지지단 보강재의 뒤틀림을 고려한 면내휨을 받는 탄성지지 보강판의 좌굴해석

Buckling Analysis of Stiffened Plates with Elastic Supports Subjected to In-Plane Bending Moment Considering Warping of End Stiffeners

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요 약

본 논문은 면내휨을 받는 2변 탄성지지 2변 단순지지 보강장방형판에 대해 유한요소법을 이용하고, 비보강 장방형판에 대해 고전적 해석법에 의해 좌굴해석한 것이다. 4변 단순지지, 2변단순지지 및 2변고정 장방형 판에 대해 기존해와 고전적 해석해 및 유한요소해를 비교하여 고전적 해석방법 및 유한요소법의 신뢰도를 입증하였다. 장방형 보강재의 뒤틀림 강성은 무시될 수 있으므로 탄성지지변의 보강재는 뒤틀림의 영향을 파악하기 위해 I형을 사용하였다. 탄성지지변을 갖는 장방형 판의 좌굴강도가 비틀림 강성 및 뒤틀림 강성에 따라 유한요소법 및 고전적 해석법에 의해 계산되고 비교되었다.

판의 지지변 사이에 보강재가 있는 경우 4변단순지지, 2변단순지지 및 2변고정 보강장방형판에 대해 유한요소해와 기존해를 비교하여 유한요소해의 정밀성을 입증하였다. 탄성지지변을 갖는 보강장방형판의 유한요소법에 의한 좌굴강도는 장방형판요소와 비틀림 및 뒤틀림을 고려한 보요소의 강성매트릭스를 조합하여 고유치 문제를 풂으로써 계산될 수 있다. 유한요소법에 의해 지지변 사이의 장방형 보강재 위치와 지지변 상의 I형 보강재의 비틀림(J) 및 뒤틀림 상수(I_W)에 따른 보강장방형판의 좌굴강도를 구하여 비교하고 효율적인 보강재의 위치를 결정하였다.

Abstract

The main objective of this paper is to analyze the rectangular stiffened plates with two opposite ends elastically restrained and the others simply supported subjected to in-plane bending by Finite Element Method. Another objective is to develope Classical Method analyzing the unstiffened rectangular plates with the above boundary conditions. In order to validate finite element and classical methods, the buckling strengths of the rectangular plates with four simply supported ends, and with two simply supported and the others fixed ends by finite element method and classical method are compared with those of references. In finite element method, elastically restrained ends can be obtained as considering torsional and warping rigidities of end stiffeners. The buckling strengths of the rectangular plates with elastically restrained ends by finite element and classical methods are calculated and compared with each other.

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In case of stiffened plates, to validate finite element method, the buckling strengths of the rectangular stiffened plates with four simply supported ends, and with two simply supported and the others fixed ends are also compared with those of references. The buckling strengths of the rectangular stiffened plates with elastically restrained ends by finite element method are calculated as solving eigenvalue problems which are obtained as assembling rectangular plate elements and beam elements considered torsional and warping rigidities. The buckling strengths of rectangular stiffened plates according to various positions of rectangular intermediate stiffener, J and I_W of end stiffeners are also obtained, which are compared to determine the efficient position of intermediate stiffener.

Keywords : stiffened plate, elastic support, buckling, warping, in-plane bending

1. Introduction

The buckling strength of plate is important in the fields of steel structures especially. In order to increase the buckling strength of plate, stiffeners can be located at the compressive side of plate. The following references and their citations include the govering equations, various methods of analysis and significant historical literature on the buckling strength of plate. The buckling of plate subjected to in-plane bending and compression is studied by S. P. Timoshenko[5]. Kazuo Terazawa and Yukio Ueda[3] used finite element method to predict the buckling strength of stiffened plate, subjected to uniformly in-plane force, with various boundary conditions. M. Uenoya[11] calculated the buckling strengths of webs with opening by finite element method. Y.S. Lee and K.D. Kim[2] studied the buckling of stiffened plate with four simply supported ends by finite element method. Tsutomu Usami[9] suggested the effective width of locally buckled plates in compression and bending. I.Z. Kim and Y.S. Lee[1] reported the buckling of opening plates with elastic supports by classical method and finite element method and herein considered the warping effect of stiffener.

When the critical stress for the web of H-section beam is decided, the web is considered as rectangular plate with two opposite ends elastically restrained and the others simply supported ends subjected to in-plane bending. In this paper, the above boundary conditions of elastically restrained are obtained as considering torsional and warping rigidities of end stiffeners. Therefore, the constant of elastically restrained is decided by torsional rigidity(GJ) and warping rigidity(EI_W) of end stiffener. In rectangular stiffener case, warping rigidity can be neglected[6]. In order to show warping effects of end stiffeners, they are assumed as I-sections. After the buckling strengths of the unstiffened rectangular plates with elastically restrained ends by classical method and finite element method are calculated, the buckling strengths of plates with four simply supported ends, and with two simply supported and the others fixed ends by these methods are compared with those of references[5),7]. When GJ of elastically restrained end is infinite, that can be considered as fixed end. Similarly, if GJ is zero, that end is simply supported. The buckling strengths of unstiffened rectangular plates according to J and I_W of end stiffeners are calculated. In case of stiffened plate, the buckling strengths by finite element method are calculated, the buckling strengths of stiffened plates with four simply supported ends, and with two simply supported and the others fixed ends are also compared with those of references[5),7)]. The stiffened rectangular plates with elastically restrained ends has three stiffeners; one rectangular intermediate stiffener and two end stiffeners considering warping effects. The buckling strengths of stiffened plates according to various positions of intermediate stiffener, J and I_W of end stiffeners are also obtained, which are compared to determine the efficient position of intermediate stiffener.

The analysis is based on the following assumptions:

1) Plates satisfy Kirchhoff's assumptions.

2) Plates are homogeneous, satisfy Hooke's law and do not have initial imperfection.

3) Axial direction of stiffeners is the same as that of in-plane forces.

4) Neutral axis of stiffeners is the same as that of plate.

