*Journal of the Ergonomics Society of Korea*
Vol.16, No.2, 1997


# EEG Model by Statistical Mechanics of Neocortical Interaction

*J. M. Choi, **M. C. Whang, *B. H. Bae, *S. Y. Kim and **C. J. Kim

## ABSTRACT

Brain potential is described using the mesocolumnar activity defined by averaged firings of excitatory and inhibitory neuron of neocortex. Lagrangian is constructed based on SMNI(Statistical Mechanics of Neocortical Interaction) and then Euler Lagrange equation is obtained. Excitatory neuron firing is assumed to be amplitude-modulated dominantly by the sum of two modes of frequency $\omega$ and $2\omega$. Time series of this neuron firing is calculated numerically by Euler Lagrangian equation. $I\omega L$ related to low frequency distribution of power spectrum, $I\omega H$ hight frequency, and Sd(standard deviation) were introduced for the effective extraction of the dynamic property in the simulated brain potential. The relative behavior of $I\omega L$, $I\omega H$, and Sd was found by parameters $\varepsilon$ and $\gamma$ related to nonlinearity and harmonics respectively. Experimental $I\omega L$, $I\omega H$, and Sd were obtained from EEG of human in rest state and of canine in deep sleep state and were compared with theoretical ones.

* Department of Physics, KAIST
** Ergonomics Lab., KRISS

## I. Introduction

Statistical mechanical approach of neocortical interaction[1-7] leads to define analytical models of EEG. Their variables and parameters are reasonably identified by ensembles of synaptic and neuronal interaction. SMNI has demonstrated its capability in describing large scale properties of STM(short term memory) and EEG phenomena[8]. The explicit algebraic form of the probability distribution for mesoscopic columnar interactions is driven by microscopic neuronal interactions. The microscopic interactions are described by electrical and chemical interaction within synaps. Their parameters all lie within experimentally observed ranges. This mesoscopic probability distribution has successfully described STM phenomena[8]. It also has described the systematic of EEG phenomena, when used as a basis derived from the most probable trajectories using the Euler-Lagrange variational equations[8]. These results provide strong quantitative support for an accurate and intuitive perspective, portraying neocortical interactions as having physical mechanisms that span disparate spatial scales and functional or behavioral phenomena. We take this approach to understand the physical mechanism generating EEG and quantify emotional states.

The purpose of this study is twofold: First, we derive a nonlinear partial differential equation from the Lagrangian for mesocolumnar neocortex interaction. This field equation governs the dynamics of the macroscopic quantities measured by EEG. Second, with respect to a particular EEG experiment with a canine in the deep sleep state and human in rest state we solve the obtained field equation analytically and numerically and prove that it is reasonably consistent with experimentally observed phenomena.

## II. Dynamic EEG-Models by Statistical Mechanics of Neocortex Interaction(SMNI) and Lagrangian

Brain EEG potential $\Phi(t)$ can be modeled by[8]

$$\Phi(t) = f(aM^E(t) + bM^I(t)) \tag{1}$$

where $M^E$ and $M^I$ stand for the mesocolumnar averaged excitatory and inhibitory neuron firings. Indices E and I represent the excitatory and inhibitory firing fields, respectively as shown in Fig. 1. Constants a and b are their contribution factors of excitatory and inhibitory neurons respectively. Eq.(1) can be expanded by

$$\Phi(t) \approx (aM^E + bM^I) + \varepsilon(aM^E + bM^I)^2 + \cdots$$

using Taylor's expansion where the order of the above equation indicates one of spectrum. Higher order terms than second order can be neglected because trispectrum from EEG signal is found to be typically much less than the bispectrum[20]. Theirfore $\Phi(t)$ is defined to be

