Estimating Curvature Change due to Layer Removal in Strips with Uniaxial Residual Stress Fields

S. H. SHIN
Mokpo Maritime University
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축방향의 잔류응력이 내재하는 STRIP의 층 가공에 따른 곡율변화

신 상 현
목포 해양대학교
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Abstract

잔류응력을 측정하는 비파괴 방법중 Neutron diffraction technique의 발달로 인하여 기존의 X-ray diffraction technique에 비하여 잔류응력 측정가능 두께가 대부분의 물질에 있어서 훨씬 깊어졌다. 비파괴방법으로 부품내의 잔류응력 분포가 측정되어진 경우, 부품의 가공시 잔류응력의 재분포로 인한 형상의 변화를 정량적으로 예측할 수 있는 방법에 대해 어떠한 경우도 논의된 바 없다. 본 연구에서는 축 방향의 잔류응력이 내재하는 strip의 층 가공시 일어나는 strip의 곡율변화를 정량적으로 예측할 수 있는 수식을 제시하였다. 간단히 컴퓨터 프로그램화 할 수 있는 수식을 전개하므로 써 현장에서 유용하게 이용할 수 있게 하였다.

INTRODUCTION

Residual stresses are self-equilibrated internal stresses remaining in a component after non-uniform plastic deformation. Curvature change arising from layer removal of residually stressed components is one of the main consumer complaints regarding manufactured products which are required to be layer-removed to very close tolerances.

Recent developments of nondestructive residual stress measurement techniques, such as X-ray diffraction, neutron diffraction, ultrasonic, and Barkhausen noise method (Hauk et al., 1990) have made it possible to determine the residual stresses without providing information on dimensional and shape changes due to material removal operations. The depth to which residual stress can be measured by neutron diffraction technique depends on a number of factors such as source intensity, coherent scattering cross section, and beam size. However, in most materials, neutrons in the normal diffraction wavelength range can several orders-of-magnitude more penetrate than X-ray (Root, J.H., et al., 1991, Prask, H.J. and Choi, C.S., 1991). The results of this research are specially useful in cases where residual stress distribution has been obtained by a non-destructive technique with no information on the effect of material removal operation on the geometrical and stress changes in the material.

Fujiwara et al.(1988) showed that a circular disk deforms into elliptical shape when it is cut out from the center of a rectangular plate containing biaxial residual stresses. They also investigated residual stress alteration and dimensional changes of a circular plate containing an axisymmetric residual stress pattern, due to material removal from the outer region of the circular plate using Electrical Discharge Machining (EDM). Though the works were performed experimentally and by FEM technique, it is believed that the works could be achieved by closed - form type analytical solutions.

Honda et al.(1988) investigated, experimentally and by FEM technique, the interaction of residual stresses with material removal by studying the dimensional changes of three circular holes drilled one after another in a residually stressed region.

The purpose of this work was to complement the existing research in the area of residual stress analysis by developing closed - form and easy - to - use expression for the prediction of curvature change due to layer removal, given the initial residual stress state and the thickness of the material to be removed. Assuming a uniaxial residual stress distribution through the thickness of a rectangular component (typical of a cold rolled strip), this research utilizes analytical technique to determine the expected curvature change of a strip as a result of repeated removal of material layers from the surface of the strip.

ANALYTICAL APPROACH

Consider a rectangular strip containing only longitudinal residual stresses of uniform magnitude in the axial and transverse (widthwise) directions as shown in Figure 1. Material properties were assumed to be elastically homogeneous and linear. It was further assumed that regardless of the deformation process by which the residual stresses were induced, the relaxation of these stresses is a linearly elastic process, and that the material was removed without creating appreciable residual stresses at the exposed material surface. Electrical discharge machining and electro - chemical machining are examples of material removal processes which would not create additional residual stresses during the material removal process.

