REAL PROJECTIVE STRUCTURES
ON THE (2,2,2,2)-ORBIFOLD

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ABSTRACT. The (2,2,2,2)-orbifold is a 2-dimensional orbifold with four order 2 cone points having 2-sphere as an underlying space. The (2,2,2,2)-orbifold admits different geometric structures. The purpose of this paper is to find some real projective structures on the (2,2,2,2)-orbifold.

1. Introduction

When a group $\Gamma$ acts properly discontinuously but do not necessarily act freely on a space $X$, the quotient space $X/\Gamma$ is called orbifold. Orbifold was first introduced by I. Satake in the name of $V$-manifold. In section 3, we give the precise definition of the orbifold and discuss its geometric structures. There are many reasons to study the orbifolds. 2-dimensional orbifolds occur naturally in the study of 3-dimensional manifolds, e.g., Seifert fibered spaces. In [T1], Thurston gave a quite complete treatment of the two dimensional case, and raised many interesting questions.

2. $(X,G)$-manifolds

Let $X$ be a manifold and $G$ a Lie group acting (transitively) on $X$. Let $M$ be a manifold of the same dimension as $X$. An $(X,G)$-atlas on $M$ is a pair $(\mathcal{U}, \Phi)$ where $\mathcal{U}$ is an open covering of $M$ and $\Phi = \{\phi_\alpha : U_\alpha \to X\}_{\alpha \in \mathcal{U}}$ is a collection of coordinate charts such that for each pair...
(U_\alpha, U_\beta) \in U \times U$ and connected components $C$ of $U_\alpha \cap U_\beta$ there exists $g_{C,\alpha,\beta} \in G$ such that $g_{C,\alpha,\beta} \circ \phi_\alpha = \phi_\beta$. An $(X,G)$-structure on $M$ is a maximal $(X,G)$-atlas and an $(X,G)$-manifold is a manifold together with an $(X,G)$-structure on it. Suppose that $M$ and $N$ are two $(X,G)$-manifolds and $f : M \to N$ is a map. Then $f$ is an $(X,G)$-map if for each pair of charts $\phi_\alpha : U_\alpha \to X$ and $\psi_\beta : V_\beta \to X$ (for $M$ and $N$ respectively) and a component $C$ of $U_\alpha \cap f^{-1}(V_\beta)$ there exists $g = g(C, \alpha, \beta) \in G$ such that the restriction of $f$ to $C$ equals $\psi_\beta^{-1} \circ g \circ \phi_\alpha$. There is a useful globalization of the coordinate charts of a geometric structure in terms of the universal covering space and the fundamental group. The proof of the following basic result can be found in Goldman [G2].

**Development Theorem.** Let $M$ be an $(X,G)$-manifold with universal covering space $p : \tilde{M} \to M$ and group of deck transformation $\pi = \pi_1(M)$. Then there exists a pair $(\text{dev}, h)$ such that $\text{dev} : \tilde{M} \to X$ is an immersion and $h : \pi \to G$ is a homomorphism such that, for each $\gamma \in \pi$,

$$\begin{array}{ccc}
\tilde{M} & \xrightarrow{\text{dev}} & X \\
\gamma \downarrow & & \downarrow h(\gamma) \\
\tilde{M} & \xrightarrow{\text{dev}} & X
\end{array}$$

commutes. Furthermore if $(\text{dev}', h')$ is another such pair, there exists $g \in G$ such that $\text{dev}' = g \circ \text{dev}$ and $h'(\gamma) = gh(\gamma)g^{-1}$ for each $\gamma \in \pi$.

We say that such a pair $(\text{dev}, h)$ is a development pair, and $\text{dev}$ the developing map and the homomorphism $h$ a holonomy representation.