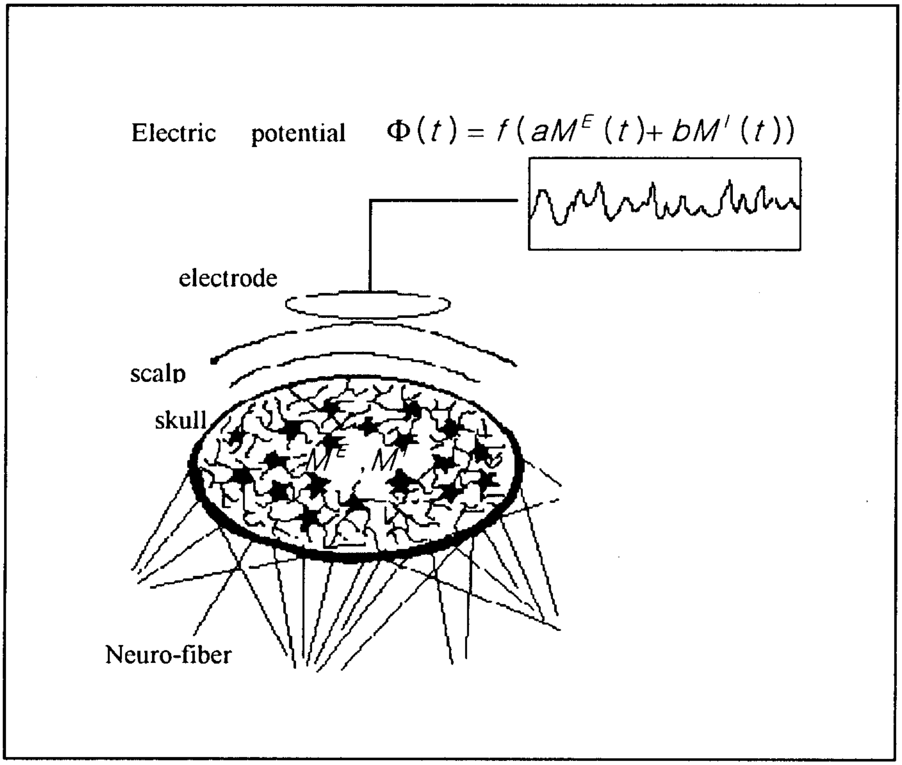


Fig. 1
Brain potential reflecting firings of neural mass in neocortex

$$\Phi(t) = (aM^E + bM^I) + \varepsilon(aM^E + bM^I)^2$$

and let $M^E(t) = cM^I(t)$, where constant c represents the ratio of excitatory and inhibitory neurons and can be determined for each electrode site. Then the model equation is obtained as the following

$$\Phi(t) = aM^E(t) + \varepsilon a^2(M^E(t))^2 \qquad (2)$$

where $\alpha = a + \frac{b}{c}$

The conditional probability distribution for the measured scalp potential $\Phi(t)$ is defined by

$$\begin{aligned} P &= \prod_G P^G[M^G(r; T+\tau) \mid M^G(r'\,'\,t)] \\ &= \sum_{\sigma_i} \delta\Big(\sum_{j\in E}\sigma_j - M^e(r; t+\tau)\Big)\delta \\ &\quad \Big(\sum_{j\in i}\sigma_j - M^i(r; t+\tau)\Big)\prod_j^N P_{\sigma_i} \\ &\approx \prod_G (2\pi\tau\sigma^2)^{-1/2}\exp(-\tau\angle) \end{aligned} \qquad (3)$$

where $P_{\sigma_i} \approx \frac{\exp(-\sigma_j F_j)}{\exp(F_j) + \exp(-F_j)}$ is the microscopic conditional probability of neuron j firing given previous firings within $\tau$ of other neurons k. $F_j = \frac{V_j - \sum_k a_{jk}\nu_{jk}}{(\pi\sum_k a_{jk}(\nu_{jk}^2 + \phi_{jk}^2))^{1/2}}$ is the threshold factor, L is Lagrangian, and G represents E and I. $V_j$ means the threshold potential of j neuron. Each neuron may contribute many synaptic interactions to many other neurons. A neuron may have as many as $10^4 \sim 10^5$ synaptic interactions. Within time $\tau \approx 5$ms, the distribution of quanta of chemical transmitter released from neuron k to neuron j has the mean $a_{jk}$, where $a_{jk} = A_{jk}(\sigma_k + 1)/2 + B_{jk}$. $A_{jk}$ is the conductivity weighting polarization transmission, dependent on k firing. $\sigma_k = 1$ if k fires but $\sigma_k = -1$ if k does not fire. $B_{jk}$ is a background including some nonsynaptic and long-range activity. This definition of $\sigma_k$ permits a decomposition of $a_{jk}$ into two physical contributions. $\nu_{jk}$ and $\phi_{jk}$ are the mean and the variance of contributions to neuronal electric potential respectively. EEG in one eletrode site is focused on in this study. The spatial change $\nabla M^E$ and external inputs[1-4] from other sites in L in Eq.(3) induced from the above mentioned conditional probability, are neglected for simplicity. Therefore the final form of Lagrangian L of EEG dynamics is given by Eq(4).

$$L = \frac{1}{2\sigma^2}(\Phi - m)^2 \tag{4}$$

where m and $\sigma$ are an averaged value and standard deviation of $\Phi(t)$ respectively, and $\Phi(t)$ is the first time derivative of $\Phi(t)$. Lagrangian L in Eq.(4) can be rewritten with Eq.(2) by

$$L(M^E, M^E, t) = \frac{1}{2\sigma^2}(\alpha M^E + 2\varepsilon\alpha^2 M^E M^E - m)^2 \tag{5}$$

The stationary states of brain function are focused in this study. It is constrained based on experimental EEG that $\sigma$, m in L are kept contant for the same stationary state, although they are the functions of $M^E$. Euler-Lagrange equation becomes

$$M^E + 2\varepsilon\alpha(M^E)^2 + 4\varepsilon\alpha M^E M^E + (2\varepsilon\alpha M^E)^2 M^E + (2\varepsilon\alpha M^E)^2 M^E = 0 \tag{6}$$

