/\delta_0^2)B\ \exp(w),\\
\partial^2\mathcal{L}/\partial\alpha_0\partial\gamma_0 &= -(1/\delta_0^2)\eta\beta_0B\ \exp(w),\\
\partial^2\mathcal{L}/\partial\alpha_0\partial\delta_0 &= -(1/\delta_0^2)\{(1+w)\exp(w)-1\},\\
\partial^2\mathcal{L}/\partial\beta_0\partial\gamma_0 &= -(1/\delta_0^2)\eta B\{\delta_0-(\delta_0+\beta_0B)\exp(w)\},\\
\partial^2\mathcal{L}/\partial\beta_0\partial\delta_0 &= -(1/\delta_0^2)B\{1-(1+w)\ \exp(w)\},\\
\partial^2\mathcal{L}/\partial\gamma_0\partial\delta_0 &= -(1/\delta_0^2)\eta\beta_0B\{(1+w)\exp(w)-1\}.
\end{aligned} \tag{13}$$

The following facts are useful to obtain Fisher information matrix.

$$\text{(i)}\ E(W) = -\theta, \qquad \text{(ii)}\ E(\exp(W)) = 1,$$
$$\text{(iii)}\ E(W\exp(W)) = 1-\theta, \qquad \text{(iv)}\ E(W^2\exp(W)) = \pi^2/6 + \theta^2 - 2\theta, \tag{14}$$

where θ is Euler's constant (θ=0.5772$\cdots$) and W has a standard extreme value distribution whose p.d.f is $\exp(w-\exp(w)),\ -\infty < w < \infty$.

The following $F(\tau,\eta)$ for an observation will be Fisher information matrix whose elements are negative expectations for the second partial derivatives in (13).

$$F(\tau,\eta) = (1/\delta_o^2)\begin{bmatrix} 1 & & symmetric & \\ -B & B^2 & & \\ \eta\beta_0 B & -\eta\beta_0 B^2 & \eta^2\beta_0^2 B^2 & \\ 1-\theta & -B(1-\theta) & \eta\beta_0 B(1-\theta) & \pi^2/6+(1-\theta)^2 \end{bmatrix} \tag{15}$$

The inverse of $F(\tau,\eta)$ is an asymtotic covariance matrix, V, for $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\gamma}$ and $\widehat{\delta}$. That is,

$$V = F^{-1}(\tau,\eta) = \begin{bmatrix} var(\widehat{\alpha}_0) & & symmetric & \\ cov(\widehat{\alpha}_0,\widehat{\beta}_0) & var(\widehat{\beta}_0) & & \\ cov(\widehat{\alpha}_0,\widehat{\gamma}_0) & cov(\widehat{\beta}_0,\widehat{\gamma}_0) & var(\widehat{\gamma}_0) & \\ cov(\widehat{\alpha}_0,\widehat{\delta}_0) & cov(\widehat{\beta}_0,\widehat{\delta}_0) & cov(\widehat{\gamma}_0,\widehat{\delta}_0) & var(\widehat{\delta}_0) \end{bmatrix} \tag{16}$$

Let H be a column vector as

$$H = \left(\frac{\partial t_q}{\partial\alpha_0}, \frac{\partial t_q}{\partial\beta_0}, \frac{\partial t_q}{\partial\gamma_0}, \frac{\partial t_q}{\partial\delta_0}\right)'$$

$$= t^*\beta_0^{-1}\exp(\gamma_0)[1, -\delta_0\beta_0^{-1}(z(q)-z(p_f)), \delta_0(z(q)-z(p_f)), z(q)]'. \tag{17}$$

Then the corresponding asymptotic variance of the MLE $\widehat{t}_q$ is of the form

$$Asvar(\widehat{t}_q) = H'VH, \tag{18}$$

where the prime denotes a vector transpose.

4. Optimum Test Plan

4.1 3-point optimum plan

In the 3-point plan $n\pi_0$ specimens are tested at the transformed test condition (0,1) and $n\pi_1$ specimens at $(1, \eta)$ and $n\pi_2$ specimens at $(\tau, 0)$. Fisher information matrix, F_0, for our 3-point plan with a sample of n independent observations is as follows ;

$$
\begin{aligned}
F_0 &= n\pi_0 F(0,1) + n\pi_1 F(1, \eta) + n\pi_2 F(\tau, 0) \\
&= (n/\delta_0^2)\begin{bmatrix} f_{11} & & & \\ f_{21} & f_{22} & Symmetric & \\ f_{31} & f_{32} & f_{33} & \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix}
\end{aligned} \tag{19}
$$

where $f_{11} = 1$, $f_{21} = -(\pi_1 C + \pi_2 \tau)$, $f_{22} = \pi_1 C^2 + \pi_2 \tau^2$, $f_{31} = \pi_1 \eta \beta_0 C$, $f_{32} = -\pi_1 \eta \beta_0 C^2$

$f_{33} = \pi_1 \eta^2 \beta_0^2 C^2$, $f_{41} = 1 - \theta$, $f_{42} = -(1-\theta)(\pi_1 C + \pi_2 \tau)$, $f_{43} = \pi_1 \eta \beta_0 C(1-\theta)$,

$f_{44} = \pi^2/6 + (1-\theta)^2$ and $C = \exp(-\eta\gamma_0)$.