### 3. Orbifold

An $n$-dimensional orbifold (without boundary) is defined to be a space equipped with a covering by open sets $\{U_i\}$ closed under finite intersections. To each $U_i$ is associated a finite group $\Gamma_i$, an action of $\Gamma_i$ on an open subset $\tilde{U}_i$ of $\mathbb{R}^n$, a homeomorphism $\phi_i : \tilde{U}_i / \Gamma_i \to U_i$. Whenever $U_i \subset U_j$, there is to be an inclusion $f_{ij} : \Gamma_i \to \Gamma_j$ and an embedding $\tilde{\phi}_{ij} : \tilde{U}_i \to \tilde{U}_j$ equivariant with respect to $f_{ij}$ such that the following diagram commutes (see Scott [Sc]):
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\[
\begin{array}{ccc}
\tilde{U}_i & \xrightarrow{\phi_{ij}} & \tilde{U}_j \\
\downarrow & & \downarrow \\
\tilde{U}_i/\Gamma_i & \xrightarrow{\phi_{ij}} & \tilde{U}_j/f_{ij}\Gamma_i \\
\downarrow & & \downarrow \\
\phi_i & & \tilde{U}_j/\Gamma_j \\
U_i & \subset & \tilde{U}_j
\end{array}
\]

A covering orbifold of an orbifold $\mathcal{O}$ is an orbifold $\tilde{\mathcal{O}}$ with a projection $p : X_{\tilde{\mathcal{O}}} \to X_{\mathcal{O}}$ between the underlying spaces, such that $p$ is a local covering, that is, each point $x \in X_{\tilde{\mathcal{O}}}$ in the domain has a neighborhood $U = \tilde{U}/\Gamma$ (where $\tilde{U}$ is an open subset of $\mathbb{R}^n$) such that $p$ restricted to $U$ is isomorphic to a map $\tilde{U}/\Gamma \to \tilde{U}/\Gamma' (\Gamma \subset \Gamma')$ and $p$ is an even covering, that is, each point $x' \in \mathcal{O}$ in the range has a neighborhood $V = \tilde{V}/\Gamma$ for which each component $U_i$ of $p^{-1}(V)$ is isomorphic to $\tilde{V}/\Gamma_i$, where $\Gamma_i \subset \Gamma$ is some subgroup. The isomorphism must respect the projections.

\[
\begin{array}{ccc}
U & \xleftarrow{\simeq} & \tilde{U}/\Gamma \\
p \downarrow & & \downarrow q \\
U' & \xleftarrow{\simeq} & \tilde{U}'/\Gamma' \\
U_i & \xleftarrow{\simeq} & \tilde{V}/\Gamma_i \\
p \downarrow & & \downarrow q \\
V & \xleftarrow{\simeq} & \tilde{V}/\Gamma
\end{array}
\]

Similarly to $(X, G)$-structures on manifolds, we can define locally homogeneous geometries on orbifolds by using in the definition of orbifolds all the mappings and group actions related to $(X, G)$-category. In that sense, we can speak about $(X, G )$-orbifold.

The cone point of order $n$ of a 2-dimensional orbifold means a point whose neighborhood is modeled on $\mathbb{R}^2/\mathbb{Z}_n$ with $\mathbb{Z}_n$ acting by rotation of order $n$. The $(2,2,2,2)$-orbifold is a 2-dimensional orbifold with four order two cone points. We will denote it by $S^2(2, 2, 2, 2)$. As defined in Scott [Sc], the Euler number of $S^2(2, 2, 2, 2)$ is zero. Then it is known that our orbifold has a Euclidean structure and the Euclidean plane $\mathbb{E}^2$
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is the universal covering space (see Thurston [T1]). For convenience, we will write $M = S^2(2,2,2,2)$ and $\tilde{M} = E^2$. Every Euclidean structure is a similarity structure which induces an obvious affine structure. Similarly every affine structure determines a projective structure, using embedding $(\mathbb{R}^n, \text{Aff}(\mathbb{R}^n)) \rightarrow (\mathbb{RP}^n, \text{PGL}(n + 1, \mathbb{R}))$. According to the Development Theorem, we can deduce that there exists a developing map $\text{dev}: \tilde{M} \rightarrow \mathbb{RP}^2$.

To express the developing image clearly, we lift $\text{dev}: \tilde{M} \rightarrow \mathbb{RP}^2$ to the universal covering $\tilde{\text{dev}}: \tilde{M} \rightarrow S^2$. The universal covering space $S^2$ of $\mathbb{RP}^2$ is realized geometrically as the sphere of directions in $\mathbb{R}^3$. Furthermore the group of lifts of $\text{PGL}(3, \mathbb{R})$ to $S^2$ equals the quotient

$$\text{GL}(3, \mathbb{R})/\mathbb{R}^+ \cong \text{SL}_+(3, \mathbb{R}) = \{ A \in \text{GL}(3, \mathbb{R}) | \det(A) = \pm 1 \}.$$ 

Hence there exists a lift of the holonomy map $h : \pi_1(M) \rightarrow \text{PGL}(3, \mathbb{R})$ to $\tilde{h} : \pi_1(M) \rightarrow \text{SL}(3, \mathbb{R})$.