2. Classical theory

The plate considered pure torsion and warping torsion of two end stiffeners can be analyzed by solving differential Eq.(2-1). The boundary conditions of elastically restrained ends are satisfied as considering various torsional and warping constants of two end stiffeners.

The coordinate system, load, and section of plate are shown in Figure 2-1.

Substituting appropriate boundary conditions in Eq.(2-1), the buckling strength of this plate can be obtained.

$$D(\partial^4 w/\partial x^4+2\partial^4 w/(\partial x^2\partial y^2)+\partial^4 w/\partial y^4) - N_x(\partial^2 w/\partial x^2) = 0 \qquad (2\text{-}1)$$

where w : displacement of z-direction,

D : flexural rigidity of plate,

N_x : in-plane force of x-direction acting simply supported ends.

Elastically restrained ends do not generate bending, but have torsional and warping rigidities. Therefore, boundary conditions are written as follows :

$$w = 0,\ M_x = -D(\partial^2 w/\partial x^2+\nu\partial^2 w/\partial y^2) = 0, \quad \text{at } x = 0, a \qquad (2\text{-}2)$$

$$w = 0,\ M_y \mp m = -D(\partial^2 w/\partial y^2+\nu\partial^2 w/\partial x^2) \mp m = 0,\ \phi = -\partial w/\partial y, \text{ at } y = \pm b/2 \qquad (2\text{-}3)$$

where $m = EI_w\frac{\partial^4\phi}{\partial x^4} - GJ\frac{\partial^2\phi}{\partial x^2}$; *uniformly torsional moment acting at the end stiffener,*

EI_w : warping rigidity,

GJ : torsional rigidity,

ν : poisson's ratio,

ϕ : torsional angle

Assuming that the displacement function satisfies the boundary conditions of Eq.(2-2), the solutions of Eq.(2-1) can be expressed as

$$w = \sin(n\pi p) * Y(q) \qquad (2\text{-}4)$$

where $p=x/a$, $q=y/b$, $0\le p\le 1$, $-0.5\le q\le 0.5$

Substituting Eq. (2-4) in Eq. (2-1) gives

$$Y^{IV} - 2\xi Y'' + \xi^2(1-\eta^2 q)\,Y = 0 \qquad (2\text{-}5)$$

where $-\frac{1}{2}\le q\le\frac{1}{2}$, $k=\frac{\sigma_{cr}}{\sigma_e}$, $\sigma_e=\frac{E\pi^2}{12(1-\nu^2)}(\frac{t}{b})^2$,

σ_{cr} ; *maxium bending stress* $\alpha=\frac{a}{b}$, t *; plate thickeness, n ; number of half waves for buckling,*

$$\xi = \left(\frac{n\pi b}{a}\right)^2,\ \eta = \frac{a}{n}\sqrt{2k}$$

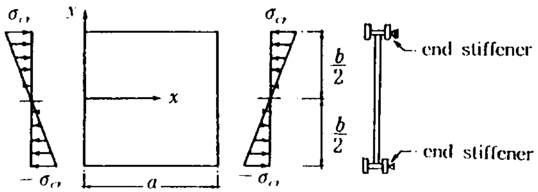


Fig. 2-1 (a) Coordinate system of plate
(b) Section of plate

Though Eq. (2-5) is a linearly homogeneous differential equation, coefficients of Y includes q and solutions Y have to be represented as

$$Y(q) = \sum_{m=0}^{\infty} c_{m+1}\, q^m$$

where c_{m+1} : coefficients m : integer (2-6)

Substituting Eq. (2-6) in Eq. (2-5) gives

$$\sum_{m=4} m(m-1)(m-2)(m-3)c_{m+1}\, q^{m-4} - 2\xi\sum_{m=2} m(m-1)c_{m+1}\, q^{m-2} + \xi^2\sum_{m=0} c_{m+1}\, q^{m-2} - \xi^2\eta^2\sum_{m=0} c_{m+1}\, q^{m+1} = 0 \qquad (2\text{-}7)$$

Eq. (2-5) is fourth order differential equation and has four basic solutions. Therefore, unknown coefficients c of Eq.(2-7) in matrix form can be expressed as

$$c_1 = R(1,1),\ c_2 = R(2,2),\ c_3 = R(3,3),\ c_4 = R(4,4),\ R(i,j) = 0 \quad \text{at } i \neq j \qquad (2\text{-}8)$$

Because the summation of coefficient of q^0 *has to be zero, an unknown coefficient c_5 becomes as follows:*

$$c_5 = -\xi^2\frac{c_1}{4\times3\times2\times1} + 2\xi\frac{c_3}{4\times3} = Q(5,1)R(1,1) + Q(5,3)R(3,3) = R(5.1) + R(5,3) \qquad (2\text{-}9)$$

The above equation can be rewritten in matrix form as

$$R(5,5) = \sum_{i=1}^{4} R(5,i)$$

where $R(5,2) = R(5,4) = 0, R(5,i) = Q(5,i)*R(i,i)$

In the same way, since the summations of coefficients c of q^1, q^2, …… have to be zero respectively, general equation of R matrix can be obtained as

$$R(i+5,j) = \xi^2\frac{\eta^2}{(i+4)(i+3)(i+2)(i+1)} R(i,j) - \frac{\xi^2}{(i+4)(i+3)(i+2)(i+1)} R(i+1,j) + 2\frac{\xi}{(i+4)(i+3)} R(i+3,j) \qquad (2\text{-}10)$$

where $i = 1,2,\cdots\cdots,\ j = 1,2,3,4$

$R(i+5,j)$ of Eq. (2-10) is included all coefficients of solution Y_j and can be expressed by $R(1,1)$, $R(2,2)$, $R(3,3)$ and $R(4,4)$, and the general solution $Y(q)$ is written as follows:

$$Y(q) = Y_1(q)R(1,1) + Y_2(q)R(2,2) + Y_3(q)R(3,3) + Y_4(q)R(4,4) = \sum_{j=1}^{4} R(j,j)Y_j(q) \qquad (2\text{-}11)$$