The asymptotic variance of a ML estimator can be obtained from the inverse of Fisher information matrix. Let p_a and p_h be the probabilities that a specimen will fail at maximum exposure time t^* under stress x_0 and stress x_2, respectively.

From (3) and (4), we have

$$
\begin{aligned}
\beta t^* \exp[-x_0\gamma] &= \delta_0(z(p_d) - z(p_f)), \\
\beta t^* \exp[-x_2\gamma] &= \delta_0(z(p_h) - z(p_f)).
\end{aligned} \tag{20}
$$

And from the relationships in formula (8), we have

$$\alpha_0 = y_0 - \delta_0 z(p_f),$$
$$\beta_0 = \delta_0 (z(p_h) - z(p_f)),$$
$$\gamma_0 = \ln[(z(p_h) - z(p_f))/(z(p_d) - z(p_f))]. \quad (21)$$

The asymptotic variance of $\hat{t}_q$ is a function of η, τ, π_0, π_1 and model parameters. The otimality criterion is to minimize the asymptotic variance of ML estimator of t_q. It can be known from our numerical searches that optimum value of τ minimizing the asymptotic variance is 1. This results is similar to minimax (D-optimal) design problem of linear model. Our design problem is induced as follows:

Given the values of q, p_f, p_d and p_h, find the values of π_0, π_1 and η minimizing the asymptotic variance of $\hat{t}_q$.

The powell algorithm(1964) for finding the minimum of a function without using derivatives is used to solve the design problem. We have chosen the values $p_f = 1*10^{-5}$, $2*10^{-5}$, $4*10^{-5}$ and $p_d = 5*10^{-5}, 1*10^{-4}, 1.5*10^{-4}$ and $p_h = 0.8$, 0.9, 0.99. We have also chosen the values $q = 0.01$, 0.05, 0.1. The optimum values of π_0, π_1, η, and asymptotic variance are in <Table 1> for the chosen values of p_f, p_d, p_h and q. It can be known from <Table 1> that 1) the larger is the value of p_f, the smaller are the values of π_1^* and η^*, and also the larger is the variance of $\hat{t}_q$, 2) the larger is the value of p_d, the larger are the values of π_1^* and η^*, and also the smaller is the variance of $\hat{t}_q$, 3) the larger is the value of p_h, the smaller are the values of π_0^*, η^* and variance of $\hat{t}_q$, and also the larger is the value of π_1^*.

4.2 Compromise test plan

Nelson(1990, p. 326) pointed out the drawbacks of optimum test plan. We consider compromise test plan to avoid these drawbacks, in which np_1 specimens are tested at the transformed test condition at (1/2, 1/2) and np_2 specimens at (1, 1/2), np_3 specimens at (1/2, 0) and np_4 specimens at (1, 0). The Fisher information matrix , F_c, for compromise test plan with a sample of n independent observations is as follows ;

$$
\begin{aligned}
F_c &= np_1F(1/2,1/2) + np_2F(1,1/2) + np_3F(1/2,0) + np_4F(1,0) \\
&= (n/\delta_0^2)\begin{bmatrix} f_{11} & & & \\ f_{21} & f_{22} & Symmetric & \\ f_{31} & f_{32} & f_{33} & \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix}
\end{aligned}
$$
(2.[illegible])

where $f_{11}=1$, $f_{21}=-(p_1\tau C+p_2C+p_3\tau+p_4)$, $f_{22}=p_1\tau^2C^2+p_2C^2+p_3\tau^2+p_4$, $f_{31}=\eta\beta_0C(p_1\tau+p_2)$, $f_{32}=-\eta\beta_0C^2(p_1\tau^2+p_2)$, $f_{33}=\eta^2\beta_0^2C^2(p_1\tau^2+p_2)$, $f_{41}=1-\theta$, $f_{42}=-(1-\theta)(p_1\tau C+p_2C+p_3\tau+p_4)$, $f_{43}=(1-\theta)\eta\beta_0C(p_1\tau+p_2)$, $f_{44}=\pi^2/6+(1-\theta)^2$ and $C=\exp(-\eta\gamma_0)$.