If we neglect the second order term in Eq.(2), i.e. $\varepsilon = 0$, Euler-Lagrange equation becomes $M^E = 0$. This reduces to a trivial non-periodic case. We can assume $M^E(t)$ for solution of nonlinear differential Eq.(6) as follows;

$$\begin{aligned} M^E(t) &= \mathrm{Re}\,[h^E + \sqrt{2h^{EE}(t)}\,(e^{i\omega t} + \gamma e^{2i\omega t})] \\ &= h^E + \sqrt{2h^{EE}}\,(\cos(\omega t) + \gamma\cos(2\omega t)) \end{aligned} \tag{7}$$

where $h^E$, $h^{EE}(t)$ and $\gamma$ are the average value of $M^E$(t), variation of $M^E$(t) and a factor related to harmonics, respectively. Note that $h^{EE}(t)$ is changed very slowly compared to $e^{i\omega t}$. The numerical solution of EEG is obtained by the substitution of Eq.(7) into Eq.(6) and its result is shown in Fig. 2(a). The comparison between the experimental EEG data in a certain interval and the

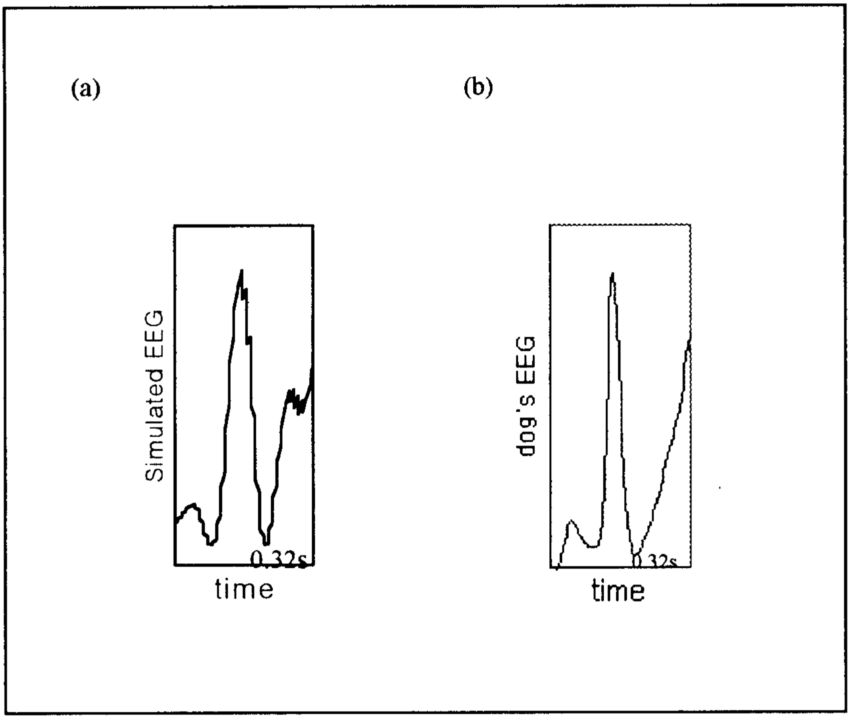


Fig. 2
Comparison of the simulated EEG(a) with the experimental EEG(b) ; The unit of vertical axis is arbitrary

simulation results shows that they are well matched with each other, as shown in Fig. 2. Substitution of Eq.(7) into Eq.(6) and taking a time-average of this results into an averaged Euler-Lagrange equation:

$$[(F_3^2+F_4^2)+2F_4F_7e^{\frac{G}{2}}+(F_7^2+F_8^2)e^G]G$$

$$+[\frac{1}{2}(F_3^2+F_4^2)+\frac{2}{3}F_4F_7e^{\frac{G}{2}}+(F_7^2+F_8^2)e^G]G^2$$

$$-[\frac{1}{2}(F_1^2+F_2^2)+\frac{3}{2}F_2F_5e^{\frac{G}{2}}-(F_5^2+F_6^2)e^G]=0$$

where

$$F_1=-\frac{W}{2\gamma}B,\ F_2=-wB,\ F_3=\frac{1}{4\gamma}B,$$

$$F_4=\frac{1}{4}B,\ F_5=-\frac{w}{2\gamma^2}B,\ F_6=-\frac{3w}{2\gamma}B,$$

$$F_7=\frac{1}{4\gamma^2}B,\ F_8=\frac{1}{2\gamma}B,\ B=\frac{B_1^2}{B_3}$$

$$B_1=\sqrt{2}\alpha+2\sqrt{2}\varepsilon\alpha^2h^E,\ B_2=\gamma B_1,$$

$$B_3=2\varepsilon\alpha^2,\ B_4=2\gamma B_3, \qquad (8)$$

$$\alpha=(a+\frac{b}{c}),\ G=In(\frac{B_4^2}{B_1^2}h^{EE})$$

We solved the EEG dynamical Eq.(8) with an arbitrary initial condition of G(0) = 2.1, and G(0) = 1.1. We consider an asymptotic behavior of G(t) after sufficient time evolution. The frequency f is fixed to be 5 Hz, that is $\omega=2\pi f=10\pi$, based on the spectral range of about $f$~$3f$, which corresponds to the experimental dominant range of about 1~20 Hz. A factor c in Eq.(3) is fixed to be 5 since excitatory neurons exist about 5 times as many as inhibitory neurons in a general neural system. Other fixed variables are determined such as a = 2.1, b = 1.5, $h^E = 10$ and m = 127 for fitting simulated range with experimental range of EEG.

