LHS기법을 이용한 불연속암반구조물의 확률유한요소해석기법개발

Development of Stochastic Finite Element Model for Underground Structure with Discontinuous Rock Mass Using Latin Hypercube Sampling Technique

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요 지

본 연구에서는 지하암반구조물의 구조해석시 불연속암반체의 물성변이를 고려할 수 있는 확률론적 해석기법을 개발하였다. 수치해석적 접근은 몬테칼로모사기법의 단점을 보완한 LHS기법을 사용하였고, 불연속면의 영향은 단층,벽개 등과 같이 불연속성이 뚜렷한 지역에서 적용성이 높은 절리유한요소모델을 사용하였다. 재료특성에 대한 확률변수는 불연속면의 수직강성과 전단강성을 다확률변수로 사용하였으며, 이들은 확률공간에서 정규분포를 갖는 경우에 대하여 고려하였다. 본 연구에서 개발된 수치해석프로그램은 검증예제를 통하여 타당성을 확인하였으며, 가상의 불연속면군이 존재하는 지하원형공동에 대한 해석을 통하여 프로그램의 적용성을 확인하였다.

Abstract

A stochastic finite element model which reflects both the effect of discontinuities and the uncertainty of material properties in underground rock mass has been developed. Latin Hypercube Sampling technique has been mobilized and compared with the Monte Carlo simulation method. To consider the effect of discontinuities, the joint finite element model, which is known to be suitable to explain faults, cleavage, things of that nature, has been used in this study. To reflect the uncertainty of material properties, multi-random variables are assumed as the joint normal stiffness and the joint shear stiffness, which could be simulated in terms of normal distribution. The developed computer program in this study has been verified by practical example and has been applied to analyze the circular cavern with discontinuous rock mass

Keywords : *stochastic finite element, latin hypercube sampling technique, underground structures, discontinuous rock mass, joint finite element model, multi-random variables*

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1. INTRODUCTION

Underground structures are widely used to overcome space shortages of aboveground structures, to secure a smooth traffic flow or to accommodate important facilities requiring special isolation.

With a growing usage of underground structures, there have been increased much concerns about the safety assessment for underground structures with discontinuous rock mass. From an engineering standpoint, the rock mass differs from general structural materials for following two points ; one, in the rock mass, there always exist discontinuities such as microfissure, joint, fracture, faults, and two, the same rock type can have a variety of physical properties. Recent numerical methods for the safety assessment of underground structure are classified into three categories ; DEM (distinct element method), BEM (boundary element method), and FEM (finite element method).

The accuracy of these methods by and large depends on the rationality of input data. However, since the design work for underground structures is generally carried out before the stage of excavation, much uncertainties are involved in physical properties for the design. Since various in-situ and laboratory tests to be performed at the site investigation are costly, moreover, it seems to be very difficult to get sufficient data for the exact stress analysis. Thus, most of numerical analyses for underground structures have not been reflected such uncertainties in physical property of rock mass, and have been performed on the assumption that all the characteristic values of rock mass at the site are constant. As such analyses do not reflect uncertainties in physical properties of rock mass, they are not simulated in real situations, that lead to the overdesign or require drastic changes of design during the construction work which result in economic losses. In order to realize more rational analysis for underground structures, therefore, it is desirable to use the stochastic model which can consider uncertainties of the physical properties in discontinuous rock mass.

The study related to the stochastic approach for uncertain structures in progress up to now has been limited to only aboveground structures. In addition, the perturbation theory or the MCS (Monte Carlo Simulation) technique applied to the stochastic finite element analysis field so far requires enormous number of calculation or complex solution procedure in producing more accurate results.

The LHS (Latin hypercube sampling) method[1], which is a sort of variance reduction technique capable of maintaining the degree of accuracy while the calculation amount is greatly reduced by preserving strong points and compensating weak points of MCS technique, has been widely used to the analysis of neutron transport in the nuclear engineering field since it was presented by the Sandia National Research Insititute of the United States in 1980.

In this study, a stochastic finite element model that reflects both the effect of discontinuities and uncertainty of physical properties of rock mass has been developed. LHS technique to make up weak points of the MCS method has been applied. Concerning discontinuities in rock mass, there has been used the joint finite element model[2] for the analysis which is known to be superior in explaining faults, cleavage, things of that nature. To reflect uncertain material properties in rock mass in this study, there are introduced two random variables such as the joint shear stiffness and the joint normal stiffness, which could be assumed in a normal distribution. New computer program has been coded and verified through the analysis of behavioral examples with discontinuous rock mass, and its practical applicability has been confirmed by using the stochastic finite element technique for the analysis of a circular rock cavern with two hypothesized discontinuities.