4. Main Computation

Now we find some examples of $\mathbb{RP}^2$-structures on $S^2(2,2,2,2)$. Let rectangle $Q$ be the fundamental domain of our orbifold in $E^2$. For computational ease we will assume the developing image of $Q$ in $S^2$ has vertices at $[0,0,1], [1,0,1], [1,1,1], [0,1,1]$ in homogeneous coordinate; i.e., $v$ is equivalent to $w$ if and only if $v = \lambda w$ for some $\lambda > 0$ for $v, w \in \mathbb{R}^3$. Let $p_i$ be the midpoints of each sides in $Q$ and $R_i$ the order two deck transformation in $S^2$ fixing $p_i$ for $i = 1,2,3,4$. If $\Gamma$ is the deck transformation group of $\tilde{M}$ with generators $R_i$'s, then $\Gamma$ admits the presentation

$$\Gamma = \left< R_1, R_2, R_3, R_4 | R_1^2 = R_2^2 = R_3^2 = R_4^2 = I, R_1R_2R_3R_4 = I \right>.$$ 

We want to find $A,B,C,D$ in $\text{SL}(3, \mathbb{R})$ acting on $S^2$ satisfying

1. $A^2 = B^2 = C^2 = D^2 = I$
2. $ABCDA = I$
3. $A[0,0,1] = [1,0,1]$
4. $B[1,0,1] = [1,1,1]$
5. $C[1,1,1] = [0,1,1]$
6. $D[0,1,1] = [0,0,1]$
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The possible \(A, B, C, D \in \text{SL}(3, \mathbb{R})\) satisfying the conditions (1) and (3) \(\sim\) (6) are easily computed. They turn out to be

\[
A = \begin{pmatrix}
-1 & a_1 & 1 \\
0 & a_2 & 0 \\
a_2^2 - 1 & a_1(1 - a_2) & 1
\end{pmatrix}
\]

with fixed points \([1, 0, 1 - a_2],\)

\[
B = \begin{pmatrix}
-b_1 - b_2 - b_1b_2 & b_2^2 - 1 & (1 + b_1)(1 + b_2) \\
-b_1 & -1 & 1 + b_1 \\
-b_1(1 + b_2) & b_2^2 - 1 & 1 + b_1 + b_1b_2
\end{pmatrix}
\]

with fixed points \([1 + b_2, 1, 1 + b_2],\)

\[
C = \begin{pmatrix}
-1 & -c_1 & 1 + c_1 \\
c_2^2 - 1 & -c_1 - c_2 - c_1c_2 & (1 + c_1)(1 + c_2) \\
c_2^2 - 1 & -c_1(1 + c_2) & 1 + c_1 + c_1c_2
\end{pmatrix}
\]

with fixed points \([1, 1 + c_2, 1 + c_2],\)

\[
D = \begin{pmatrix}
-d_1 & 0 & 0 \\
-d_2 & -1 & 1 \\
-d_2(1 + d_1) & d_1^2 - 1 & 1
\end{pmatrix}
\]

with fixed points \([0, 1, 1 + d_1].\)

Since \(\det A = -a_2^3 > 0, \det B = b_2^3 > 0, \det C = c_2^3 > 0\) and \(\det D = d_1^3 > 0\), we see that

\[
(7) \quad a_2 < 0, b_2 > 0, c_2 > 0, d_1 > 0.
\]