## III. Comparison of the simulated EEG with the experimental one in human and Canine

To extract the unknown dynamic property from the simulated and the experimental EEG, $I\omega L$, $I\omega H$ and Sd are introduced. $I\omega L$ is defined by

$$I\omega L = -\frac{\log P(2\omega) - \log P(\omega)}{\log 2\omega - \log \omega} = \frac{\log\left(\frac{P(\omega)}{P(2\omega)}\right)}{\log 2} \tag{9}$$

which means the slope for range between $\omega$ and $2\omega$ in the log-log power spectral space ($\log\omega$, $\log P(\omega)$). $I\omega H$ is defined by and

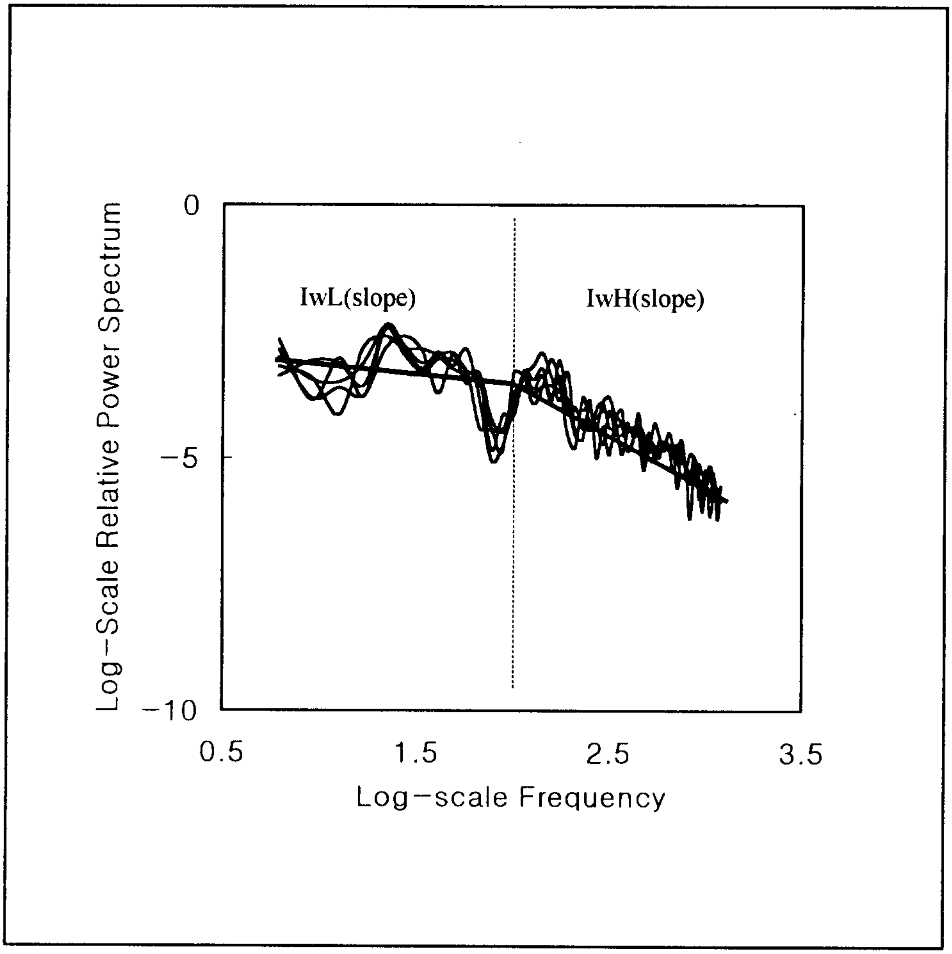


Fig. 3 Power spectrum of EEG plotted with log-log axis

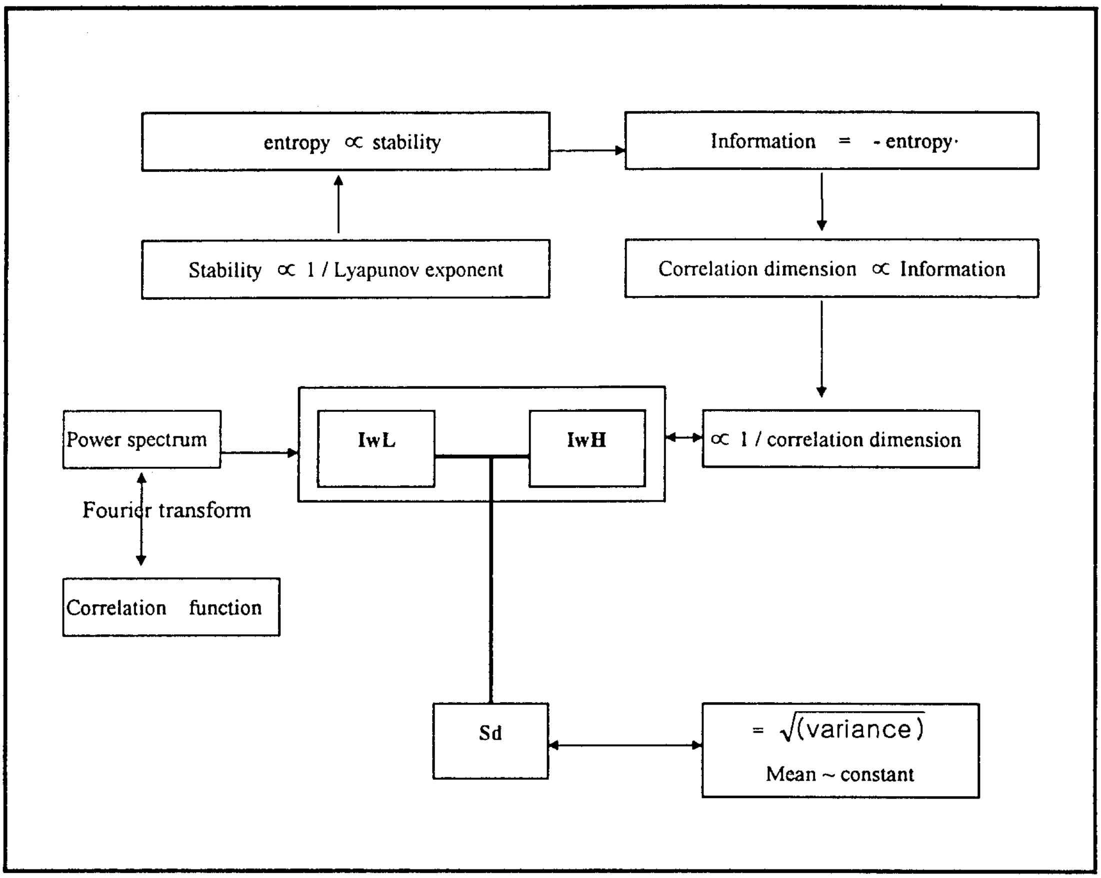


Fig. 4 Block diagram representing the relations of $I\omega L$, $I\omega H$, Sd to other dynamic variables

$$I\omega H = -\frac{\log P(3\omega) - \log P(2\omega)}{\log 3\omega - \log 2\omega} = \frac{\log\left(\frac{P(2\omega)}{P(3\omega)}\right)}{\log 1.5} \tag{10}$$

which means the slope of range between $2\omega$ and $3\omega$. Sd means the standard deviation of brain potential $\Phi(t)$ and is described by

$$Sd = \frac{\sqrt{\sum_{i=1}^{N}(\Phi_i - m)^2}}{N} \tag{11}$$

where N and m are the total sampling number and the average of brain potential $\Phi$(t) over N, respectively. $I\omega L$ and $I\omega H$ can provide some information related to spectral distribution of low frequency ranges and of high frequency ranges respectively from EEG as shown in Fig. 3. In general, $I\omega L$ is different from $I\omega H$ in EEG. Sd can provide some information related to the amplitude from EEG. Three variables $I\omega L$, $I\omega H$, $Sd$ can find the temporal change of dynamic properties because they are related indirectly to other dynamic variables as shown in Fig.4. Note that $I\omega L$ and $I\omega H$ are inversely proportional to the correlation dimension[18] and inversely proportional to the negative of Shannon entropy[19] approximately in low and high frequency components of EEG respectively. That is, correlation dimension is proportional indirectly to Shannon information. The information is defined as the negative of Shannon entropy which is a measure of the lack of knowledge. The information can be regarded as a measure for the knowledge of which event of the sample set is to be expected, if the probability distribution in phase space is only known.

The $I\omega L$-$I\omega H$-$Sd$ relation is investigated from the simulated EEG in various conditions of $\varepsilon$, $\gamma$ parameters, and then compared with the experimental EEG. $I\omega L$, $I\omega H$, $Sd$ can be constructed from the simulated EEG as follows.