2. STOCHASTIC FINITE ELEMENT ANALYSIS

2.1 Generation of random deviates from distribution

In this study, the power residue method[3] has been adopted to generate pseudo-random numbers denoted by $\{u_n\}$, n=0, 1, 2, ⋯. If cumulative distribution function(CDF) of a random variable X is $F_X(x)$ and the cumulative probability of X is given in the form of $F_X(x)=z$, the set of random deviates x defining the distribution of X is represented as the following equation using the inverse method[4].

$$x_i = F_X^{-1}(z_i),\ i = 1, 2, 3, \cdots, n \qquad (1)$$

As pseudo-random numbers are in the form of uniform distribution on the interval (0,1), it becomes $z_i = u_i$ when the CDF of pseudo-random numbers and that of desired distribution homologize 1:1.

When pseudo-random numbers are not used in sufficiently great size, they may be biased partially between 0 and 1, and are potentially unable to represent the desired distribution. If the desired distribution is discretized into m nonoverlapping intervals of equal probability and one value from each interval is selected at random, more optimal sampling can be ensured. In order to generate distributions of multi-variables, this procedure is repeated for each variable, each time working with the corresponding CDF. The next step involves pairing the selected values. If two variables$(X_1,\ X_2)$ are sampled independently and paired randomly, the sample correlation coefficient of the two random deviates$(S_{x1},\ S_{x2})$ of each pair shall be considered the effect by the sampling fluctuations. In this study, the following correlation factor used widely in the LHS technique is applied[5].

$$\rho_{12} = \frac{\sum_{i=1}^{n}\left(S_{x_1^i} - \frac{n+1}{2}\right)\left(S_{x_2^i} - \frac{n+1}{2}\right)}{\left\{\sum_{i=1}^{n}\left(S_{x_1^i} - \frac{n+1}{2}\right)^2 \sum_{i=1}^{n}\left(S_{x_2^i} - \frac{n+1}{2}\right)^2\right\}^{\frac{1}{2}}} \qquad (2)$$

where, S_{x1} indicates the random number obtained in i intervals of a variable X_1, and S_{x2} indicates the random number obtained in i intervals of a variable X_2. If ρ_{12} is larger, statistical correlation between two random deviates is higher. So, in LHS technique, when this value is less than 1, a pair of random numbers is considered to be relatively reasonable[1].

2.2 Spatial variations of material properties

The spatial variation of material properties is assumed to be a 2D homogeneous stochastic process in this study. The fluctuating component $g(x)$ of a material property is then assumed to have mean zero $E[g(x)]=0$ and the autocorrelation function $R_i(\xi) = E(g\ (x)_i \cdot$

$g(x+\xi)_j$, where $x=\{x,\ y\}^T$ indicates the position vector, j denotes the random variable number, and $\xi=[\xi_x\ \xi_y]^T$ means the separation vector between two points x and $x+\xi$. If the randomness of the spatial variation is isotropic, the autocorrelation function of the spatial variation is supposed to be a function only of the distance $|\xi|$. The following form of an isotropic autocorrelation function is considered for this study ;

$$R_j(\xi) = \sigma_j^2 \ \mathrm{EXP}\left[-\left(\frac{|\xi|}{d}\right)^2\right] \tag{3}$$

where, σ_j is the standard deviation of random variable j and d can be defined the scale of correlation such that the larger it is, the more slowly the correlation disappears[6]. If the discontinuity is discretized by m joint elements, the fluctuating component $g_i(x)$ is composed of m material property values associated with these m element correlation each other. Their correlational characteristics can be specified in terms of the covariance matrix $Cov_j[g_X, g_Y]_{i,k} = E_j[g_X, g_Y]_{i,k} = R_j(\xi_{X,Y})_{i,k}$, where the subscript j indicates the random variable number and $\xi_{X,Y}$ is the separation between the centroids of elements i and k. Therefore, the final distribution of random variable can be obtained by multiplying equation (1).

$$G_j(x,\ Cov)_i = Cov_j[g_X,\ g_Y]_i \cdot [x_u]_j \tag{4}$$

2. 3 Finite element analysis

In this study, a constant strain 4 node joint element suggesetd by Goodman[2] was used for the stochastic finite element analysis of underground structure. It has a uniform thickness simulating the irregular and variable region between joint walls. Beginning from the potential energy formulation and minimizing with respect to nodal displacements, the element stiffness for the four-node joint element can be derived. In the derivation of the stiffness matrix, it is assumed that the joint element has length ℓ and each of two pairs of nodes, $(I,\ L)$ and $(J,\ K)$, which can reflect the effect of joint thickness by an iterative solution procedure[2], initially have identical coordinates.

