손상이 증가하는 아스팔트 콘크리트의 점탄성 구성모델

Constitutive Modeling of Asphalt Concrete With Time-Dependent Damage Growth

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요 지

본 논문에서는 반복하중에 의해 손상을 입은 아스팔트 콘크리트의 점탄성 구성모델에 대한 역학적 접근방법을 제시하였다. 모의변수로 나타낸 탄성-점탄성 일치원리는 아스팔트 콘크리트의 점탄성과 시간의존 손상의 증가를 별도로 평가하도록 적용되었다. 선형-점탄성 파괴역학에 사용되고 있는 미소균열의 증가법칙이 물체내 손상증가를 나타내는데 성공적으로 사용되었다. 응력과 모의변형도로 나타나는 구성방정식은 먼저 변형도조절에 대해 세워졌으며, 응력과 모의변형도를 모의응력과 변형도로 간단하게 대체함으로서 응력조절 구성방정식으로 변형되었다. 모의응력으로 나타낸 변형된 구성방정식은 응력조절모드에서 파괴에 이르는 아스팔트 콘크리트의 모든 역학적 거동을 충분히 예측하고 있다.

Abstract

Mechanical behavior of asphalt concrete that accounts for viscoelasticity and damage evolution under cyclic loading conditions is modeled and presented in this paper. An elastic-viscoelastic correspondence principle in terms of pseudo variables is applied to separately evaluate viscoelasticity and time-dependent damage growth in asphalt concrete. A microcrack growth law, which is commonly employed in linear viscoelastic fracture mechanics, is successfully used for describing the damage growth in the body. A constitutive equation in terms of stress and pseudo strain is first established for controlled-strain mode, and then transformed to controlled-stress constitutive equation by simply replacing stress and pseudo strain with pseudo stress and strain. The transformed constitutive equation in terms of pseudo stress satisfactorily predicts the mechanical behavior of asphalt concrete all the way up to failure under controlled-stress modes.

Keywords : *asphalt concrete, correspondence principle, constitutive mode, cyclic test, damage, fatigue, viscoelasticity*

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1. INTRODUCTION

Asphalt concrete is a viscoelastic composite material composed of asphalt binder and aggregate. The viscoelasticity of asphalt concrete is mainly due to the presence of asphalt binder, which is a well-known viscoelastic material. Thus, it is essential to employ theory of viscoelasticity for accurate prediction of constitutive behavior of asphalt concrete. When asphalt concrete is subjected to traffic loading, hysteretic stress-strain relationship is usually observed. The hysteretic behavior of asphalt concrete under traffic loading is attributed to three major mechanisms[1,2] damage growth ; relaxation of stresses in the system due to the viscoelasticity of asphalt concrete ; and microcrack healing. Since these three mechanisms take place simultaneously, it is important to model the damage growth separately from the viscoelastic effect to simplify the modeling of the mechanical behavior of asphalt concrete.

Lee and Kim[2] applied elastic-viscoelastic correspondence principle[3] in terms of pseudo strain to transform the viscoelastic analysis to an elastic case. A time-dependent damage parameter based on the microcrack growth law[4] was then employed to model the damage growth in asphalt concrete. The resulting constitutive model satisfactorily predicted the hysteretic behavior of asphalt concrete under the controlled-stress and -strain modes. In this study, the same methodology was used to model the constitutive behavior of asphalt concrete.

Although Lee and Kim[2] successfully modeled the fatigue damage evolution of asphalt concrete under both modes-of-loading, use of pseudo strain in the controlled-stress mode is not a practical approach because of the intrinsic difficulty in the calculation of pseudo strains[5]. In this study, therefore, pseudo stress approach was employed in the controlled-stress mode to increase the applicability of the model to actual pavement design and analysis.

2. CONSTITUTIVE THEORY

2. 1 Correspondence Principle

Schapery[3] proposed an elastic-viscoelastic correspondence principle (CP) that can be applicable to both linear and nonlinear viscoelastic materials, with or without aging.