$Y_j(q)$ of Eq.(2-11) can be represented as

$$Y_j(q) = \sum_{p=0}^{\infty} Q(p+1,j)q^p \qquad (2\text{-}12)$$

$$\text{where} \quad Q(i,j) = \begin{cases} 1,\ i=j, & i,j=1,\cdots,4 \\ 0,\ i \neq j & i,j=1,\cdots,4 \end{cases}$$

Substituting Y(q) in the boundary conditions of Eq. (2-3), the result is as follows:

$$\begin{bmatrix} Y_1(1/2) & Y_2(1/2) & Y_3(1/2) & Y_4(1/2) \\ Y_1(-1/2) & Y_2(-1/2) & Y_3(-1/2) & Y_4(-1/2) \\ S_1 & S_2 & S_3 & S_4 \\ T_1 & T_2 & T_3 & T_4 \end{bmatrix} \begin{bmatrix} R(1,1) \\ R(2,2) \\ R(3,3) \\ R(4,4) \end{bmatrix} = 0 \quad (2\text{-}13)$$

where $S_i = Y_i^{'}(1/2) + \xi^2\gamma\frac{Y_i(1/2)}{b^3} + \xi\beta\frac{Y_i(1/2)}{b}$

$$T_i = Y_i^{'}(-1/2) - \xi^2\gamma\frac{Y_i^{'}(-1/2)}{b^3} - \xi\beta\frac{Y_i^{'}(-1/2)}{b}$$

$$\gamma = \frac{EI_w}{D}, \quad \beta = \frac{GJ}{D}$$

The stress σ_{cr} that satisfies Eq.(2-13) is the maximum in-plane stress of the plate with two opposite ends elastically restrained and the others simply supported.

3. Finite Element Method

The stiffened plate analyzed by this method has three stiffeners: one compressive intermediate stiffener and two end stiffeners with torsional and warping rigidities. The global stiffness matrix of the stiffened plate can be obtained from assembling global stiffness matrix of plate subjected to in-plane bending moment and those of stiffeners subjected to in-plane forces.

3.1 Stiffness matrix of plate element

Figure 3-1 shows global coordinate system of rectangular plate. Since shears and in-plane forces on the surfaces parallel to x-axis are not considered, τ_{xy} and σ_y equal to zero and σ_x equals to $(1\text{-}2y/b)\sigma_{cr}$, where σ_x is in-plane stress by in-plane bending moment: a and b are respectively lengths of global axes x and y directions for plate, and y varies zero to b. Considering the symmetry of the plate and load, one half of plate which is shaded in Figure 3-1 is analyzed. Figure 3-2 represents local coordinate system, displacement, and forces of rectangular plate element. The element variation of strain energy and virtual work of the plate for out of plane deflections[4),8),10)] are expressed as

$$\delta U_p = \{\delta^e\}[[A]^{-1T}\int[C]^T[D][C]dx\,dy\,[A]^{-1}]\,\delta\{\delta^e\} \quad (3\text{-}1)$$

$$\delta W_p = \{\delta^e\}[t[A]^{-1T}\int[G]^T[\sigma][G]dx\,dy\,[A]^{-1}]\,\delta\{\delta^e\} \quad (3\text{-}2)$$

where $\{\delta^e\}$: nodal displacement

$[A]$: matrix for relations of nodal displacement and coefficients matrix of displacement w

$[C]$: matrix for relations of curvature and coefficients matrix of displacement w

$[D]$: flexural rigidity of plate

t : thickness of plate

$[G]$: matrix for relations of slope of deflection θ and coefficients matrix of displacement w

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \quad (3\text{-}3)$$

Bending stiffness matrix and geometrical stiffness matrix are obtained from Eqs.(3-1) and (3-2).

$$[K_P^e] = [A]^{-1T}\int [C]^T[D][C]dx\,dy\,[A]^{-1}$$

$$[K_\sigma^e] = t[A]^{-1T}\int [G]^T[\sigma][G]dx\,dy\,[A]^{-1}$$

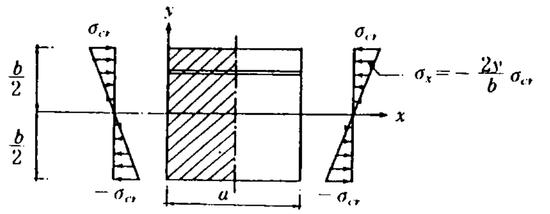
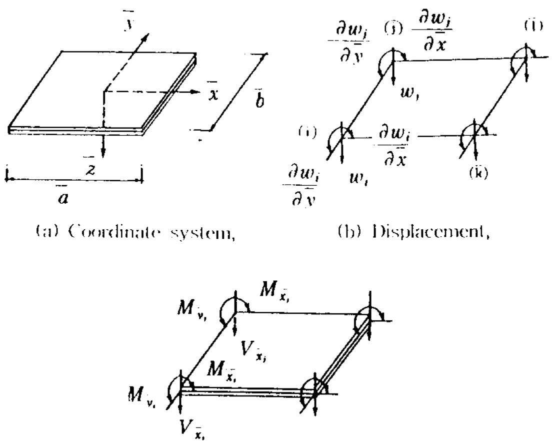


Fig. 3-1 Coordinate system of rectangular plate



(a) Coordinate system, (b) Displacement,

(c) Forces

Fig. 3-2 Stiffness matrix of stiffener element considering pure torsion and warping torsion

3.2 Stiffness matrix of stiffener element considering pure torsion and warping torsion

Assuming displacement function as third order polynomial, and substituting boundary conditions for displacement and slope of deflection in this polynomial, the displacement function can be represented as

$$w = [H_b]\{w_x\} \qquad (3\text{-}4)$$

where $[H_b] = [(2\xi^3-3\xi^2+1)(\xi^3\bar{a}-2\xi^2\bar{a}-\xi\bar{a})$
$(3\xi^2-2\xi^3)(\xi^3\bar{a}-\xi^2)] \qquad (3\text{-}5)$