The relationship between displacements at nodes of the joint element and the corresponding increments of external forces can be expressed as follows:

$$\{\Delta F\} = [K_j]\ \{u\} \tag{5}$$

where, $\{\Delta F\} = \{\Delta F_{s,I}\ \Delta F_{n,I}\ \Delta F_{s,I}\ \Delta F_{n,I}\ \Delta F_{s,K}\ \Delta F_{n,K}\ \Delta F_{s,L}\ \Delta F_{s,L}\}^T$ (5a)

$$[K_j] = \frac{\ell}{4}\begin{bmatrix} k_s & 0 & k_s & 0 & -k_s & 0 & -k_s & 0 \\ 0 & 2k_n & 0 & 0 & 0 & 0 & 0 & -2k_n \\ k_s & 0 & k_s & 0 & -k_s & 0 & -k_s & 0 \\ 0 & 0 & 0 & 2k_n & 0 & -2k_n & 0 & 0 \\ -k_s & 0 & -k_s & 0 & k_s & 0 & k_s & 0 \\ 0 & 0 & 0 & -2k_n & 0 & 2k_n & 0 & 0 \\ -k_s & 0 & -k_s & 0 & k_s & 0 & k_s & 0 \\ 0 & -2k_n & 0 & 0 & 0 & 0 & 0 & 2k_n \end{bmatrix} \tag{5b}$$

$$\{u\} = \{u_I\ v_I\ u_J\ v_J\ u_K\ v_K\ u_L\ v_L\}^T \tag{5c}$$

When the normal stiffness(k_s) and shear stiffness(k_n) of discontinuites are considered as random variables, the element stiffness matrix of a joint element can be represented in the following form.

$$[K_j]_{i,j} = [K_j]_{io,j}\ (1+g_{i,j}(k_{s,n})) \tag{6}$$

where $[K_j]_{io,j}$ is the matrix to which the mean value of joint stiffness used for element stiffness matrix defined by equation (5b) is applied. And, in equation (6), $g_{i,j}$ $(k_{s,n})$ indicate the distribution function to be considered the correlation between joint normal stiffness and joint shear stiffness, which can be obtained by reflecting the $G_j(x, Cov)_i$ calculated by expression (4).

The solution (5) provides the forces and displacements at nodal points for the next iterative calculation. Iterative procedures depend on nonlinear deformation laws appropriate for a discontinuity in a rock mass. In this study, to modify the behaviors of joint opening and joint closure, the following equation suggested by Goodman[2] was used.

$$F_n = \frac{\Delta v}{V_{mc} - \Delta v} F_{n,o} \qquad (7)$$

where, $F_{n,o}$ is the initial external force at a nodal point, Δv is the difference of normal displacements between the individuals of a nodal point pair caused by an increment of normal force $(F_n - F_{n,o})$, V_{mc} is the joint maximum closure beginning from initial load $F_{n,o}$.

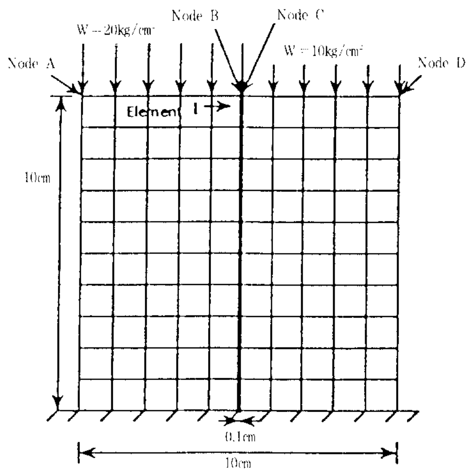
3. A NUMERICAL ANALYSIS PROGRAM OF UNDERGROUND STRUCTURES

3.1 Composition and function of the program

A 2D FEM program has been developed by which we could estimate the response variability due to the physical property variation of discontinuities within rock mass. This program consists of a main program and 21 subprograms coded by Fortran language. The element library of this code includes three different element types : (1) plane strain finite element with 3 or 4 nodes to model intact rock masses, (2) joint finite element to model discontinuities, and (3) truss finite element to model reinforcemenet members such as rockbolts. To realize an efficient inverse matrix calculation of global stiffness matrix, the modified Cholesky method and the skyline algorithm have been adopted. As for the method of considering the nonlinearity of discontinuities, the load transfer method[7] suggested by Zienkiewicz has been employed.