According to his CP, a viscoelastic problem can be transformed to an elastic case by using physical stresses and *pseudo strains* or physical strains and *pseudo stresses.*

For linear viscoelastic materials, a uniaxial stress-strain relationship is

$$\varepsilon = \int_0^t D(t-\tau)\frac{d\sigma}{d\tau}\,d\tau \tag{1}$$

which can be written as

$$\sigma^R = E_R\,\varepsilon \tag{2}$$

if we define

$$\sigma^R = E_R\int_0^t D(t-\tau)\,\frac{d\sigma}{d\tau}\,d\tau \tag{3}$$

where E_R is reference modulus which is an arbitrary constant. σ^R is so-called the *pseudo stress* and D(t) is uniaxial creep compliance. A correspondence can be found between Eq. (2) and a linear elastic stress-strain relationship.

Similarly, using an inverse form,

$$\sigma = \int_0^t E(t-\tau)\frac{d\varepsilon}{d\tau}\,d\tau \tag{4}$$

which can be written as

$$\sigma = E_R\,\varepsilon^R \tag{5}$$

if we define

$$\varepsilon^R = \frac{1}{E_R}\int_0^t E(t-\tau)\,\frac{d\varepsilon}{d\tau}\,d\tau \tag{6}$$

where ε^R is pseudo strain and E(t) is uniaxial relaxation modulus.

2. 2 Damage Parameter

Schapery[4] proposed the following time-dependent damage parameter based on a generalization of microcrack growth law :

$$S_p = \left(\int_0^t \left|\sigma^{\frac{1}{N}}\right|^p dt\right)^{\frac{1}{p}} \tag{7}$$

where p=(1+N)k, and N is the exponent in $\sigma-\varepsilon^R$ relationship (N=1 for linear viscoelastic materials) ; and k=(1+1/m) or k=1/m depending on characteristics of a failure zone. m is the exponent in the power law relationship between creep compliance and time. In earlier work[2], it was observed that p=2(1+1/m) was better assumption for the controlled-strain condition while p=2/m was adequate for the controlled-stress case.

To predict strains from known stresses, the damage parameter based on stress in Eq. (7) is useful. However, if the strains are known, ε^R should be used instead of σ in Eq. (7) to calculate the values of S_p.

3. CALCULATION OF PSEUDO VARIABLES

Pseudo strain or pseudo stress is an essential parameter for applying Schapery's correspondence principle[3] to the hysteretic stress-strain behavior of asphalt concrete. As shown in Eq. (6), the computation of the pseudo strain requires the acquisition of strain values from time zero to time t. This method is impractical to be used in the long-term fatigue analysis due to limitations in computer memory capacity and computing time. An alternate way to calculate the pseudo strain is to present the relaxation modulus and the strain as analytical functions of time and integrate the product of these functions.

In the controlled-strain mode, the haversine strain history can be simply represented in an analytical function as follows :

$$\varepsilon(t) = [\varepsilon_0 + \varepsilon_0 \sin(\omega t + \theta)]H(t) \tag{8}$$

where ε_0 is a strain amplitude, ω is an angular velocity of strain input, t is a current θ, is a regression constant, and H(t) is heaviside step function that H=0 when t<0, and H=1 when t>0. From Eqs. (6) and (8), we can obtain

$$\varepsilon^R = \frac{1}{E^R}\left[\varepsilon_0 E(t) + \varepsilon_0 |E^*| \sin(\omega t + \theta + \phi)\right] \tag{9}$$

where ϕ is a phase angle, E(t) is the uniaxial relaxation modulus, and $|E^*|$ is absolute value of complex modulus. More detailed steps involved in deriving Eq. (9) can be found in Lee[6].

Similarly, the stress input and pseudo stress response can be represented as follows :

$$\sigma(t) = [\sigma_0 + \sigma_0 \sin(\omega t + \theta)]H(t) \tag{10}$$

$$\sigma^R(t) = E_R\left[\sigma_0 D(t) + \sigma_0 |D^*| \sin(\omega t + \theta + \phi)\right] \tag{11}$$

where σ_0 is a stress amplitude, D(t) is the uniaxial creep compliance, and $|D^*|$ is absolute value of complex compliance. For the simplicity of analysis, $E_R=1.0$ was used in all the calculation of pseudo variables.