$\{w_x\} = \{w_i, \dot{w}_i, w_j, \dot{w}_j\}^T \qquad (3\text{-}6)$

$\xi = \bar{x}/\bar{a}, \quad \dot{w}_i = \partial w/\partial\bar{x}$

$\bar{a}$, $\bar{b}$: lengths of plate element in the local axes x,y directions

Equilibrium equation of stiffener subjected to torsional moment at two ends is

$$EI_w\frac{\partial^4\phi}{\partial x^4} - GJ\frac{\partial^2\phi}{\partial x^2} = 0 \qquad (3\text{-}7)$$

where EI_W : Warping rigidity, GJ : torsioned rigidity, ϕ : torsional angle

Boundary conditions of stiffener element are as follows:

$$\phi = -\partial w/\partial\bar{y} = \phi_i,\ \partial\phi/\partial\bar{x} = \dot{\phi}_i \quad \text{at } \bar{x} = 0$$
$$\phi = -\partial w/\partial\bar{y} = \phi_j,\ \partial\phi/\partial\bar{x} = \dot{\phi}_j \quad \text{at } \bar{x} = \bar{a} \qquad (3\text{-}8)$$

where $\bar{a}$; length of stiffener element

Substituting Eq. (3-8) in general solution of Eq. (3-7), torsional displacement function becomes as

$$\phi = [H_t]\{\phi_x\} \qquad (3\text{-}9)$$

where $[H_t] = [f_1(\bar{x}), f_2(\bar{x}), f_3(\bar{x}), f_4(\bar{x})]$

$\{\phi_x\} = [\phi_i, \dot{\phi}_i, \phi_j, \dot{\phi}_j]^T$

$[H_t]$: shape function for torsion

$\{\phi_x\}$: nodal displacement by torsion

$\alpha = \sqrt{GJ/EI_W}$

$f_1(\bar{x}) = (1-B_1/A_1)+\alpha(B_5/A_1)\bar{x}+(B_1/A_1)\mathrm{Cosh}\alpha\bar{x}+(B_6/A_1)\mathrm{Sinh}\alpha\bar{x}$

$f_2(\bar{x}) = -(B_1/A_1)+(B_1/A_1)\bar{x}+(B_3/A_1)\mathrm{Cosh}\alpha\bar{x}+(B_7/A_2)\mathrm{Sinh}\alpha\bar{x}$

$f_3(\bar{x}) = (B_1/A_1)-\alpha(B_5/A_1)\bar{x}-(B_1/A_1)\mathrm{Cosh}\alpha\bar{x}+(B_5/A_1)\mathrm{Sinh}\alpha\bar{x}$

$$f_4(\bar{x}) = -(B_4/A_1) - \alpha(B_8/A_2)\bar{x} + (B_4/A_1)\cosh\alpha\bar{x} + (B_8/A_2)\sinh\alpha\bar{x}$$

$$A_1 = \alpha(\cosh\alpha l - 1)^2 - (\sinh\alpha l - \alpha l)\sinh\alpha \ell$$

$$A_2 = -A_1$$

$$B_1 = \alpha(1 - \cosh\alpha l)$$

$$B_2 = -B_1$$

$$B_3 = (\sinh\alpha l - \alpha l\cosh\alpha l)$$

$$B_4 = \alpha l - \sinh\alpha l$$

$$B_6 = -B_5$$

$$B_7 = -\alpha l\sinh\alpha l - 1 + \cosh\alpha l$$

$$B_8 = 1 - \cosh\alpha l$$

Bending strain energy for stiffener element can be obtained as

$$U_{sb} = \frac{1}{2}\int \{M_b\}^T\{\chi_b\}d\bar{x} \qquad (3\text{-}10)$$

where $\{M_b\} = EI\{\chi_b\}$

$\{M_b\}$: bending moment vector of stiffener element

EI : flexural rigidity of stiffener element

$\{\chi_b\}$: curvature of stiffener element

Curvature $\{\chi_b\}$ is expressed as follows:

$$\{\chi_b\} = (-\partial^2 w/\partial\bar{x}^2) = -[H_b''][w_x]$$

Substituting $\{\chi_b\}$ in Eq. (3-10) gives

$$U_{sb} = \frac{1}{2}\{w_x\}^T[EI\int [H_b'']^T[H_b'']d\bar{x}\ ]\{w_x\} \qquad (3\text{-}11)$$

Bending stiffness matrix $[K_{sb}^e]$ for the stiffener element is obtained by the variation of U_{sb}.

$$[K_{sb}^e] = EI\int [H_b'']^T[H_b'']d\bar{x}$$

Torsional strain energy is divided into two parts as follows:

$$U_{st} = U_{sv} + U_{sw} \qquad (3\text{-}12)$$

where U_{sv} and U_{sw} are strain energy of pure torsion and warping torsion, respectively.

Figure 3-3 shows warping of I-section that warping is restrained, where ϕ, h is torsional angle and depth of I-section, respectively.

Strain energy for pure torsion and warping torsion are expressed, respectively, as follows:

$$U_{sv} = \frac{1}{2}\{\phi_x\}^T[\int GJ[H_t']^T[H_t']d\bar{x}\ ]\{\phi_x\} \qquad (3\text{-}13)$$

$$U_{sw} = \frac{1}{2}\int EI_f(\frac{\partial^2\bar{y}}{\partial\bar{x}^2})^2 d\bar{x} \qquad (3\text{-}14)$$

From Eq.(3-13), the stiffness matrix $[K_{sv}^e]$ for pure torsion can be obtained as

$$[K_{sv}^e] = \int GJ[H_t']^T[H_t']d\bar{x}$$

Assuming stiffener as I-section, I_f becomes moment of inertia of flange for y-axis. Substituting $y = h*\phi/2$ in Eq.(3-14), the strain energy for warping torsion becomes as

$$U_{sw} = \frac{1}{2}EI_w\int\{\frac{\partial^2\phi}{\partial\bar{x}^2}\}^T\{\frac{\partial^2\phi}{\partial\bar{x}^2}\}d\bar{x} \qquad (3\text{-}15)$$

where $I_w = \frac{I_f}{2}h^2$

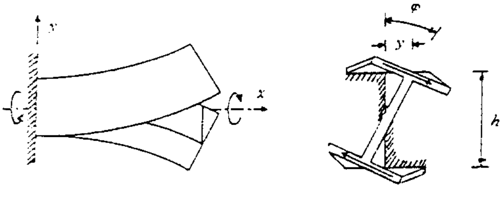


Fig. 3-3 Warping of I-section that warping is restrained

Substituting Eq.(3-9) in Eq.(3-15), Eq.(3-15) is rewritten as

$$U_{sw} = \frac{1}{2}\{\phi_x\}^T[EI_w\int[H_t'']^T[H_t'']d\bar{x}\]\{\phi_x\} \qquad (3\text{-}16)$$