3.2 Verification examples

In order to verify the program prepared by this study, a stochastic analysis was performed on the analysis model indicated in Figure 1. Figure 1, which refers to the deterministic model presented by Van Dillen and Ewing[8], is intended to grasp the joint slip

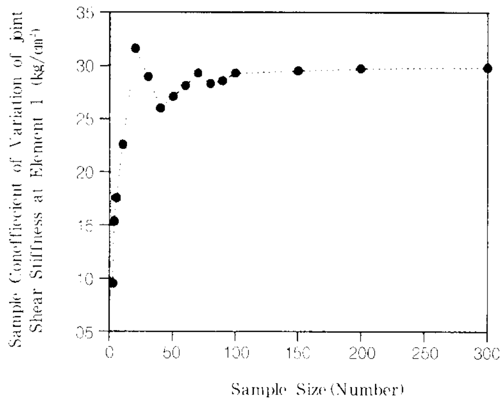


Joint Element :
Normal stiffness=100kg/cm²
T_o/q_u(the ratio of tensile to compressive strengh of wall rock)=0.1
B_o(the ratio of residual to peak strength)=0.6
Φ_μ(friction angle)=0°
i_o(dilatancy angle)=0°
V_{mc}(maximum joint closure)=0.1cm
Plane strain Element :
E=500kg/cm²
ν=(poisson's ratio)=0

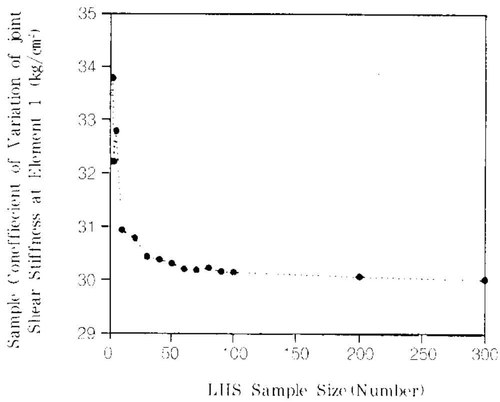
Fig. 1 Model for verification

causing discontinuity by the changes in the joint shear stiffness. If there is no shear stiffness of discontinuity in the figure, the right and left parts of the discontinuity behave separately against vertical load. Thus, the vertical displacement of left part is twice as much as that of right part, as uniform loads of 20kg/cm^2 and 10kg/cm^2 are applied to the left upper part and right upper part respectively. Moreover, with the increase in the size of joint shear stiffness, the difference of vertical displacement between the left and right parts adjacent to the discontinuity will reduce. Referring to such tendency of deterministic analysis results, a stochastic analysis has been performed for the case when the mean value of joint shear stiffness is 100kg/cm^2 and the standard deviation is the normal distribution of 30kg/cm^2 in this example. For this analysis, the scale of correlation(d) was assumed to be 1.0, and the physical properties except joint shear stiffness were assumed to be of deterministic value indicated in Figure 1. Iteration number to simulate the nonlinear behavior of discontinuity was used 30 times through the convergence tests[9] of vertical displacement at the loaded point. In this case, tolerance limit in increment is within 1.0%.

For the purpose of selecting a reasonable sample size, the sample coefficient of variation(Standard deviation/Mean ; SCOV) of the joint shear stiffness sampled at joint element 1 was examined according to the increase in the sample size. Figure 2 indicates the variance of the joint shear stiffness sampled from joint element 1 ; (a) is the analysis result by the MCS method and (b) is that by LHS method. The SCOV of the joint shear stiffness obtained by the MCS method shows unstable tendency with the changes in the