4. CONSTITUTIVE MODELING OF ASPHALT CONCRETE

4.1 Uniaxial Viscoelastic Constitutive Model under the Controlled-Strain Mode

Since a linear relationship between the stress and pseudo strain has been found when the damage was negligible in the previous work[2], it was assumed that the material was linearly viscoelastic and that any deviation from the linear viscoelastic behavior was due to damage. Thus, the following form of the constitutive model was employed in this study :

$$\sigma = I(\varepsilon^R)\, D(S_m) \tag{12}$$

where I=initial pseudo stiffness, $D(S_m)$= damage function, and S_m=internal state variables (ISV). The initial pseudo stiffness was used to minimize the sample-to-sample variability.

To study the damage growth in asphalt concrete, the controlled-strain fatigue tests were conducted with two strain amplitudes. These strain levels were high enough to induce some damage to the specimens. Fig. 1 shows typical stress-pseudo strain hysteresis loops at different numbers of cycles. In this figure, the loading and unloading paths in each cycle are nonlinear due to the damage. In addition, the stiffness reduction of the material due to damage growth results in the change of the slope of each cycle as cyclic loading continues.

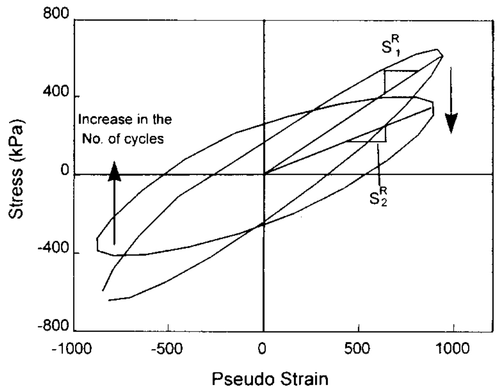

Fig. 1 Stress-pseudo strain behavior of asphalt concrete under the controlled-strain mode.

To represent this change of the slope, *pseudo stiffness*, denoted by S^R, was defined as follows :

$$S^R = \sigma / \varepsilon^R_m \tag{13}$$

where ε^R_m is a pseudo strain value at the peak pseudo strain of each cycle and σ is a stress value corresponding to ε^R_m.

It is found from the detailed experimental study that the following three ISV's are needed to describe the hysteretic stress-pseudo strain behavior described above :

a. Ratio of current ε^R to the largest ε^R during the ε^R history up to that time : $\varepsilon^R/\varepsilon^R_L$

b. Amplitude of pseudo strain : ε^R_0

c. Damage parameter : S_p

The first ISV $\varepsilon^R/\varepsilon^R_L$ is used to differentiate whether a current stress is on the loading or unloading path for the same value of pseudo strain. The amplitude of pseudo strain is employed to account for the change in the size of stress-pseudo strain hysteresis loop that depends on the strain amplitude. The change in pseudo stiffness due to fatigue damage could be described by the damage parameter S_p because S_p represents microcrack growth in the body.

Thus Eq. (12) becomes :

$$\sigma = I(\varepsilon^R)D(S_p, \frac{\varepsilon^R}{\varepsilon_L^R}, \varepsilon_0^R) \tag{14}$$

To obtain the explicit form of the damage function D, several forms of the constitutive model were attempted using the experimental data, and the following additive form was found to yield the best result :

$$\sigma = I(\varepsilon^R)\left[F(S_p) + G\left(\varepsilon_0^R, \frac{\varepsilon^R}{\varepsilon_L^R}\right)\right] \tag{15}$$

The function F represents the change in the secant pseudo stiffness (S^R) during cyclic loading, and the function G accounts for the hysteretic behavior of stress-pseudo strain relationship.

4. 2 Characterization of the Material Functions F and G

Two different types of asphalt-aggregate mixtures denoted by AAD and AAM were used in this study to characterize and validate the constitutive model. Detailed information on the mechanical properties of these two mixtures can be found in Lee[6]. In this paper, focus is placed on the theoretical framework of developing a constitutive model for asphaltic mixtures. Also, only one mixture data is presented in this paper, and additional data can be found in Lee (1996).

Using experimental data, pseudo strain values were calculated, and the values of S^R/I were calculated from Eq. (13) and plotted against S_p determined from Eq. (7) in Fig. 2(a).

The data presented in this figure includes the data from the entire loading path of the first cycle and the values of the remaining cycles up to failure. The discrepancies between two strain-level data in the first loading path could be due to experimental errors where some minor adjustments occur within the test setup. However, after the first loading path, all the data from two different strain amplitudes fall on the same curve up to failure. Since the same loading time (0.1 second) for the two strain amplitudes produced different strain rates for the two tests, S_p demonstrates its ability of accounting for the effect of strain rates on damage growth.