$$I\omega L = -\log\left[\frac{B_1}{B_2 + B_3\sqrt{E_1 e^G}}\right]\log 2$$

$$I\omega H = -\log\left[\frac{\dfrac{B_2}{\sqrt{E_1 e^G}} + B_3}{B_4}\right]\log 2 \qquad (12)$$

$$Sd = S_1 e^G + S_2 e^{\frac{G}{2}} \quad \text{where } S_1 = \frac{B_3 + B_4}{\sqrt{2}}\sqrt{E_1},$$

$$S_2 = \frac{B_1 + B_2}{\sqrt{2}}\sqrt{E_1},\ E_1 = \frac{B_1^2}{B_4^2}$$

$I\omega L$, $I\omega H$, $Sd$ are calculated using the

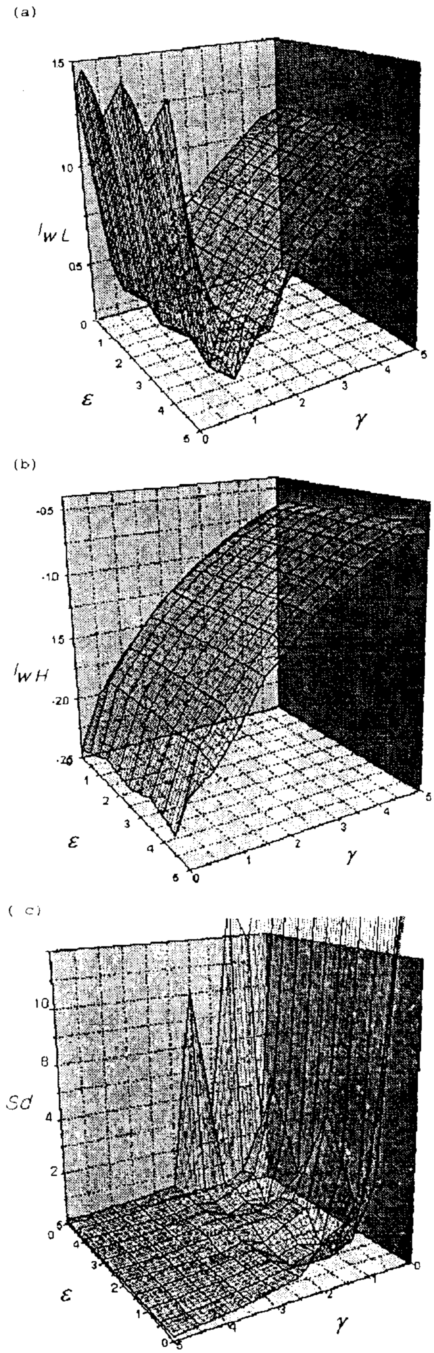


Fig. 5 Behavior of $I\omega L$, $I\omega H$, Sd with the increment of $\varepsilon$ and $\gamma$

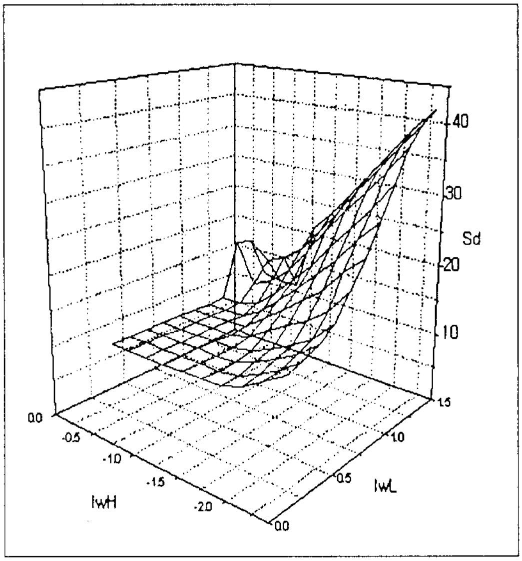


Fig. 6 $I\omega L$-$I\omega H$-$Sd$ relation for the simulated EEG signals

solution, G of Eq.(8), and their behaviors of $I\omega L$, $I\omega H$, $Sd$ are investigated with the increment of $\varepsilon$ and $\gamma$ parameters from 0 to 5 by 0.2 unit step as shown in Fig. 5. In Fig. 5(a), $I\omega L$ has a minimum value when $\gamma$ is nearly 1. Its decrease with $\gamma$ before 1 is steeper than its increase with $\gamma$ after 1. However, $I\omega L$ is horizontally flat and turns out to be insensitive with $\varepsilon$. In Fig. 5(b), $I\omega H$ has no minimum peak and increases slowly with the increment of $\varepsilon$ less than about 1. In Fig. 5(c), Sd decreases exponentially with the increment of $\gamma$ and $\varepsilon$. The combined result of $I\omega L$-$I\omega H$-$Sd$ relation is given in Fig. 6. It is observed that Sd becomes larger as $I\omega L$ becomes smaller and $I\omega H$ becomes larger.

Computerized electroencephalograph was used to measure EEG signal from 4 persons with eye closed and from a canine in a deep sleep state. The instrument consists of EEG-amplifier, 8-bit analog to digital converter, EEG-computer interface, which sampled scalp voltage at 21 electrode at a rate of 204.8Hz for human and sampled scalp voltage at 1 electrode at a rate of 250Hz for canine. The silver chloride cup electrodes were placed, using a conductive paste, on the 10/20 international electrode system for human and on the left part 4 cm distant from the vertex and at the vertex as a reference for canine. In order to compare experimental results with simulation, $I\omega L$, $I\omega H$, and $Sd$ are calculated from each EEG signal of 21 channels with 1024 sampled data from 4 subjects according to the definition given by Eqs.(9), (10), and (11). The $I\omega L$-$I\omega H$-$Sd$ relations from 4 subjects are shown in Fig. 7 and similar to that of the simulation as shown in Fig. 6.