(a) MCS Solution

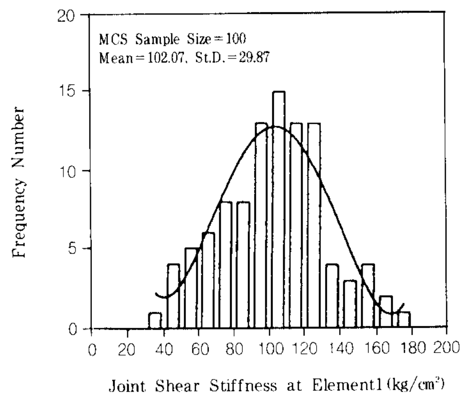


(b) LHS Solution

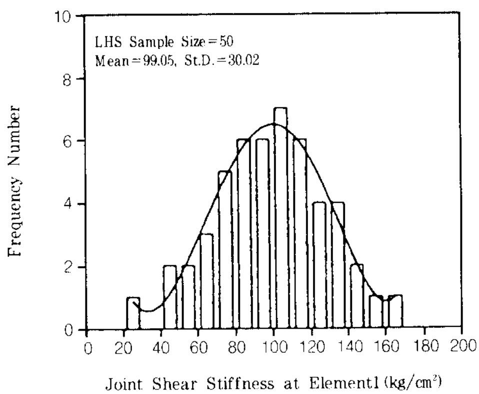
Fig. 2 Sample coefficient of variation of joint shear stiffness at element 1 as function of sample size

sample size which is less than 100. However, in the case of LHS method, stability was indicated when the sample size was more than 30. In the MCS method with 100 samples, the mean value characteristics of the sampled joint shear stiffness was about 2.1% greater than the input characteristics, and the standard deviation was about 2.5% smaller. On the other hand, when the LHS method with 100 samples was employed, the mean value was about 0.5% smaller compared to input characteristics and the standard deviation was about 0.1% greater. In this example, the estimated error in the mean value and the standard deviation obtained by the MCS method with 100 samples was almost the same as the error obtained by the LHS method with 20 samples.

The frequency distribution of the sampled shear stiffness in joint element 1 is indicated in Figure 3. In Figure 3, (a) indicates the result of analysis by the MCS method with 100 samples, and (b) shows the result of analysis by the LHS technique with 50 samples. As indicated in the figure, the distribution characteristics by two different methods show approximately the normal distribution form defined as input. However, it is noted that the result by the LHS technique indicates the normal distribution form defined as input more accurately even with less samples than that by MCS technique. As observed in Figure 3, the result by the LHS method has an error of less than 0.5% of input, while the MCS method requires about 250 samples to realize an error of 0.5%. In addition, the time of calculation required by the MCS method with 250 samples with pentium model 90 MHz is about 4.2 times that of the LHS method. Therefore, it is demonstrated that the LHS technique is much more efficient than the MCS method in terms of both time and size of calculation.



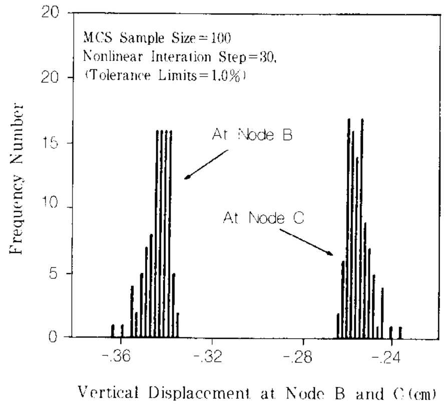
(a) MCS results



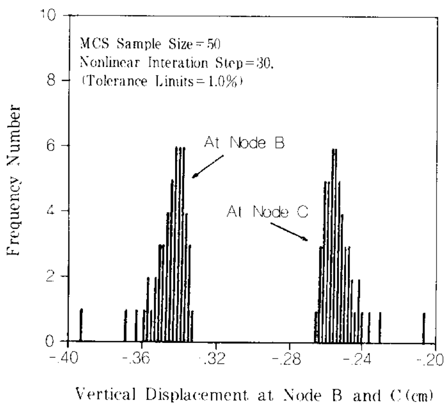
(b) LHS results

Fig. 3 Comparison of LHS results with MCS results about distribution characteristics of sampled joint shear stiffness at element 1

The frequency distribution of vertical displacement at node B and C obtained by stochastic analysis is indicated in Figure 4. As indicated in this figure, the frequency distribution of vertical displacement at nodes B and C obtained by two different methods are similar to each other. Nodes B and nodes C are the left and the right side of discontinuity composing joint elements, and the difference of vertical displacements at the two points means the shear displacement generated at joint element. The vertical displacement at node B indicates a concentrated tendency in the range of about -0.35cm~-0.34cm and the vertical displacement at node C indicates a concentrated tendency in the range of about -0.26cm~-0.25cm. The vertical displacement distribution at both nodes are of



(a) MCS Results



(b) LHS Results

Fig. 4 Distribution shape of sampled displacements at nodes B and C

the Log-Normal type.