The optimum proportions at four test conditions are determined for the same values p_f, p_d, p_h and q in Subsection 4.1. We obtained optimum plan for compromise test in <Table 2>. It can be known from <Table 2> that 1) the asymptotic variance of ML estimator $\widehat{t_q}$ is larger than that of optimal 3-point test plan, 2) the larger is the value of p_f, the larger are the value of p_1^* and the variance of $\widehat{t_q}$, and also the smaller is the value of p_2^*, 3) the larger is the value of p_d, the smaller are the values of p_1^* and the variance of $\widehat{t_q}$, and also the larger is the value of p_2^*, and 4) the larger is the value of p_h, the smaller is the variance of $\widehat{t_q}$.

4.3 An illustrative example

Nelson(1981) gave measurement data on dielectric breakdown strength of insulation specimens. The design stress is 150℃ in the ADTs. We considered first 3-point plan. If we want to estimate the 10th percentile of lifetime distribution at design stress, and the pre-estimates of p_h, p_d, p_f are 0.8, 1.0×10^{-4}, $4\times10^{[illegible]}$ respectively, then optimal low stress level η^* is 0.408 and optimum proportions are $\pi_0^*=0.288$, $\pi_1^*=0.584$ and $\pi_2^*=0.128$ from <Table 1>. When high stress is specified as 275℃, optimum low stress is 224℃. Asymptotic variance of ML estimator of 10th percentile in this optimum design is 50.6. In the case of compromise test plan, the optimum proportions are optained from <Table 2> as $p_1^*=0.132$, $p_2^*=0.456$, and $p_3^*=0.312$ at the same values of pre-estimates in 3-point test plan. And asymptotic variance of ML estimator of 10th percentile in this design is 98.1, which is larger than those of optimum 3-point plan.

〈 Table 1 〉 Optimum test plans for given values of p_f, p_d, p_h and q.

q	p_h	p_a	$p_f=1\times10^{-5}$				$p_f=2\times10^{-5}$				$p_f=4\times10^{-5}$			
			π_0	π_1	η	var	π_0	π_1	η	var	π_0	π_1	η	var
.01	.80	.00005	29.5	.594	.497	43.3	.295	.576	.398	57.8	.299	.550	.259	114.1
		.0001	.290	.619	.606	32.7	.288	.603	.513	38.7	.288	.584	.408	50.6
		.00015	.273	.699	.892	21.6	.267	.681	.796	22.9	.262	.663	.697	24.7
	.90	.00005	.292	.596	.490	42.4	.292	.578	.393	56.0	.297	.551	.256	108.9
		.0001	.285	.621	.595	32.2	.284	.605	.505	37.8	.285	.586	.402	49.0
		.00015	.266	.700	.870	21.5	.261	.682	.777	22.8	.256	.665	.681	24.4
	.99	.00005	.285	.599	.477	40.6	.286	.581	.384	52.8	.293	.554	.252	99.9
		.0001	.277	.625	.576	31.3	.276	.609	.490	36.3	.278	.589	.393	46.2
		.00015	.253	.702	.830	21.4	.248	.686	.743	22.5	.244	.669	.654	24.0
.05	.80	.00005	.320	.573	.497	52.9	.317	.558	.398	79.0	.315	.537	.259	180.4
		.0001	.321	.592	.606	36.0	.317	.578	.513	47.4	.314	.562	.408	70.7
		.00015	.324	.649	.892	18.1	.318	.633	.796	21.0	.313	.617	.697	25.1
	.90	.00005	.317	.575	.490	51.4	.314	.560	.393	76.0	.313	.538	.256	171.4
		.0001	.317	.594	.595	35.2	.314	.580	.505	46.0	.312	.564	.402	67.9
		.00015	.318	.650	.870	18.0	.313	.634	.777	20.7	.308	.618	.681	24.7
	.99	.00005	.311	.577	.477	48.6	.310	.562	.384	70.8	.310	.540	.252	155.9
		.0001	.310	.596	.576	33.7	.308	.582	.490	43.5	.306	.566	.393	63.1
		.00015	.307	.652	.830	17.8	.302	.637	.743	20.3	.299	.621	.654	23.8
.10	.80	.00005	.327	.567	.497	59.6	.324	.553	.398	91.9	.320	.534	.259	217.3
		.0001	.331	.584	.606	39.5	.326	.571	.513	53.8	.322	.556	.408	83.3
		.00015	.340	.635	.892	18.0	.333	.619	.796	21.7	.328	.604	.697	27.3
	.90	.00005	.325	.568	.490	57.7	.321	.554	.393	88.3	.318	.535	.256	206.3
		.0001	.327	.585	.595	38.5	.323	.572	.505	52.1	.320	.557	.402	79.9
		.00015	.334	.635	.870	17.9	.328	.620	.777	21.5	.323	.605	.681	26.7
	.99	.00005	.319	.570	.477	54.4	.317	.556	.384	82.0	.315	.536	.252	187.5
		.0001	.320	.587	.576	36.7	.317	.574	.490	49.0	.315	.559	.393	73.9
		.00015	.323	.637	.830	17.6	.318	.622	.743	20.9	.314	.608	.654	25.6