The regression analysis on the controlled-strain data resulted in the following equation for the function F :

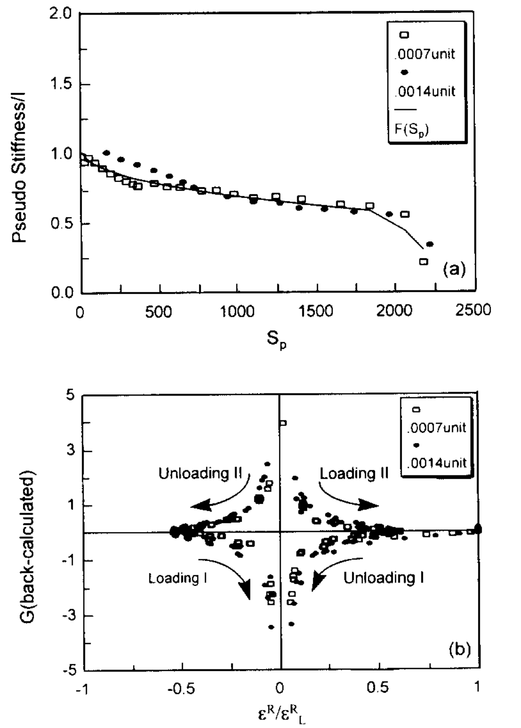

Fig. 2 Damage evolution in asphalt concrete under the controlled-strain mode : (a) S^R-S_p ; (b) back-calculated $G-\varepsilon^R/\varepsilon^R_L$

$$F(S_p) = F_0 - F_1(S_p)^{F_2} \qquad (16)$$

The regression coefficients in Eq. (16) are presented in Table 1.

To find the explicit form of G, $F(S_p)$ was subtracted from $\sigma/(I\varepsilon^R)$ (hereinafter, called back-calculated G) and then plotted against $\varepsilon^R/\varepsilon^R_L$. As shown in Fig. 2(b), a strong power relationship was observed between the back-calculated G and $\varepsilon^R/\varepsilon^R_L$. In this figure, the value of $\varepsilon^R/\varepsilon^R_L$ increases during loading (Loading I for the negative values of $\varepsilon^R/\varepsilon^R_L$ and Loading II for the positive values of $\varepsilon^R/\varepsilon^R_L$) while it decreases during unloading (Un loading I for the positive values of $\varepsilon^R/\varepsilon^R_L$ and Unloading II for the negative values of $\varepsilon^R/\varepsilon^R_L$). The study of this data produced the following form of function G :

$$G = G_0 + G_1\left|\frac{\varepsilon^R}{\varepsilon^R_L}\right|^{G_2(\varepsilon^R_0)} \qquad (17)$$

where $$G_2(\varepsilon^R_0) = G_{2R}\left[\beta_0 + \frac{\beta_1}{\varepsilon^R_0}\right] \qquad (18)$$

Table 1 Coefficients in Equation (16)

F_0	F_1	F_2	Boundary Condition
1.008	0.00046	0.93	$S_p \leq 1,820$
3.174	0.00146	1.0	$S_p < 1,820$

The regression coefficients, G_0, G_1, G_{2R}, β_0, and β_1 are summarized in Table 2. Since the size of the stress-pseudo strain hysteresis loop is dependent upon the amplitudes of pseudo strain incurred in the specimens, the exponent G_2 in Eq. (17) is function of ε^R_0 instead of constant values.

4.3 Uniaxial Viscoelastic Constitutive Model under the Controlled-Stress Mode

Before the controlled-strain constitutive equation (15) is applied to the controlled-stress mode, a careful investigation was made on the controlled-stress fatigue test data to identify the basic difference in hysteretic behavior and damage growth under both modes of loading. Fig. 3 presents a typical pseudo stress-strain relationship. It can be seen from Fig. 3 that the slope of the σ^R-ε curve decreases due to damage growth in the body as cyclic loading continues. Unlike the controlled-strain case, negligible hysteresis loop is observed in the controlled-stress mode. Recall that *pseudo stiffness* S^R was employed in the controlled-strain case to describe the change in the slope of σ-ε^R cycle due to damage growth. We will use the same terminology in the controlled-stress case, but