Therefore, the stiffness matrix for warping torsion is obtained as

$$[K_{sw}^e] = EI_w\int[H_t'']^T[H_t'']d\bar{x}$$

Geometrical stiffness matrix for stiffener element can be obtained by work of the external forces. If displacement $\triangle_b$ is generated by external forces to act to stiffener element, work of external forces is expressed as follows:

$$V = \int_A \Delta_b\,\sigma\,dA \qquad (3\text{-}17)$$

where Δ_b ; displacement of stiffener element
A ; sectional area of stiffener,
σ ; stress

In Fig. 3-4, small length ds after deformation is

$$ds = (d\bar{v}^2+d\bar{w}^2+d\bar{x}^2)^{\frac{1}{2}} \approx [\frac{1}{2}(\frac{\partial\bar{v}}{\partial\bar{x}})^2 + \frac{1}{2}(\frac{\partial\bar{w}}{\partial\bar{x}})^2+1]d\bar{x} \qquad (3\text{-}18)$$

Displacement $\triangle_b$ is the difference between before and after deformations of the total length.

$$\Delta_b = s-\bar{a} = \frac{1}{2}\int[(\frac{\partial\bar{v}}{\partial\bar{x}})^2+(\frac{\partial\bar{w}}{\partial\bar{x}})^2]d\bar{x} \qquad (3\text{-}19)$$

In Figure 3-4, considering movement and rotation of line element,

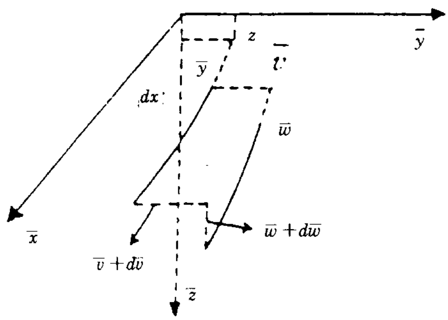


Fig. 3-4 Lateral deformation of stiffener element

$$v = \bar{v} - \phi\bar{z}, \quad w=\bar{w}+\phi\bar{y} \qquad (3\text{-}20)$$

where $\bar{w}, \bar{v}$; translations of shear center
$-\phi\bar{z}, -\phi\bar{y}$; translations due to the rotation of the fiber about the shear center

Substituting Eq.(3-20) in Eq.(3-19) and then substituting this result in Eq.(3-17), the work of the external forces is rewritten as

$$V = \frac{1}{2}\int_0^{\bar{a}}\int_A\sigma[(\frac{\partial v}{\partial\bar{x}})^2+(\frac{\partial w}{\partial\bar{x}})^2+(\bar{y}^2+\bar{z}^2)(\frac{\partial\phi}{\partial\bar{x}})^2 -2\bar{z}(\frac{\partial v}{\partial\bar{x}})(\frac{\partial\phi}{\partial\bar{x}})+2y(\frac{\partial w}{\partial\bar{x}})(\frac{\partial\phi}{\partial\bar{x}})]dAd\bar{x}$$

Neglecting lateral displacement and substituting Eqs.(3-4) and (3-9) in the above formula, the work of the external forces becomes as

$$V = \frac{1}{2}\{w_x\}^T[\sigma A\int[H_b']^T[H_b']d\bar{x}\]\{w_x\} + \frac{1}{2}\{\phi_x\}^T[\sigma I_p\int[H_t']^T[H_t']d\bar{x}\]\{\phi_x\} + \{w_x\}^T[\sigma s_z\int[H_b']^T[H_t']d\bar{x}\]\{\phi_x\} \qquad (3\text{-}21)$$

where $I_p = \int(\bar{y}^2+\bar{z}^2)dA, \quad s_z = \int\bar{y}\,dA$

From Eq.(3-21), the geometrical stiffness matrices for bending and torsion are expressed

$$[K^e_{s\sigma1}] = \sigma A\int [H_b']^T[H_b']d\overline{x}$$
$$[K^e_{s\sigma2}] = \sigma I_p\int [H_t']^T[H_t']d\overline{x}$$
$$[K^e_{s\sigma3}] = \sigma S_z\int [H_b']^T[H_t']d\overline{x}$$

where $[K^e_{s\sigma3}]$ is null matrix if the stiffeners are assumed to be symmetrical relative to the plate middle plane

Assembling the stiffness matrices of Eqs. (3-11), (3-13), (3-16) and (3-21) gives the element stiffness matrices of Eqs.(3-22) and (3-23).

$$[K^e_s] = \begin{bmatrix} [K^e_{sb}] & 0 \\ 0 & [K^e_{sv}]+[K^e_{sw}] \end{bmatrix} \tag{3-22}$$

$$[K^e_{s\sigma}] = \begin{bmatrix} [K^e_{s\sigma1}] & [K^e_{s\sigma3}]^T \\ [K^e_{s\sigma3}] & [K^e_{s\sigma2}] \end{bmatrix} \tag{3-23}$$

3.3 Eigenvalue equation

Considering warping torsion of the stiffener, nodal displacement for torsion can be expressed by torsional angle $\phi = -\partial w/\partial y$ and $\partial\phi/\partial x$.