The fact that all \(R_i\) have order two implies (2) is equivalent to

\((2')\) \quad \(CD = BA.\)

\[
CD = \begin{pmatrix}
d_1 - d_2 - (1 + c_1)d_1d_2 & -1 + (1 + c_1)d_1^2 & 1 \\
d_1(1 - c_2^2) - d_2 - (1 + c_1)(1 + c_2)d_1d_2 & -1 + (1 - c_1)(1 + c_2)d_2^2 & 1 \\
d_1(1 - c_2^2) - d_2 - d_1d_2(1 + c_1 + c_1c_2) & -1 + (1 + c_1 + c_1c_2)d_1^2 & 1
\end{pmatrix}
\]

\[
BA = \begin{pmatrix}
-1 + (1 + b_1)(1 + b_2)a_2^2 & a_1 - a_2(1 - b_2^2) - a_1a_2(1 + b_1)(1 + b_2) & 1 \\
-1 + (1 + b_1)a_2^2 & a_1 - a_2 - a_1a_2(1 + b_1) & 1 \\
-1 + (1 + b_1 + b_1b_2)a_2^2 & a_1 - a_2(1 - b_2^2) - a_1a_2(1 + b_1 + b_1b_2) & 1
\end{pmatrix}
\]

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From (2'), we get the following 6 equations with 8 unknowns.

(8) \[ d_1 - d_2 - (1 + c_1)d_1d_2 = -1 + (1 + b_1)(1 + b_2)a_2^2 \]
(9) \[ d_1(1 - c_2^2) - d_2 - (1 + c_1)(1 + c_2)d_1d_2 = -1 + (1 + b_1)a_2^2 \]
(10) \[ d_1(1 - c_2^2) - d_2 - d_1d_2(1 + c_1 + c_1c_2) = (1 + b_1 + b_1b_2)a_2^2 - 1 \]
(11) \[ -1 + (1 + c_1)d_1^2 = a_1 - a_2(1 - b_2^2) - a_1a_2(1 + b_1)(1 + b_2) \]
(12) \[ -1 + (1 + c_1)(1 + c_2)d_1^2 = a_1 - a_2 - a_1a_2(1 - b_1) \]
(13) \[ -1 + (1 + c_1 + c_1c_2)d_1^2 = a_1 - a_2(1 - b_2^2) - a_1a_2(1 + b_1 + b_1b_2) \]

To determine A, B, C, D, we only need to solve the above equations with \( a_2 < 0, \ b_2 > 0, \ c_2 > 0, \) and \( d_1 > 0. \)

Subtract (11) from (12), (9) from (8), (13) from (12), (9) from (10), (11) from (13), and (10) from (8) respectively.

(14) \[ c_2(1 + c_1)d_1^2 = a_1a_2b_2(1 + b_1) - a_2b_2^2 \]
(15) \[ d_1c_2^2 + c_2(1 + c_1)d_1d_2 = b_2(1 + b_1)a_2^2 \]
(16) \[ c_2d_1^2 = -a_2b_2^2 + a_1a_2b_1b_2 \]
(17) \[ c_2d_1d_2 = b_1b_2a_2^2 \]
(18) \[ c_1c_2d_1^2 = a_1a_2b_2 \]
(19) \[ c_2^2d_1 + c_1c_2d_1d_2 = b_2a_2^2 \]

Because \( c_2d_1 \neq 0 \) by (7),

(18') \[ c_1 = \frac{a_1a_2b_2}{c_2d_1^2} \]
(17') \[ d_2 = \frac{b_1b_2a_2^2}{c_2d_1} \]

Firstly, assume

(assumption 1) \[ b_1 = 0 \]

which implies

(20) \[ d_2 = 0 \]
(21) \[ d_1c_2^2 = b_2a_2^2 \]

by (17'),

by (16),

by (15) and (20).
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Multiplying the above two equations gives

(22) \[ c_2d_1 = -a_2b_2 \]

Substituting the above into (21) gives

(23) \[ c_2 = -a_2 \quad \text{by (7)} \]

which yields

(24) \[ d_1 = b_2 \quad \text{by (22)} \]

Thus

(25) \[ c_1 = -\frac{a_1}{b_2} \quad \text{by (18')} \]

Finally, put (24) and (25) into (11), we obtain

\[ -1 + (1 + (-\frac{a_1}{b_2}))b_2^2 = a_1 - a_2(1 - b_2^2) - a_1a_2(1 + b_2) \]

which becomes

\[ (a_2 - 1)(b_2 + 1)(b_2 - (1 + a_1)) = 0 \]

Since \(a_2 < 0\) and \(b_2 > 0\) by (7),

(26) \[ b_2 = 1 + a_1 > 0 \]

which yields in turn

(27) \[ d_1 = 1 + a_1 \quad \text{by (24)} \]

(28) \[ c_1 = -\frac{a_1}{1 + a_1} \quad \text{by (25)} \]

Putting (20), (26), (27) and (assumption 1) into (8) gives

\[ (2 + a_1) = (2 + a_1)a_2^2 \]

Because \(1 + a_1 = b_2 > 0\) and \(a_2 < 0\) by (7) and (26),

(29) \[ a_2 = -1 \]

which implies

(30) \[ c_2 = 1 \quad \text{by (23)} \]

In summary, we get the solution of our 6 equations under \(b_1 = 0\) as follows.