It is suggested that the deep sleep state of a canine may be characterized by $\varepsilon$, $\gamma$ parameters estimated by comparison with the simulated $I\omega L$-$I\omega H$-$Sd$ relations. $I\omega L$, $I\omega H$, and $Sd$ are calculated from 32 sequential sets with 1024 sampled data of EEG from a canine.

The resulting $I\omega L$-$I\omega H$, $I\omega L$-$Sd$ and $I\omega H$-$Sd$ relationships are shown in Fig. 8(a), respectively. On the other hand, we calculated $I\omega L$, $I\omega H$, Sd from simulated EEG with $\varepsilon$ fixed to be 2.0 and $\gamma$ varied from 0 to 5 by 0.2 unit step. Whatever $\varepsilon$ is fixed to be any constant near about 2.0, the general trends of $I\omega L$, $I\omega H$, Sd did not change much, as shown in Fig. 8(b). The

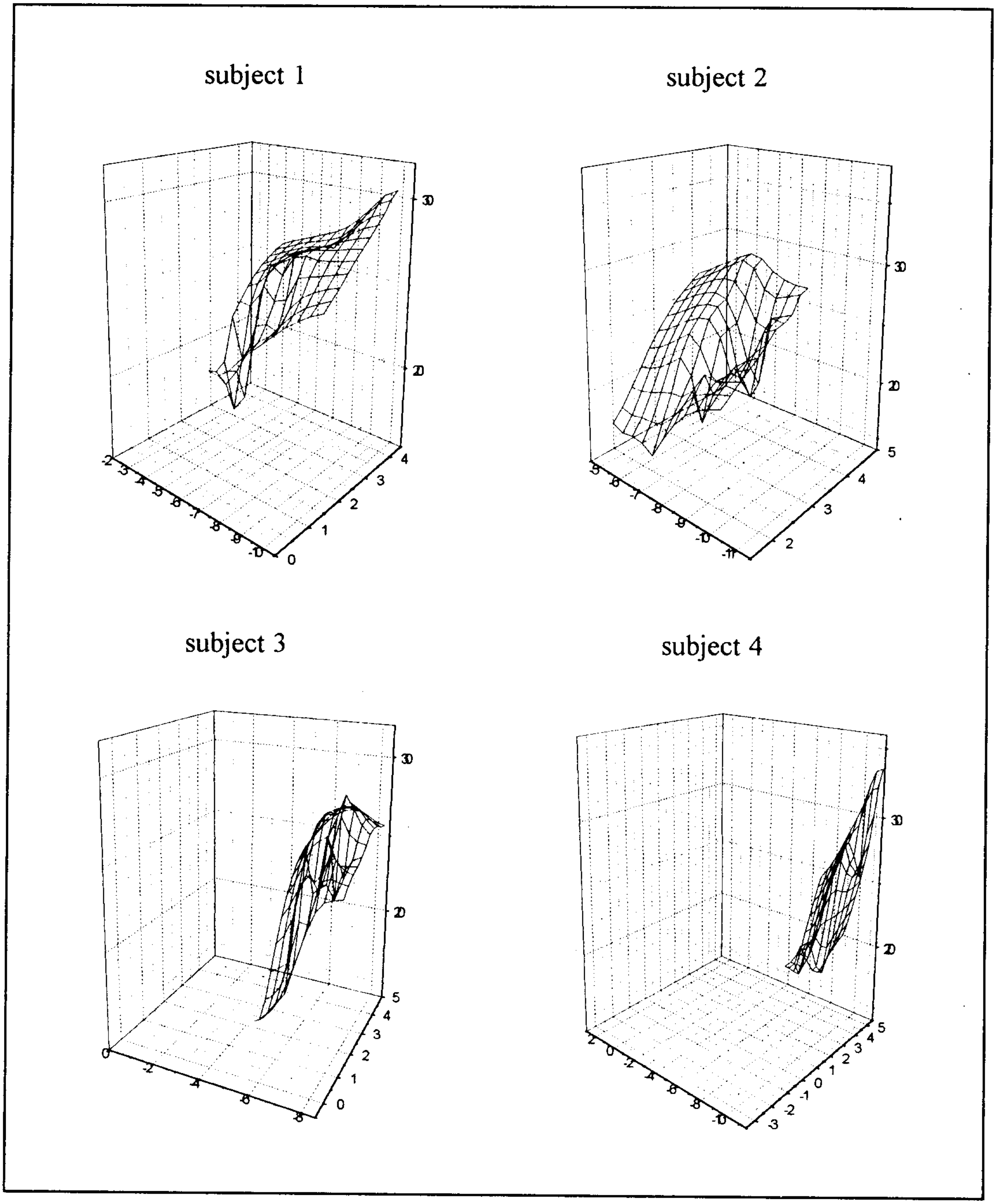


Fig. 7 *IωL-IωH-Sd* relation for the experimental EEG signals of 4 subjects

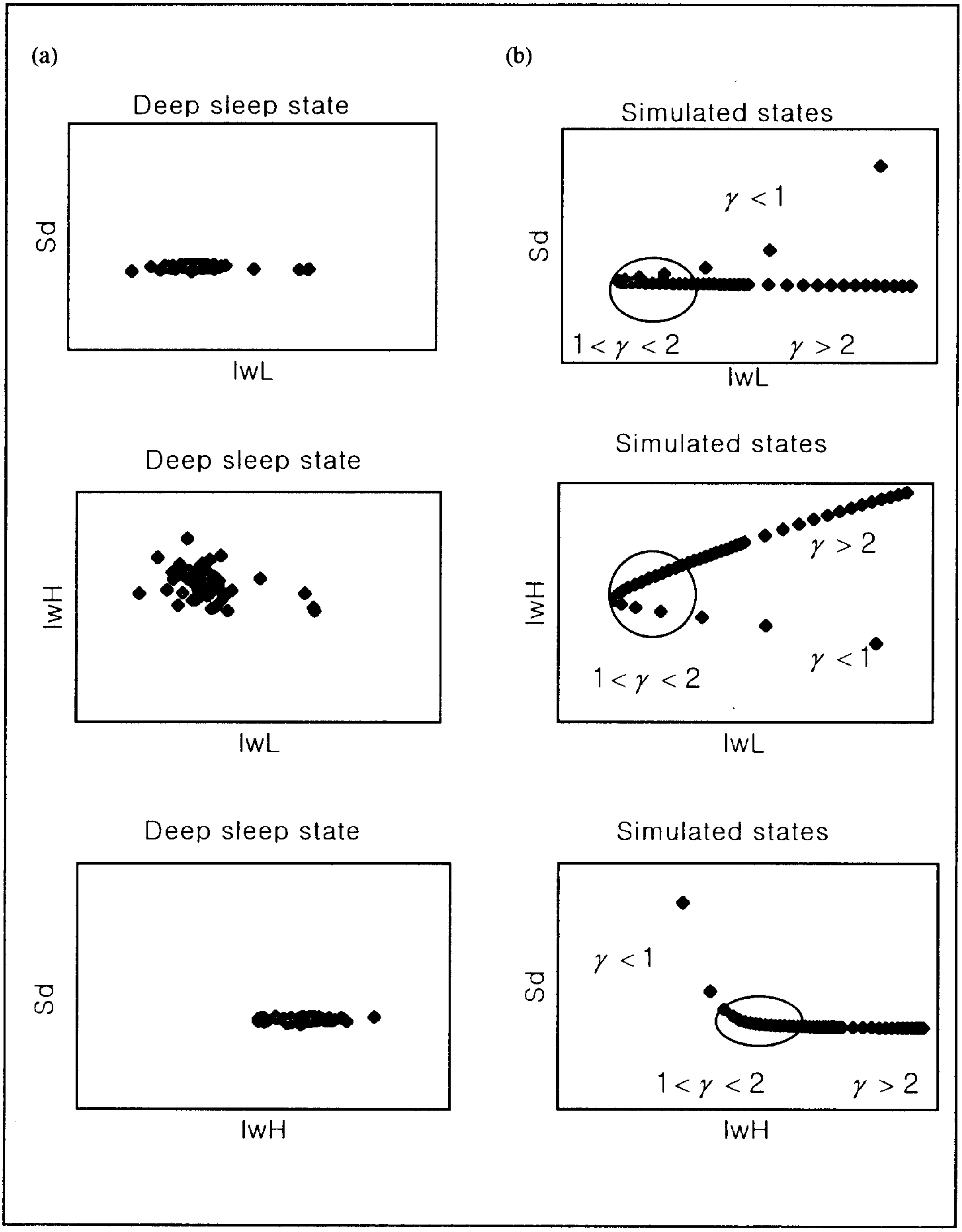


Fig. 8 Comparison of the experimental results with simulation in canines deep sleep state ; Each axis has arbitrary unit.

result in Fig. 8(b) which indicates the slope in the marked ellipse area with $\gamma$ value range about 1~2 is similar to that of Fig.4, thereby It is indicated that the sleep stage of a canine refers to $\varepsilon = 2.0$ and $\gamma = 1 \sim 2$ of simulation model.

## V. Conclusion

The realistic physical modeling of EEG was performed in this study. Much more work in this model is needed and the investigation in the case of various brain function must be carried. This model is valuable in that it shows the possibility of physiological explanation of EEG reflecting the brain function.

## Acknowledgment

The authors are very thankful to Dr. Lester Ingbers and Prof. Ernst Nieburs fruitful discussions. This research is supported by G7-Project from the Ministry of Science and Technology, KOSEF grant, and Biotech 2000 Project.

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