The number of degree of freedom per node becomes four.

$$\{\delta^e\} = \{w, -\partial w/\partial y, \partial w/\partial x, \partial\phi/\partial x\}^T \tag{3-24}$$

Eigenvalue equation for stiffened plate is obtained from assembling stiffness matrices of plate elements and those of stiffener elements.

$$[([K_p]+[K_s])+([K_{\sigma p}]+[K_{\sigma s}])]\{\delta\} = 0 \tag{3-25}$$

where $[K_p]$: bending stiffness matrix for plate

$[K_s]$: bending and torsional stiffness matrix for stiffeners

$[K_{\sigma p}]$: geometrical stiffness matrix for plate

$[K_{\sigma s}]$: geometrical stiffness matrix for stiffeners

4. Numerical analysis

4.1 Conditions of application

Two types of plates(A-type, Fig. 4-1, and B-type, Fig. 4-2) subjected to in-plane moment at the simply supported ends are analyzed. The other two ends are assumed elastically restrained. The boundary conditions of elastically restrained ends are obtained by means of torsional and warping rigidities of I-section stiffeners. Considering the symmetry of the plate and load, one half of plate, which is shaded in Figures 4-1 and 4-2, is analyzed. Finite element mesh is 12 x 5. Though these plates almost buckled by one or two half waves, the solutions for two half waves are only obtained in this analysis.

Section properties of I-section stiffeners are shown in Table 4-1, and all the informations for two types of plates can be found in Figures 4-1 and 4-2. In order to validate finite element and classical methods, the buckling strengths of the rectangular plates(A-type) with four simply supported ends, and with two simply supported and the others fixed ends are compared with those of references[5,7]. The buckling strengths of the rectangular stiffened plates(B-type) with the above boundary conditions are also compared with those of references[5,7]. To check the effects on J and I_W of end stiffener and to determine the efficient position of rectangular intermediate stiffener, it is assumed that J value of the end stiffener is varied from 0.0001 to 0.1 and I_w, 0.1J to 1000.0J, and the position of intermediate stiffener from the center line of stiffened plate is located from b/12 to 5b/12.

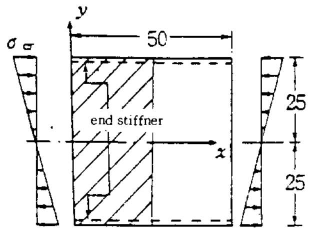


Fig. 4-1 A-type plates
(without intermediate stiffener)

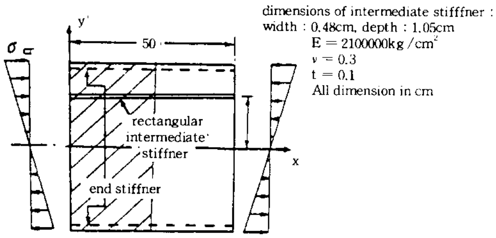


Fig. 4-2 B-type plates
(with intermediate stiffener)

Table 4-1 Section properties of I-section stiffeners(two end stiffeners)

$J(cm^4)$	$\alpha = I_w / J$	I	I_p
10^{-4}	0.1	0.00157	0.00187
	1.0	0.00646	0.00744
	10.0	0.02557	0.02901
	100.0	0.09825	0.11068
	1000.0	0.37475	0.42068
10^{-3}	0.1	0.00752	0.00925
	1.0	0.03212	0.03749
	10.0	0.12979	0.14815
	100.0	0.50385	0.56923
	1000.0	1.93006	2.16969
10^{-2}	0.1	0.03593	0.00463
	1.0	0.15697	0.18703
	10.0	0.65053	0.74921
	100.0	2.56792	2.91324
	1000.0	9.91543	11.6916
10^{-1}	0.1	0.17440	0.23929
	1.0	0.75364	0.92673
	10.0	3.21891	3.75744
	100.0	12.99860	14.8363
	1000.0	50.61871	57.1851

4.2 Validation of finite element and classical solutions

Table 4-2 shows the buckling strengths of A-type plates with four simply supported ends, and two simply supported and the others fixed ends. It can be seen that the solutions by classical and finite element methods are close to those of handbook[7] and Timoshenko[5]. In case of the rectangular plate with four simply supported ends, finite element solution in comparison with Timoshenko's solution has the error of 0.69 %. Because Timoshenko's solutions are obtained by solving three simultaneous equations, they do not have about one-third of 1% in comparison with the solutions of four simultaneous equations.[5]

Table 4-2 Comparison of buckling strengths of A-type plates(two half-waves, unit ; kg/cm^2,(   ) : error(%))

a/b=1	Classical method	F. E. M	Hand-book[7]	Timosh-enko[5]
four simply supported ends	193.8113 (0.11,-0.28)	193.0091 (-0.30,-0.69)	193.5960	194.3552
two simply supported and the others fixed ends	301.1885 (-0.07,-0.07)	299.6834 (-0.57,-0.57)	301.4024	301.4024

Table 4-3 Variation of buckling strengths of A-type plate according to the number of finite element mesh (two half-waves, $J=10^{-4}cm^4$, $I_w=10^{-4}cm^6$)

No. of element	6×3	8×4	10×5	12×5	Classical method
$\sigma_{cr}(kg/cm^2)$	182.1781	188.5852	193.2947	195.0981	195.8093
error(%)	6.96	3.69	1.28	0.36	

Table 4-4 Comparison of buckling strengths for B-type plates(two half waves, unit ; kg/cm^2)

a/b=1	four simply supported ends			a/b=0.8	two simply supported and two fixed ends		
$I(cm^4)$	F. E. M.	Handbook [7]	error(%)	$I(cm^4)$	F. E. M.	Handbook [7]	error(%)
I=0.0046	228.3553	234.5928	−2.66	I=0.0046	351.2211	346.1952	1.45
I=0.023	264.6847	265.1126	−0.16	I=0.0092	363.0844	366.6936	−0.98

Table 4-3 shows a variation of buckling strengths of A-type plate according to the number of finite element mesh. In case of 10×5 and 12×5 mesh, the errors of finite element solutions in comparison with the classical ones are 1.28 % and 0.36 %, respectively. If the finite element solutions are calculated by use of more mesh, more accurate solutions can be obtained. But the finite element solutions are calculated by use of 12×5 mesh because of capacity of personal computer(Pentium75).

It is assumed that an rectangular intermediate stiffener lies on the range of b/12 to 5b/12 from the center line of stiffened plate. Boundary conditions are assumed as four simply supported ends, and two simply supported and two fixed ends. The results of finite element analysis are shown in Table 4-4 with those of handbook[7]. In case of four simply supported, the error of finite element solution in comparison with handbook's solution is about 2.66 %.