In order to verify the feasibility of this stochastic output in Figure 4, we conducted deterministic analyses of many cases with various joint stiffness in the 99.8% range of normal distribution. The joint shear stiffness which belongs to the 99.8% of normal distribution with the mean value of 100kg/cm^2 and the standard deviation 30kg/cm^2 is substituted into the equation of normal distribution function, and then 7.3kg/cm^2~192.7kg/cm^2 can be computed. The vertical displacements at nodes B and C obtained by deterministic analyses are indicated in Figure 5. As shown in Fig. 5, with the decrease of the joint shear stiffness, the right and the left sides of discontinuity behave separately against vertical load. Thus, when the joint shear stiffness approaches to zero, the vertical displacement at node B converges twice as much as that of node C. Also, the difference of vertical displacements at the two nodes tends to decrease with the increase in the joint shear stiffness. In this example, the vertical displacements at nodes B and C are generated in the value of -0.396cm and -0.204cm respectively when joint shear stiffness is 10kg/cm^2, -0.347cm and -0.254cm when joint shear stiffness is 100kg/cm^2, and -0.332cm and -0.268cm when joint shear stiffness is 190kg/cm^2. If joint shear stiffnesses were uniformly sampled from each range of normal distribution, the vertical displacement at node B would indicate more concentrated tendency in the range of -0.347cm~-0.332cm rather than the range of -0.396cm~-0.347cm, and the vertical displacement at node C would show more concentrated tendency in the range -0.254cm~-0.268cm than in the range of -0.204cm~-0.254 cm. When the stochastic results indicated in

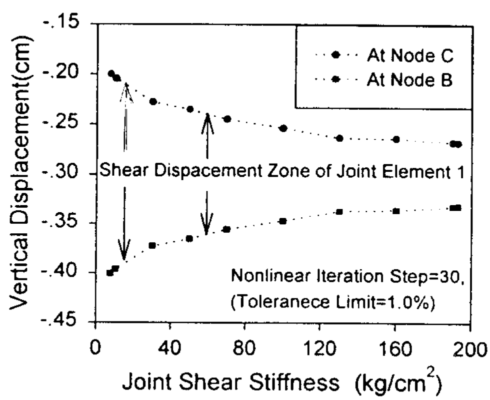


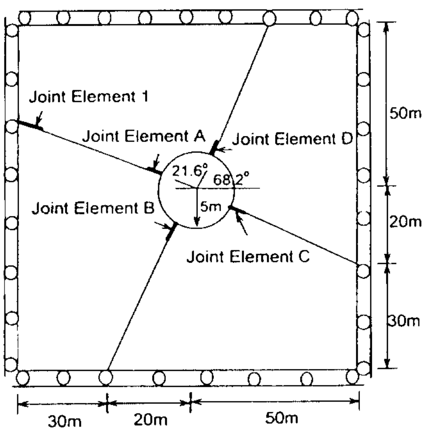
Fig. 5 Deterministic anlaysis output

Fig. 4 are compared with deterministic results in this way, it may be understood that the ranges of vertical displacement distribution and the concentration tendency exist within a reasonable scope.

4. STOCHASTIC ANALYSIS FOR A CIRCULAR CAVERN IN DISCONTINUOUS ROCK MASS

In order to confirm the applicability of the program, a stochastic analysis was performed on the underground structure in discontinous rock mass indicated in Figure 6. Rock mass was assumed to be granite with unit weight of 2.6t/m³, and hydrostatic initial stress was only applied as load.

The joint shear stiffness and the joint normal stiffness were considered for random variables simulated in terms of normal distribution with the mean value of 500kg/cm² and the standard deviation of 32% against mean value. Iteration number to simulate the nonlinear behavior of discontinuity was used 50 times through the convergence tests of displacements around cavern. At this time, the increment tolerance limit in increment is within 2.0%. In addition, 50 LHS samples were used for the analysis.