〈Table 2〉 Compromise test plans for given values of p_f, p_d, p_h and q.

q	p_h	p_a	$p_f = 1\times10^{-5}$				$p_f = 2\times10^{-5}$				$p_f = 4\times10^{-5}$			
			p_1	p_2	p_3	var	p_1	p_2	p_3	var	p_1	p_2	p_3	var
.01	.80	.00005	.082	.473	.348	68.3	.141	.447	.309	120.6	.230	.418	.248	575.0
		.0001	.000	.504	.404	42.5	.052	.477	.370	57.1	.132	.456	.312	98.1
		.00015	.000	.664	.291	24.5	.000	.659	.327	25.1	.000	.617	.355	27.3
	.90	.00005	.072	.469	.357	66.7	.141	.447	.309	117.1	.229	.418	.249	556.0
		.0001	.000	.505	.402	41.6	.053	.478	.368	55.7	.125	.452	.319	95.1
		.00015	.000	.667	.288	24.2	.000	.663	.324	24.7	.000	.620	.352	26.8
	.99	.00005	.071	.470	.356	63.6	.140	.448	.308	110.9	.228	.418	.248	522.4
		.0001	.000	.507	.399	40.0	.056	.482	.362	53.1	.124	.453	.317	89.8
		.00015	.000	.674	.282	23.5	.000	.669	.318	24.0	.000	.626	.346	25.9
.05	.80	.00005	.113	.457	.334	101.2	.172	.441	.290	195.7	.241	.415	.242.	1000.5
		.0001	.051	.479	.378	58.2	.100	.461	.344	87.0	.158	.441	302	167.1
		.00015	.000	.604	.378	24.0	.006	.565	.385	28.4	.000	.515	.406	35.7
	.90	.00005	.111	.456	.336	98.4	.172	.441	.290	189.8	.240	.415	.243	967.7
		.0001	.051	.480	.377	56.7	.100	.461	.343	84.5	.158	.441	.301	162.0
		.00015	.000	.604	.377	23.5	.006	.565	.384	27.7	.000	.516	.405	34.7
	.99	.00005	.112	.457	.333	93.4	.172	.442	.288	179.3	.240	.415	.242	909.7
		.0001	.049	.479	.377	53.9	.099	.462	.341	80.0	.157	.442	.300	152.5
		.00015	.000	.496	.487	23.4	.004	.566	.382	26.5	.000	.517	.401	32.9
.10	.80	.00005	.129	.455	.324	120.5	.174	.436	.292	237.6	.243	.414	.242	1228.8
		.0001	.061	.471	.375	68.5	.114	.457	.336	105.2	.172	.440	.292	207.2
		.00015	.003	.573	.391	26.7	.000	.532	.406	32.8	.008	.494	.411	42.8
	.90	.00005	.127	.454	.325	117.1	.174	.436	.292	230.4	.243	.414	.242	1188.7
		.0001	.062	.471	.373	66.7	.114	.457	.335	102.1	.172	.441	.291	200.5
		.00015	.003	.573	.389	26.0	.000	.532	.404	31.9	.008	.494	.409	41.5
	.99	.00005	.128	.455	.323	111.1	.174	.436	.291	217.6	.243	.414	.241	1117.8
		.0001	.064	.472	.370	63.3	.112	.456	.335	96.5	.172	.441	.290	188.7
		.00015	.002	.573	.387	24.8	.000	.533	.401	30.3	.005	.494	.409	39.2

5. Concluding Remarks

We presented optimum ADTs plans in which the performance value of a test specimen has Weibull distribution. we have optained optimum low stress, exposure time and proportions to each test condition which are determined numerically to minimize the asymptotic variance of the ML estimator for a stated quantile of the lifetime distribution at use condition.

We have also obtained the compromise test plan. In the case of compromise test plan the proportions at four test conditions are determined numerically by the same optimality criterion in 3-point plan.

Optimum ADTs plans can be used in a kind of destructive tests in which the performance of a test specimen is measured only at a particular inspection time.

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