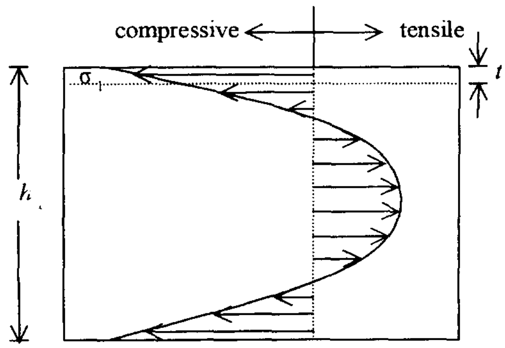

Fig. 1 Typical residual stress distribution in a cold rolled strip

If the first layer of thickness, t, and of average residual stress of is removed, a new equilibrium configuration will be established. The relationship between the moment created due to the removal of the layer and curvature change is given by:

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{6h_1 t_1 \sigma_1}{E(h_2)^3} \qquad (1)$$

where h_1 and h_2 are the height of the body before and after the removal of the first layer, respectively, E is Young's modulus, and R_1, R_2 are radii of curvatures of the mid-section of the remaining body, before and after removal of the first layer, respectively. It should be noted that throughout this analysis positive curvature is assumed when the center of curvature is above the object while negative curvature is assumed when the center of curvature is below the object. Since the distribution of the residual stress in the remaining body changes due to the removal of the first layer, the curvature change resulting from the removal of the second layer will be a function of the new state of residual stress as opposed to the original state of residual stress. In order to compute the curvature change due to the removal of the second layer, the redistributed residual stress in the second layer after the removal of the first layer must be first determined. This can be done by computing the location of the neutral axis as shown in Figure 2.

The shaded triangles shown in Figure 2 represent the stress state in the block which would

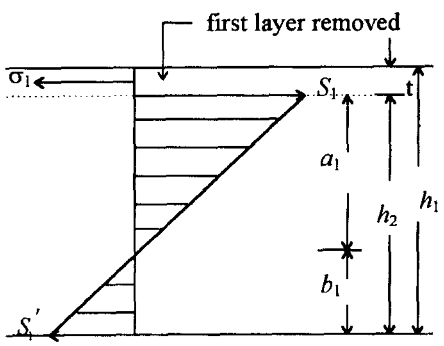

Fig.2 Location of neutral axis after the first layer removal

be created by bringing the remaining body back to its pre-removal shape. In Figure 2, equilibrium of forces requires that

$$\sigma_1 t + \frac{a_1 S_1}{2} + \frac{b_1 S_1'}{2} = 0 \tag{2}$$

where a_1 is the height from the neutral axis to the upper surface of the remaining body after the removal of the first layer, b_1 is the height from the neutral axis to the bottom surface of the remaining body, and S_1 and S_1' are stresses which would be produced at the upper and bottom surfaces of the remaining body, respectively, when the remaining body is brought back to its shape prior to the removal of the first layer. Referring to Figure 2,

$$a_1 + b_1 = h_2 \tag{3}$$

$$(-S_1') = \frac{b_1 S_1}{a_1} \tag{4}$$

The relationship between the stress, S_1, and the curvature change caused by the removal of the first layer can be expressed as:

$$S_1 = -a_1 E\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{5}$$

Utilizing equations (1) through (5), one can solve for the stress, S_1, as well as the distance, a_1. The result is :

$$S_1 = -\frac{E h_2(3h_1 + h_2)}{6h_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{6}$$

$$a_1 = -\frac{h_2(3h_1 + h_2)}{6h_1} \tag{7}$$

where the term $(1/R_1 - 1/R_2)$ is known in equation (1).

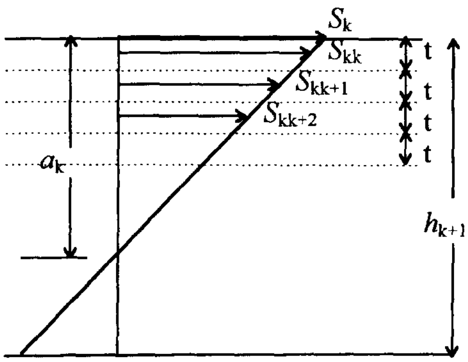

Fig.3 The correction terms at the middle of each layer

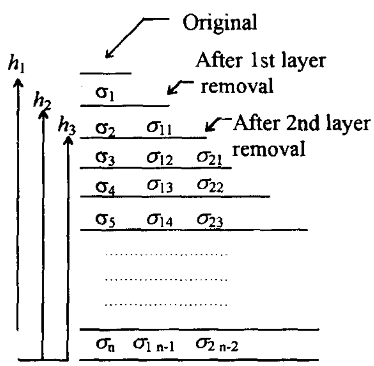

Fig.4 Original and redistributed residual stresses in each layer

The correction term, S_{11} (refer to Figure 3), is the stress which would be created in the middle of the second layer (i.e. the uppermost layer of the remaining body after the first layer removal) when the remaining body after the removal of the first layer is brought back to its shape before the removal of the first layer. Hence, it should be taken into account for computing the redistributed residual stress, σ_{11} (see Figure 4), in the second layer, such as:

$$\sigma_{11} = \sigma_2 - S_{11} \tag{8}$$

where σ_2 is the original residual stress in the second layer of the original body. The correction term, S_{11}, can be calculated by geometric consideration, as shown in Figure 3,

$$S_{11} = \frac{S_1}{a_1}\left(a_1 - \frac{t}{2}\right) \tag{9}$$

In order to compute the curvature change due to the removal of the second layer, the redistributed residual stress, σ_{11}, must be used in an equation similar to the equation (1), as follows:

$$\frac{1}{R_2} - \frac{1}{R_3} = \frac{6th_2\sigma_{11}}{E(h_3)^3} \tag{10}$$

Generalizing this procedure at a certain stage of the successive layer removal, suppose $(i+1)$th layer is now being removed, and assuming each layer has a uniform thickness of t, we can have an equation similar to equation (10):

$$\frac{1}{R_{i+1}} - \frac{1}{R_{i+2}} = \frac{6t\sigma_{i1}h_{i+1}}{E(h_{i+2})^3} \tag{11}$$

where is the average redistributed residual stress in the uppermost layer ($(i+1)$th layer) of the remaining body after the removal of the first layer through the (i)th layer (see Figure 4), h_{i+1} is the height of the remaining body after the removal of the first layer through the (i)th layer, R_{i+1} and R_{i+2} are radii of curvature of the mid - section of the body, before and after the removal of the $(i+1)$th layer, respectively. However, the redistributed residual stress, σ_{i1}, is not known yet, and this has been changed from the original residual stress (σ_{i+1}) in the ($i+1$)th layer due to the removal of the first layer through the (i)th layer. Thus, in order to compute the stress, σ_{i1}, the correction terms accounting for the removals of the first layer through the (i)th layer should be calculated. For computing the correction terms, the stress,

S_k for $1 \leq k \leq i$, which would be produced by bringing back the remaining body after the removal of the (k)th layer to the shape prior to the removal of the (k)th layer, should be first determined. Since the stress, S_1, and hence the correction term, S_{11}, for computing the curvature change due to the removal of the second layer were solved in equation (6) and (9),

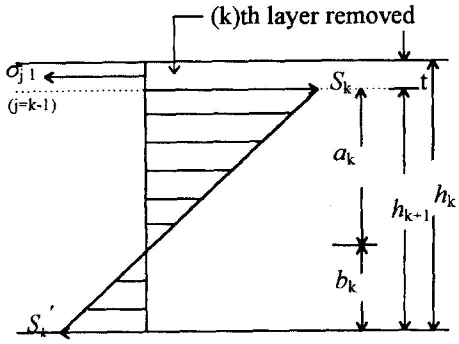

Fig.5 Location of neutral axis after subsequent layer removal

Suppose the stress, S_k for $2 \leq k \leq i$, is to be computed. In Figure 5, similar to equation (2), force equilibrium condition requires that

$$\sigma_{j1}t+\frac{a_k S_k}{2}+\frac{b_k S_k'}{2}=0, \qquad (j=k-1) \qquad (12)$$

where σ_{ji} is the redistributed residual stress in the (k)th layer prior to the removal of the (k)th layer, a_k is the height from the neutral axis to the upper surface of remaining body after the removal of (k)th layer, b_k is the height from the neutral axis to the bottom surface of the remaining body, S_k and S_k' are the stresses which would be created at the top and bottom surfaces of the remaining body after the removal of (k)th layer when the remaining body is forced to assume its shape prior to the removal of the (k)th layer, respectively.

The curvature change before and after the removal of the th layer can be related to the redistributed residual stress in the (k)th layer prior to the removal of the (k)th layer, σ_{ji}, as:

$$\frac{1}{R_k}-\frac{1}{R_{k+1}}=\frac{6th_k\sigma_{j1}}{E(h_{k+1})^3} \qquad (13)$$

where $j=k-1$.