Idealization :
Rock : 240 Isoparametric Quadrilateral Elements
Discontinuity : 40 Joint Elements

Material Properties(Deterministic Value) :
Rock :
E : 500,000kg/cm²
ν : 0.25
ε(unit weight) = 2.6t/m³
Discontinuity :
T_o/q_u(the ratio of tensile to compressive strengh of wall rock) = 0.1
B_o(the ratio of residual to peak strenght) = 0.6
Φ_μ(friction angle) = 0°
i_o(dilatancy angle) = 0°
V_{mc}(maximum joint closure) = 1cm

Fig. 6 A circular cavern within discontinuous rock mass

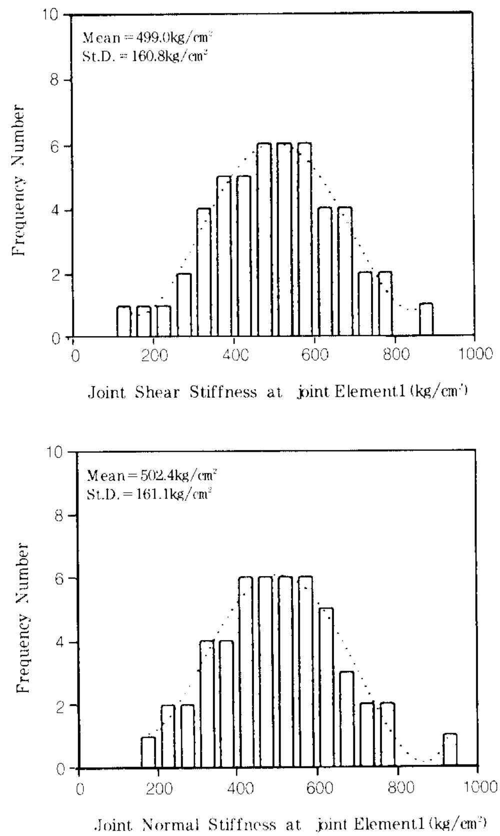
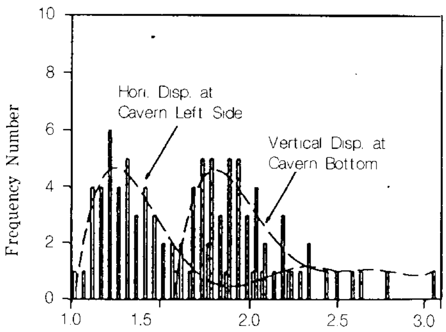


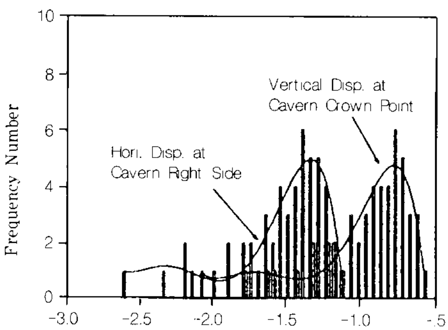
Fig. 7 Sampled joint stiffness of joint element 1

Distribution form of the joint stiffnesses sampled at element 1 is indicated in Figure 7. Figure 7 expresses reasonably the normal distribution form defined as input. In this example, the mean value of the sampled joint shear stiffness at element 1 is about 0.2% smaller compared with the input value and the standard deviation was about 0.7% greater. In the case of joint normal stiffness, the mean value was about 0.5% greater and the standard deviation was about 0.2% greater compared with the input.

The sampled displacement at 4 representative points around cavern are indicated in Table 1 and Figure 8. The displacements after excavation occur toward the inner part of the cavern due to the release of initial stress and are of log-normal distribution form in general. It is noticeable that the magnitude and variance of displacements vary depending on the locations reviewed. In this example, the variance of vertical displacement at cavern crown point is about 2.1 times that at the cavern bottom and about 1.4 ~1.6 times that at the cavern left and right side. Therefore, the top of cavern has the greater sensitive response to the displacement caused by changes in the stiffness of discontinuities in comparison with 3 other points.



(a) Positive direction



(b) Negative direction

Fig. 8 Sampled displacements at 4 representative points around cavern

Table 1 The distribution characteristics of displacement at 4 representative points around cavern

	Vertical Disp. at Cavern Crown Point	Vertical Disp. at Cavern Bottom	Horizontal Disp. at Cavern Left Side	Horizontal Disp. at Cavern Right Side
Mean (mm)	−0.966	1.991	1.435	−1.522
St.D. (mm)	±0.312	±0.301	±0.300	±0.309
Variance	0.322	0.151	0.211	0.203