Table 2 Coefficients in Equation (17) and (18)

Function G				Function G_2		
Boundary Condition	G_0	G_1	G_{2R}	Boundary Condition	β_0	β_1
$\varepsilon^R/\varepsilon^R_L < 0$, and increases	0.1917	−0.1765	−1.00	$\varepsilon^R_L \leq 200$	0.0	0.0
$\varepsilon^R/\varepsilon^R_L > 0$, and increases	−0.0933	0.0912	−1.30	$\varepsilon^R_L > 200$	1.246	−247.1
$\varepsilon^R/\varepsilon^R_L > 0$, and decreases	0.1573	−0.1653	−1.00	−	−	−
$\varepsilon^R/\varepsilon^R_L < 0$, and decreases	−0.6945	0.5713	−0.63	−	−	−

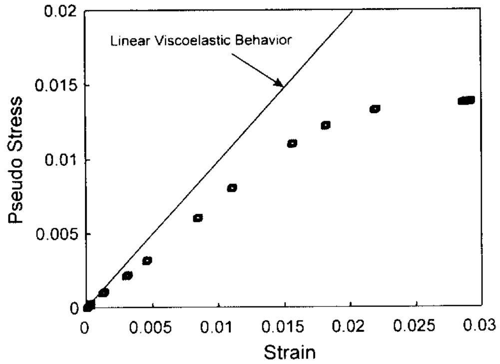

Fig. 3 Pseudo stress-strain behavior of asphalt concrete under the controlled-stress mode

physical stress and pseudo strain are replaced with respective pseudo stress and physical strain in the definition of pseudo stiffness in Eq. (13).

Using the controlled-stress fatigue test data, the S_p values were calculated with $p=2/m$. Since the reduction in S^R was observed from both modes of loading, the characteristics of S^R-S_p relationship were compared in Fig. 4(a). It is observed from this figure that the S_p value of the controlled-strain mode is larger than the one of the controlled-stress mode for a given value of S^R. This discrepancy may be attributed to several factors. In the controlled-strain fatigue tests, both tensile (a positive sign) and compressive (a negative sign) stresses occur in a specimen, while only tensile stresses occur in the specimen under the controlled-stress mode. Schapery[4] proposed the damage parameter in terms of $|\varepsilon^R|$ instead of ε^R to include some damage due to the compressive stresses. However, most of the cracks observed during the controlled-strain fatigue tests conducted in this study were tensile cracks, which propagated in the direction perpendicular to the loading direction. This implies that the compressive stresses caused little damage to the specimen. Furthermore, the compressive stresses have beneficial effects on the closing of tensile cracks. Therefore, the use of $|\varepsilon^R|$ in the controlled-strain mode may result in the overestimation of the damage.

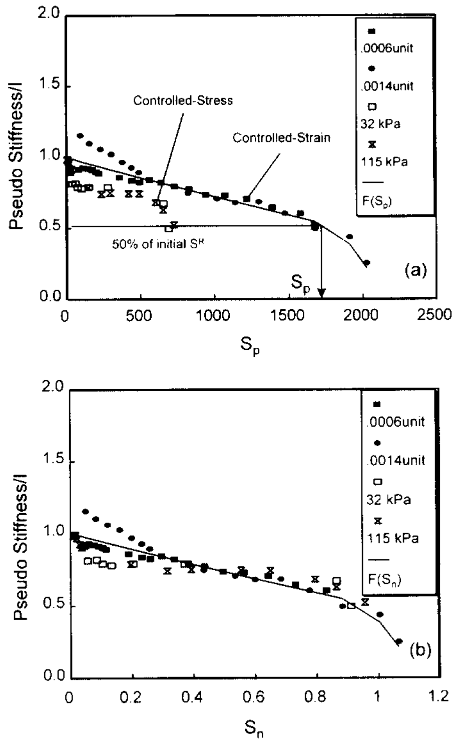

Fig. 4 Reduction in S^R under both modes of loading : (a) S^R-S_p ; (b) S^R-S_n

For the above reasons, S_p was normalized by using a value of S_p at failure, denoted by S_f. To determine the value of S_f, the failure criterion of a specimen should be established a priori. The most common failure criterion of asphalt concrete under the controlled-strain mode is a 50 percent reduction in the initial stiffness[7]. Knowing the stiffness reduction is due to both relaxation and damage growth of the materials, this failure criterion is modified to the 50 percent reduction in the *initial pseudo stiffness* to eliminate the viscoelasticity of the materials. Then, the value of S_f is the one corresponding to the 50 percent of the initial secant pseudo stiffness as shown in Fig. 4(a). In the controlled-strain mode, the S_f value was 1,900, and in the controlled-stress mode, 760. *The normalized damage parameter* S_n is defined as follows :