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Solution (1)
\[
\begin{align*}
 a_1 & \quad \text{independent variable greater than -1} \quad \text{by (26)} \\
 a_2 & = -1 \quad \text{by (29)} \\
 b_1 & = 0 \quad \text{by (assumption1)} \\
 b_2 & = 1 + a_1 \quad \text{by (26)} \\
 c_1 & = -\frac{a_1}{1 + a_1} \quad \text{by (28)} \\
 c_2 & = 1 \quad \text{by (30)} \\
 d_1 & = 1 + a_1 \quad \text{by (27)} \\
 d_2 & = 0 \quad \text{by (20)}
\end{align*}
\]

Secondly, assume

(assumption 2) \quad b_1 \neq 0 .

Note that \( d_1 \neq 0 \) by (7). Then (16) becomes

\[
 a_1 = \frac{c_2d_1^2}{a_2b_1b_2} + \frac{a_2b_2^2}{a_2b_1b_2} \\
 = \frac{d_1b_1b_2a_2^2}{d_2a_2b_1b_2} + \frac{b_2}{b_1} \quad \text{by (17)} \\
 = \frac{d_1a_2}{d_2} + \frac{b_2}{b_1} .
\]

Substitute the above into (18), then we get

\[
 c_1 = \frac{a_1a_2b_2}{c_2d_1^2} = \left( \frac{d_1a_2}{d_2} + \frac{b_2}{b_1} \right) \frac{a_2b_2}{c_2d_1^2} \\
 = \frac{a_2b_2}{c_2d_1d_2} + \frac{b_2^2a_2}{b_1c_2d_1^2} = \frac{a_2^2b_2}{b_1b_2a_2} + \frac{b_2^2a_2}{b_1c_2d_1^2} \quad \text{by (17)} \\
 = \frac{1}{b_1} + \frac{b_2^2a_2}{b_1c_2d_1^2} .
\]

Putting (17) and (32) into (19) gives

\[
 c_2^2d_1 + \left( \frac{1}{b_1} + \frac{b_2^2a_2}{b_1c_2d_1^2} \right)b_1b_2a_2^2 = b_2a_2^2 .
\]
Simplifying the above, we have

\[(33) \quad a_2 b_2 = -d_1 c_2 .\]

Then (31) and (32) become

\[(34) \quad a_1 = \frac{d_1}{d_2} \left( -\frac{c_2 d_1}{b_2} \right) + \frac{b_2}{b_1} = \frac{b_2}{b_1} - \frac{d_1^2 c_2}{b_2 d_2} ,\]

\[(35) \quad c_1 = \frac{1}{b_1} - \frac{b_2}{b_1 d_1} .\]

Substituting the above two equations and (33) into (18) gives

\[\left( \frac{1}{b_1} - \frac{b_2}{b_1 d_1} \right) c_2 d_1^2 = \left( \frac{b_2}{b_1} - \frac{c_2 d_1^2}{b_2 d_2} \right) (-d_1 c_2) ,\]

which simplified as

\[(36) \quad b_2 d_2 = b_1 c_2 d_1 .\]

Thus

\[(37) \quad a_1 = \frac{b_2}{b_1} - \frac{d_1}{b_1} \quad \text{by (34).}\]

Putting (35) and (36) into (8) gives

\[b_1 c_2 d_1 (c_2 d_1 + b_2 c_2 d_1 + b_2 + b_2 d_1) \]
\[- ((c_2 d_1 + d_1 + 1)b_2^2 - (d_1 + c_2 d_1)c_2 d_1 b_2 - c_2^2 d_1^2) \]
\[= b_1 c_2 d_1 (c_2 d_1 + b_2 c_2 d_1 + b_2 + b_2 d_1) \]
\[- ((c_2 d_1 + d_1 + 1)b_2 + c_2 d_1)(b_2 - c_2 d_1) \]
\[= (c_2 d_1 + b_2 c_2 d_1 + b_2 + b_2 d_1)(b_1 c_2 d_1 - b_2 + c_2 d_1) = 0 .\]

