Comm. Korean Math. Soc. **12** (1997), No. 4, pp. 1065–1067

A SHORT PROOF OF BAILEY'S FORMULA

ARJUN K. RATHIE AND JUNESANG CHOI

ABSTRACT. The aim of this research is to derive an interesting formula due to Bailey by a very short method.

1. Introduction and Results Reguired

Professor Bailey [1] had obtained the following very interesting and useful formula involving the product of generalized hypergeometric series:

$$\begin{aligned} &{}_0F_1(-;\rho;x)\times {}_0F_1(-;\sigma;x)\\ &={}_2F_3\left(\frac{1}{2}(\rho+\sigma),\,\frac{1}{2}(\rho+\sigma-1);\,\rho,\,\sigma,\,\rho+\sigma-1;\,4x\right). \end{aligned} \tag{1.1}$$

Very recently Rathie [3] has given a very short proof of the well-known Preece's identity by utilizing the formula (1.1).

The following formulas will be required in our present proof.

Kummer [2]:

$$e^{-x}\times {}_1F_1(\alpha;\,\rho;\,x)={}_1F_1(\rho-\alpha;\,\rho;\,-x); \tag{1.2}$$

$$e^{-x/2}\times {}_1F_1(\alpha;\,2\alpha;\,x)={}_0F_1\left(-;\,\alpha+\frac{1}{2};\,\frac{x^2}{16}\right). \tag{1.3}$$

Bailey [1]:

$$\begin{aligned} &{}_1F_1(\alpha;2\alpha;\,x)\times {}_1F_1(\beta;2\beta;\,-x)\\ &={}_2F_3\left(\frac{1}{2}(\alpha+\beta),\,\frac{1}{2}(\alpha+\beta+1);\,\alpha+\frac{1}{2},\,\beta+\frac{1}{2},\,\alpha+\beta;\,\frac{x^2}{4}\right). \end{aligned} \tag{1.4}$$

Received February 28, 1997. Revised August 30, 1997.
1991 Mathematics Subject Classification: primary 33C20, secondary 33C10, 33C15.
Key words and phrases: Hypergeometric series, A Bailey's formula.

It is not out of place to mention here that recently Rathie and Nagar [4] have given two interesting formulas contiguous to (1.3).

The aim of this research is to derive the Bailey's formula (1.1) by a very short method.

2. Proof of the formula (1.1)

In order to prove (1.1), it is sufficient to verify the following formula

$$
\begin{aligned}
&{}_0F_1\left(-\,;\,\rho;\,\frac{x^2}{16}\right)\times\left(-\,;\,\sigma;\,\frac{x^2}{16}\right)\\
&= {}_2F_3\left(\frac{1}{2}(\rho+\sigma),\,\frac{1}{2}(\rho+\sigma-1);\,\rho,\,\sigma,\,\rho+\sigma-1;\,\frac{x^2}{4}\right).
\end{aligned}
\tag{2.1}
$$

Indeed, replacing x^2 by $16x$ in (2.1) immediately reaches at our desired formula (1.1).

Start with the first part of (2.1):

$$
{}_0F_1\left(-\,;\,\rho;\,\frac{x^2}{16}\right)\times{}_0F_1\left(-\,;\,\sigma;\,\frac{x^2}{16}\right)
$$

using (1.3)

$$
\begin{aligned}
&= \left[e^{-x/2}{}_1F_1(\rho-\frac{1}{2};\,2\rho-1;\,x)\right]\left[e^{-x/2}{}_1F_1(\sigma-\frac{1}{2};\,2\sigma-1;\,x)\right]\\
&= {}_1F_1(\rho-\frac{1}{2};\,2\rho-1;\,x)\left[e^{-x}{}_1F_1(\sigma-\frac{1}{2};\,2\sigma-1;\,x)\right]
\end{aligned}
$$

using (1.2) in the second expression

$$
= {}_1F_1(\rho-\frac{1}{2};\,2\rho-1;\,x){}_1F_1(\sigma-\frac{1}{2};\,2\sigma-1;\,-x)
$$

using (1.4)

$$
= {}_2F_3\left(\frac{1}{2}(\rho+\sigma),\,\frac{1}{2}(\rho+\sigma-1);\,\rho,\,\sigma,\,\rho+\sigma-1;\,\frac{x^2}{4}\right),
$$

which completes the proof of (2.1).

References

[1] W. N. Bailey, *Products of generalized hypergeometric series*, Proc. London Math. Soc. **28** (1928), 242–254.

[2] E. E. Kummer, *Über die hypergeometrische Reihe* $F(a;\, b;\, x)$, J. Reine Angew Math. **15** (1836), 38–83.

[3] Arjun K. Rathie, *A short proof of Preece's identities and other contiguous results, submitted for publication*, 1977.

[4] Arjun K. Rathie and Vishakha Nagar, *Kummer's second theorem involving product of generalized hypergeometric series*, Le Matematiche (Catania), **50** (1995), 35–38.

Arjun K. Rathie
Department of Statistics
University of Brasilia
70910-900, Brasilia-D.F. (Brasil).
On leave from Department of Mathematics
Dungar College (MDS University), BIKANER-334001
Rajasthan State, INDIA

Junesang Choi
Department of Mathematics
College of Natural Sciences
Dongguk University
Kyongju 780-714, Korea