The distribution characteristics of the shear slip and normal closure of joint elements (A, B, C, D of Figure 6) intersecting cavern are indicated in Table 2 and Figure 9. As indicated in the Table and Figure, the distribution

Table 2 The distribution characteristics of sampled displacements at 4 joint elements

Joint Element		A	B	C	B
Joint shear slip (mm)	Mean value	0.350	0.427	0.167	0.245
	St. D.	±0.207	±0.216	±0.193	±0.202
	Variance	0.591	0.506	1.155	0.825
Joint normal closure (mm)	Mean value	1.580	1.526	1.751	1.516
	St. D.	±0.763	±0.748	±0.752	±0.733
	Variance	0.483	0.490	0.429	0.484

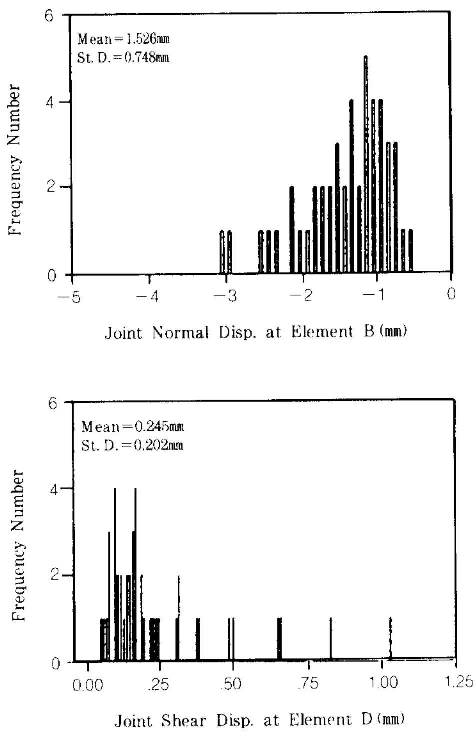


Fig. 9 The distribution characteristics of displacement at 2 representative joint elements

characteristics of the sampled joint slip and the sampled joint normal closure are different beacause the size and direction of the initial stress act differently due to the location of discontinuity. In this example, the sampled joint slip obtained from 4 elements are generally log-normal distribution type. However, the sampled joint normal closure obtained from the same elements indicate the normal or the log-normal distribution type with relatively smaller variance than the joint slip. Joint slips tend to concentrate in the range of 0.13~0.50mm at element A, 0.2~0.5mm at element B, and less than 0.25mm at elements C and D. The largest mean value of joint slip of 4 elements was found in element B, and the largest variance against the mean value was found element C. The results of Table 2 indicated the most sensitive response at element C to the joint slip, while similar sensitive response at all elements to the joint normal closure. In the case of joint normal closure, the concentration was found in the range of 1.0~2.0mm in general at 4 elements.

It took about 46 minutes to calculate this example with Pentium 90MHz. When the Monte Carlo simulation was employed, it took about 2,500 minutes with 100 samples and about 5,000 minutes with 200 samples, revealing increase of time in proportion to the sample size. Moreover, in comparison with the distribution characteristic defined as the input, the shear stiffness distribution sampled from joint element 1 by the MCS method indicated about 1.0% difference in the mean value and 2.2% difference in the standard deviation when the sample size of 100 were used and 1.0% difference in the mean value and 0.5% difference in the standard deviation when the sample size of 200 were used. In this example, therefore, the LHS method is expected to reduce the calculation time required by the MCS method by 50~100 times.

5. CONCLUSION

In this study, a stochastic finite element model that reflects the uncertainty of physical properties in discontinuous rock mass has been proposed by applying the LHS technique. Major performances are summarized as ; 1) From the numerical results through verification examples, it has been observed that the analytic results by LHS method were similar to those by the MCS method. When the stochastic results were compared with the deterministic results, the variance and the concentration tendency of vertical displacement was within a reasonable value. Therefore, it could be concluded that the numerical model developed by this study was rational and reasonable. 2) Judging from the analytic results on the model of Figure 7, the largest variance among 4 representative points around the cavern was found at the crown point, the value of which was 2.1 times that at the bottom and about 1.4 to 1.6 times that at the left and the right side. Among 4 joint elements intersecting the cavern, element C was most sensitive to the joint slip, while the variance of the joint normal displacement was found to be similar at all elements. The sampled joint normal closure was determined to be in a type of log-normal distribution with relatively smaller variance than the joint slip. In addition, it was noted that the LHS technique significantly reduced the computation time to be required for the MCS method by about 50 to 100 times.

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