$$S_n = \frac{S_p}{S_f} \tag{19}$$

The S_p in Eq. (16) is then replaced with S_n :

$$F(S_n) = F_0 - F_1 (S_f\ S_n)^{F_2} \tag{20}$$

The S^R values shown in Fig. 4(a) were plotted against the S_n values in Fig. 4(b). As can be seen in this figure, all the S^R values for both modes of loading fall on the same curve.

To take advantage of the constitutive relationship developed from the controlled-strain tests, the constitutive equation (15) is transformed to account for the controlled-stress case. It should be reminded that the constitutive equation (15) is extended version of the linear viscoelastic constitutive equation (5) to account for the damage in the body. The linear viscoelastic constitutive equation (2) is identical to Eq. (5) but has an inverse form. Based on this observation, it is assumed that the inverse form of the constitutive equation (15) could be obtained by simply replacing the physical stress and pseudo strain in Eq. (15) with pseudo stress and physical strain as follows :

$$\sigma^R = I(\varepsilon)\left[F(S_n) + G\left(\varepsilon_0, \frac{\varepsilon}{\varepsilon_L}\right)\right] \tag{21}$$

where ε_L is the largest value of strain up to current time. It can be seen from the typical strain response as shown in Fig. 6 that the values of $\varepsilon/\varepsilon_L$ are mostly larger than 0.6. Thus, as can be seen in Fig. 2(b), $G \approx 0$ when $\varepsilon/\varepsilon_L > 0.6$. Then, equation (21) may be reduced to

$$\varepsilon = \frac{\sigma^R}{IF(S_n)} \tag{22}$$

Except that pseudo stress is employed in Eq. (22) instead of pseudo strain, one of the major differences between the controlled-strain and -stress constitutive equations is that the hysteresis function G, which accounts for the hysteretic behavior of stress-pseudo strain, is not needed in the controlled-stress case. This can be proved by the σ^R-ε behavior shown in Fig. 3. There is no significant hysteresis loop observed in this figure that supports the validity of the constitutive equation (22). Now, one may predict strain response for sinusoidal stress input from Eq. (22) using pseudo stress obtained from Eq. (11) and damage function F obtained from the controlled-strain cyclic test data.

5. VERIFICATION OF THE CONSTITUTIVE MODELS

For the verification purposes, uniaxial tensile fatigue tests were performed at 25℃ under the controlled-strain and -stress modes. Fig. 5 demonstrates the verification of the constitutive equation (15) for the controlled-strain fatigue tests. To validate the constitutive model for entire fatigue life of asphalt concrete, 4 cycles were randomly selected for presentation, which represent initial, early,