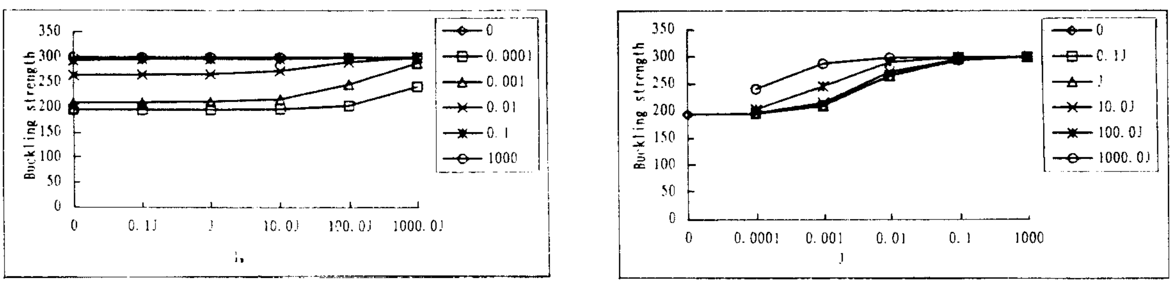
Table 4-5 Buckling strengths of A-type plates(two half waves)( ) ; classical method, { } ; error(%), unit ; kg/cm^2

I_w \ J	0	0.0001	0.001	0.01	0.1	∞
0	193.0091 (193.8113) {-0.41}	194.8733 (195.7319) {-0.44}	208.6712 (210.4367) {-0.84}	264.2975 (263.8220) {0.18}	293.1742 (295.8218) {-0.89}	
0.1J		195.0936 (195.7396) {-0.39}	209.5272 (210.4949) {-0.46}	265.2202 (263.9229) {0.49}	295.7549 (295.8426) {-0.03}	
J		195.0981 (195.8093) {-0.36}	210.9486 (211.0133) {-0.03}	265.9909 (264.8066) {0.45}	296.0646 (296.0235) {0.01}	299.6834 (301.1885) {-0.50}
10.0J		195.9486 (196.5014) {-0.28}	215.6057 (215.9000) {-0.14}	272.8255 (271.7876) {0.38}	295.8035 (297.3302) {-0.51}	
100.0J		203.1400 (202.9705) {-0.09}	245.9694 (246.1046) {-0.05}	290.3710 (291.1397) {-0.26}	299.0245 (300.0958) {-0.36}	
1000.0J		240.9014 (240.8730) {-0.36}	287.9070 (289.2175) {-0.45}	298.6527 (299.8647) {-0.40}	299.6516 (301.0522) {-0.47}	

4.3 Parametric study for A-type plates

Table 4-5 shows the buckling strengths of A-type plates with two opposite ends elastically restrained and the others simply supported subjected to in-plane bending by finite element method and classical method. It can be seen that two kinds of methods agree with each other well with maximum error 0.89%.

Figure 4-3(a) represents the buckling strength of A-type plate according to I_w with constant J. This figure shows that, the buckling strength increases rapidly, in case that J equals to 0.001 and I_w varies from 10.0 J to 1000.0 J, and nearly constant for all I_w in case that J is more than 0.1. It means that the boundary conditions of elastically restrained end which J is more than 0.1 are close to those of fixed end, and buckling strength is not concerned with I_w.



(a) J=constant (b) Iw=constant

Fig. 4-3 Buckling strengths of A-type plates according to J and I_W by finite element method.

The buckling strength of A-type plate according to J with constant I_w is expressed in Figure 4-3(b). This figure represents that, in case that J is less than 0.0001 or greater than 0.1, the buckling strength is nearly constant.

4.4 Parametric study for B-type plates

The buckling strengths of B-type plates are given in Table 4-6. This table and Figure 4-4 show that the most efficient position of rectangular intermediate stiffener is b/4 to b/3 from the center line of B-type plate. Figure 4-5 represents the buckling strength of B-type plate according to I_w, J and the position of intermediate stiffener. This figure shows that, I_w is almost no effects on the buckling strength in case that J is more than 0.01, or J is less than 0.001 and the position of intermediate stiffener from the center line of the plate is more than 4b/12.

Table 4-6 Buckling strengths in kg/cm^2 of B-type plates for J and I_w(two half waves)

J	I_w \ y	5b/12	4b/12	3b/12	2b/12	b/12
0.0001	0	482.3510	1075.3850	771.2876	479.0846	390.0647
	0.00001	506.1968	1080.3960	789.4000	487.8877	396.2182
	0.0001	506.2082	1080.3260	798.7912	491.8037	396.9440
	0.001	506.5230	1064.0620	857.9727	497.2795	399.2691
	0.01	508.2275	1066.0140	981.7842	576.6351	426.8192
	0.1	509.9763	1070.6490	1012.8120	659.4620	474.7161
0.001	0	482.5043	1075.5870	784.8292	486.0938	394.2361
	0.00001	507.0880	1031.5180	903.1176	544.1195	428.1510
	0.0001	507.2943	1064.1130	898.2346	548.4091	430.0286
	0.001	508.9343	1065.2070	996.9371	588.8175	451.5993
	0.01	510.5148	1070.8640	1012.6690	662.5528	473.6325
	0.1	512.5112	1073.3620	995.1342	660.6133	480.7160
0.01	0	504.0640	1063.8906	972.4200	645.4243	466.5533
	0.00001	509.7293	1069.249	976.3553	653.1417	468.9156
	0.0001	510.1506	1070.3720	984.3939	647.0693	471.4009
	0.001	511.4722	1041.4320	994.4908	660.9146	479.1650
	0.01	512.4894	1074.1060	995.1163	660.4901	480.6761
	0.1	513.4147	1074.9350	1011.9650	659.5884	480.0262
0.1	0	494.9645	1038.0170	994.2623	662.1094	479.4953
	0.00001	510.7407	1074.3630	995.0178	662.7393	479.8447
	0.0001	512.8967	1074.4690	995.1148	663.2925	480.1256
	0.001	510.8871	1074.6900	995.0091	659.9181	480.2752
	0.01	511.6270	1074.9420	1011.9550	659.5753	480.0169
	0.1	511.3976	1030.4470	994.8279	659.4791	479.9487