Similar to equations (3), (4) and (5) we can have:

$$a_k+b_k=h_{k+1} \qquad (14)$$

$$(-S_k')=\frac{b_k S_k}{a_k} \qquad (15)$$

$$S_k=-a_k E\left(\frac{1}{R_k}-\frac{1}{R_{k+1}}\right) \qquad (16)$$

Again, utilizing equations (12) through (16) one can solve for the stress, S_k, as well as the distance from the upper surface to the neutral axis (a_k) of the remaining body. The result is :

$$S_k=-\frac{Eh_{k+1}(3h_k+h_{k+1})}{6h_k}\left(\frac{1}{R_k}-\frac{1}{R_{k+1}}\right) \qquad (17)$$

$$a_k=\frac{h_{k+1}(3h_k+h_{k+1})}{6h_k} \qquad (18)$$

The correction terms S_{ki} ($i=k, k+1, k+2, \ldots$) due to the removal of the (k)th layer are the stresses which would be created in the middle of each layer in the remaining body by bringing the remaining body back to its shape prior to removal of the (k)th layer (referring to Figure 3), which can be expressed as:

$$S_{ki}=\frac{S_k}{a_k}\{a_k-(i-k+0.5)t\} \qquad (19)$$

In order to compute the redistributed residual stress, σ_{i1}, the sum of the correction terms, S_{ki}, should be taken into account. The correction terms following removal of the 1st, 2nd 3rd and 4th layer will be S_{11}, $S_{12}+S_{22}$, $S_{13}+S_{23}+S_{33}$ and $S_{14}+S_{24}+S_{34}+S_{44}$, respectively Hence, the redistributed residual stress,

in equation (11), can be obtained by subtracting the sum of the correction terms (S_{1i}, S_{2i} S_{3i}, ..., S_{ii}) from the original residual stress in that layer ($(i+1)$th layer) in the original body. For the removal of the first layer, the relationship between the average residual stress in the first layer and the predicted curvature change is given by equation (1). For the removals of the second layer and all other subsequent layers, the correction terms should be taken into account for determining the redistributed residual stress in those layers. Thus, the redistributed residual stress in the uppermost layer of the remaining body after the removal of a layer can be written as follows:

$$\begin{aligned} \sigma_{11} &= \sigma_2 - S_{11} \\ \sigma_{21} &= \sigma_3 - (S_{12} + S_{22}) \\ \sigma_{31} &= \sigma_4 - (S_{13} + S_{23} + S_{33}) \\ \sigma_{41} &= \sigma_5 - (S_{14} + S_{24} + S_{34} + S_{44}) \end{aligned} \tag{20}$$

where σ_{i1} refers to the redistributed residual stress in the uppermost layer of the remaining body due to the removal of the first layer through (i) th layer, σ_{i+1} refers to original residual stress in the ($i+1$)th layer of original body. In general, equation (20) can be written as:

$$\sigma_{i1} = \sigma_{i+1} - \sum_{k=1}^{i} S_{ki} \qquad \text{for } i \geq 1 \tag{21}$$

Utilizing equation (11), the residual stress, σ_{i1}, can be related to the curvature change due to the removal of the corresponding layer by :

$$\frac{1}{R_{i+1}} - \frac{1}{R_{i+2}} = \frac{6th_{i+1}}{E(h_{i+2})^3}\left(\sigma_{i+1} - \sum_{k=1}^{i} S_{ki}\right) \qquad \text{for } i \geq 1 \tag{22}$$

Therefore, the subsequent curvature change following successive removal of layers can be predicted for a known original distribution of residual stress of a rectangular strip.

CONCLUSIONS

Equations (1) and (22) provide a convenient and simple - to - use means of determining the curvature change due to removal of layers of a residually stressed component. Conversely, if it is required to have a specific post - machining curvature, the above equations can be used to determine the required thickness to be removed in order for the remaining body to acquire the desired curvature. The developed equations can easily be computer - programed and used in real time, provided that the original residual stress state through thickness can be evaluated utilizing non - destructive technique such as neutron diffraction method. A potential use of the method is for manufacturing industries as the industries strive to meet the increasing demands for precisely - shaped components such as wafers, diaphragms miniature probes and optical devices with tight tolerances.

However, in reality, the curvature change due to layer - removal should include the effect of the stresses induced by the layer - removal process itself. Therefore, for more accurate results one should account for residual stresses caused by actual layer - removal process, specially when a component is layer - removed by conventional technique.

The variations in width and length of the strip due to Poisson' s effect were not considered in this paper, thus the method is limited to bending. Though, it is believed that the estimation of the variations is quite simple. Also, the change in thickness by Poisson' s effect of layers to be removed was neglected. Hence, future research should be directed towards analysis of cases in which the Poisson' s effect on the thickness change is not negligible.

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