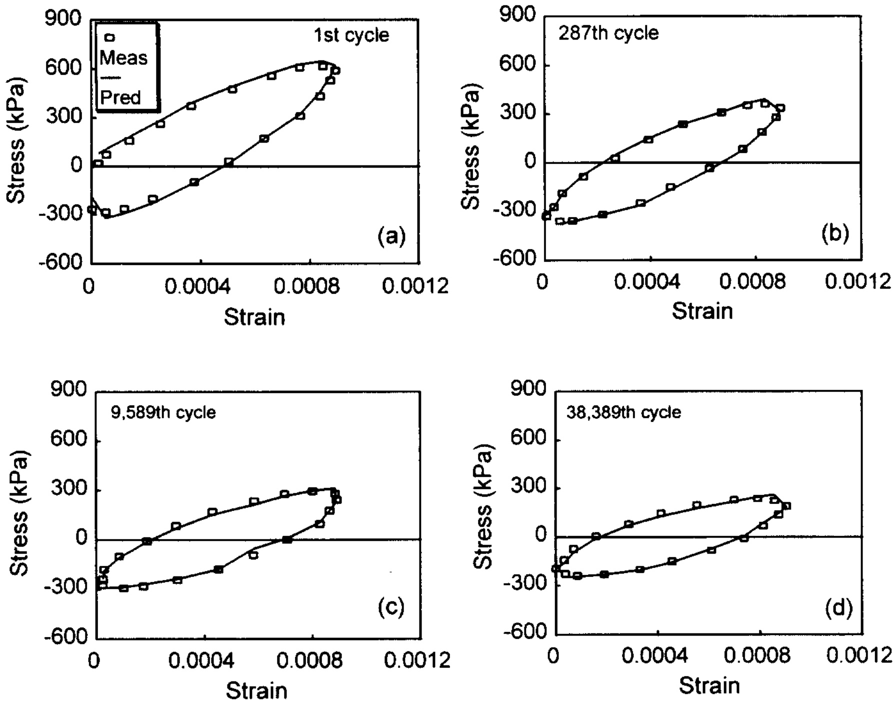

Fig. 5 Validation of the constitutive model under the controlled-strain mode with the strain amplitude of 0.0009unit. (N_f=44,000)

intermediate, and final states of fatigue life of the specimen. Fig. 6 represents the validation results of the controlled-stress constitutive model (22). In general, an excellent agreement is found between the measured and predicted values.

In Fig. 5, the constitutive model successfully predicts the reduction in stresses in the controlled-strain test all the way up to failure. Since the strain amplitude used in the verification test was not used in the determination of the coefficients in the constitutive equation (15), it is proved again that S_p is an excellent means of eliminating the strain-level-dependence of the materials.

In Fig. 6, the accuracy of prediction in the controlled-stress fatigue test is not as good as the prediction in the controlled-strain case. It must be noted here that the coefficients in the function F used in the prediction were obtained from the controlled-strain fatigue tests.

The extrapolation from the controlled-strain mode to the controlled-stress mode might have affected the accuracy. Although the prediction is not as good as the controlled-strain case, the constitutive model reasonably predicts the increase in strain up to failure, indicating the validity of the constitutive model (22).

6. CONCLUSIONS

Mechanical behavior of asphalt-aggregate mixtures accounting for viscoelasticity and damage growth under uniaxial loading condi-

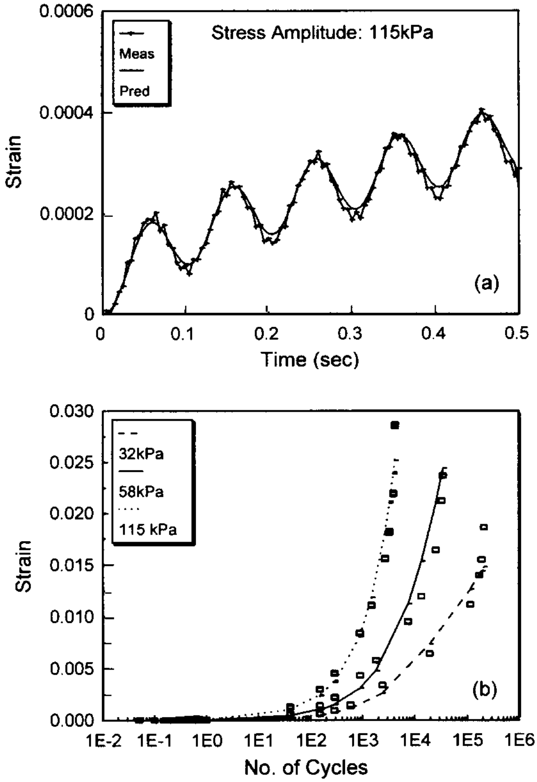

Fig. 6 Validation of the constitutive model under the controlled-stress mode with various stress amplitudes : (a) prediction of strain in the first 5 cycles; (b) prediction of average strain histories up to failure

tions was modeled. A controlled-strain constitutive model (15) in terms of pseudo strain and damage parameter was first developed.

This constitutive equation was then transformed to the controlled-stress constitutive equation (22) by simply replacing physical stress and pseudo strain with pseudo stress and physical strain. In the verification study, the both constitutive equations (15) and (22) satisfactorily predicted the stress-strain behavior of asphalt concrete all the way up to failure under the controlled-strain and -stress modes, respectively.

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