y : distance of position of intermediate stiffener from the center line of plate

5. Conclusions

The plate with two opposite ends elastically

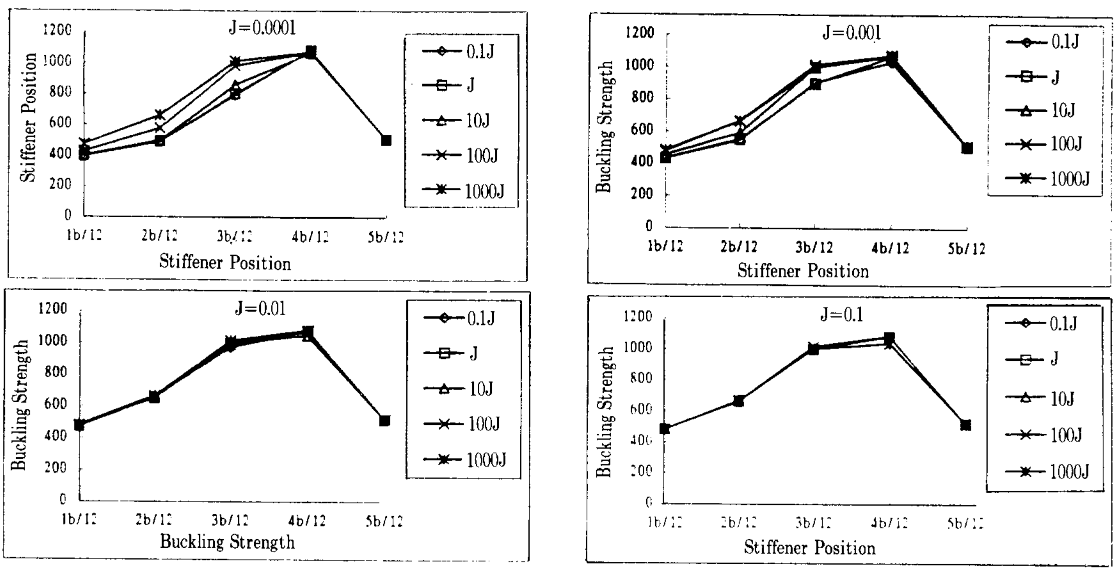


Fig. 4-4 Variation of buckling strengths of B-type plates according to position of intermediate stiffener

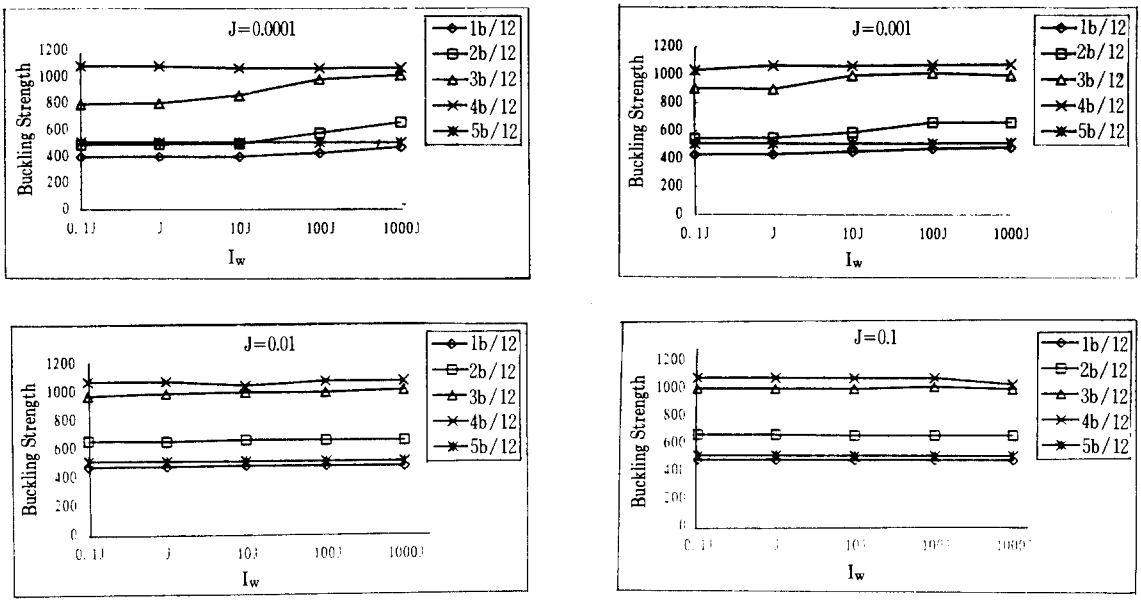


Fig. 4-5 Variation of buckling strengths of B-type plates according to J and I_w(J ; constant)

restrained and the other simply supported ends is analyzed by classical method and finite element method. The buckling strengths for the in-plane moment at the simply supported ends are obtained by the above two kinds of methods. The buckling strengths of the plates with four simply supported ends, and with two simply supported and two fixed ends are compared with those of Timoshenko and handbook. The solutions of stiffened plates with one intermediate rectangular stiffener on the center line of plate and the same boundary conditions as unstiffened plates are also compared with those of handbook. From the buckling strengths of stiffened plates, the efficient position of the intermediate stiffener is determined.

The conclusions of this analysis are as follows:

1) In the case of A-type plates for $J=10^{-4}$ (cm^4) and $I_w=10^{-4}$(cm^6), the buckling strength by the finite element method approaches that of the classical theory as the number of finite elements increases. With 12×5 elements, the error is about 0.36%.(Table 4-2)

2) In the case of A-type plate, the maximum error of the finite element solutions in comparison with classical solution is 0.89 %.

3) In the case of B-type plates, the most efficient position of intermediate stiffener is b/4 to b/3 from center line of the plate.

4) I_w is almost no effects on the buckling strength of B-type plate in case that J is more than 0.01, or J is less than 0.01 and the position of intermediate stiffener is more than 4b/12.

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