Therefore \(c_2 d_1 + b_2 c_2 d_1 + b_2 + b_2 d_1 = 0\), or \(b_1 c_2 d_1 - b_2 + c_2 d_1 = 0\). By (7), only the second is true. Hence

\[(38) \quad b_2 = c_2 d_1 (1 + b_1) \neq 0 .\]
which yields in turn

\begin{align*}
(39) \quad a_1 &= \frac{c_2 d_1 (1 + b_1) - d_1}{b_1} = \frac{d_1}{b_1} (c_2 + c_2 b_1 - 1) \quad \text{by (37),} \\
(40) \quad c_1 &= \frac{d_1 - (1 + b_1) c_2 d_1}{b_1 d_1} = -\frac{c_2 + b_1 c_2 - 1}{b_1} \quad \text{by (35),} \\
(41) \quad a_2 &= -\frac{c_2 d_1}{b_2} = -\frac{1}{1 + b_1} \quad \text{by (33),} \\
(42) \quad d_2 &= \frac{b_1 c_2 d_1}{c_2 d_1 (1 + b_1)} = \frac{b_1}{1 + b_1} \quad \text{by (36).}
\end{align*}

Then (12) becomes

\begin{align*}
(b_1 (1 + b_1) - (1 + b_1) (c_2 + b_1 c_2 - 1)) (1 + c_2) d_1^2 \\
- 2(1 + b_1) (c_2 + b_1 c_2 - 1) d_1 - b_1 (1 + b_1) - b_1
\end{align*}

Since $c_2, d_1 > 0$ and $1 + b_1 > 0$ by (41) and (7), $(1 + c_2) (1 + b_1) d_1 + b_1 + 2 \neq 0$. Hence

\[(1 - c_2) (1 + b_1) d_1 - b_1 = 0.\]

Note that $c_2 = 1$ implies $b_1 = 0$ which is contradicted to (assumption 2). Moreover $1 + b_1 > 0$ by (41) and (7). Therefore

\begin{align*}
(43) \quad d_1 &= \frac{b_1}{(1 - c_2)(1 + b_1)}.
\end{align*}

Putting the above into (38) and (39) gives

\begin{align*}
(44) \quad b_2 &= \frac{b_1 c_2}{1 - c_2}, \\
(45) \quad a_1 &= \frac{c_2 + c_2 b_1 - 1}{(1 - c_2)(1 + b_1)},
\end{align*}

respectively. Under $b_1 \neq 0$, we get the other solution as follows.
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Solution (2)

\[
\begin{aligned}
a_1 &= \frac{c_2 + c_2 b_1 - 1}{(1 - c_2)(1 + b_1)} \\
a_2 &= \frac{-1}{1 + b_1} \\
b_1 &= \text{independent variable greater than } -1 \text{ not equal to } 0 \\
b_2 &= \frac{b_1 c_2}{1 - c_2} \\
c_1 &= \frac{-c_2 + b_1 c_2 - 1}{b_1} \\
c_2 &= \text{positive independent variable not equal to } 1 \\
d_1 &= \frac{b_1}{(1 - c_2)(1 + b_1)} \\
d_2 &= \frac{b_1}{1 + b_1}
\end{aligned}
\]

by (45) 

by (41) 

by (44) 

by (40) 

by (43) 

by (42)

Note that \(d_1 > 0\) implies either \(-1 < b_1 < 0, c_2 > 1\) or \(b_1 > 0, 0 < c_2 < 1\).

Pictured below are the developing images in \(E^2\) using the stereographic projection from \((0, 0, -1)\) with various choice of value of the each parameters. The equator in \(S^2\) is drawn as the circles in the pictures.
Solution(1)

\[ a_{1} = -0.7 \]

Solution(2)

\[ b_{1} = -0.6 \]
\[ c_{2} = 2 \]
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\[ \text{Solution(2)} \]
\[ b_{-1} = 1 \]
\[ c_{-2} = 0.4 \]

References


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