⊠연구논문

임의 절단된 자료의 평균잔여수명 검정에 관한 연구

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A Study on the Test of Mean Residual Life with Random Censored Sample

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Abstract

The mean residual life(MRL) function gives the expected remaining life of a item at age t. In particular F is said to be an increasing initially then decreasing MRL(IDMRL) distribution if there exists a turning point $t^* \geq 0$ such that $m(s) \leq m(t)$ for $0 \leq s \leq t < t^*$, $m(s) \geq m(t)$ for $t^* \leq s \leq t$. If the preceding inequality is reversed, F is said to be a decreasing initially then increasing MRL(DIMRL) distribution. Hawkins, et al.(1992) proposed test of H_0 : F is exponential versus H_1 : F is IDMRL, and H_0 versus H_1' : F is DIMRL when turning point is unknown. Their test is based on a complete random sample $X_1, \cdots, X_n$ from F. In this paper, we generalized Hawkins-Kochar-Loader test to random censored data.

1. Introduction

Reliability engineers and biostatisticians find it useful to categorize life distributions according to different aging properties. These categories are useful for modeling situations where items improve or deteriorate with age. Let F be a continuous life distribution (i.e., $F(t)=0$ for $t \leq 0$) with the finite first moment. The mean residual life(MRL) function is defined as

$$m(t)=E(X-t\mid X>t)=\frac{\int_t^{\infty}\overline{F}(x)\,dx}{\overline{F}(t)} \quad \text{for each } t\geq 0,$$

where $\overline{F}(t)=1-F(t)$. Each $F(t)$ is uniquely determined by $m(t)$, via the relation

$$\overline{F}(t)=\frac{m(0)\exp\left\{-\int_0^t[m(u)]^{-1}du\right\}}{m(t)}, \quad t\geq 0.$$

Theoretical properties of the MRL function are given in Cox(1962), Kotz and Shambhag(1980), Hall and Wellner(1981) and Bhattacharjee(1982). Applications of it are surveyed in Guess and Proschan(1988), where it is seen that various families of life distributions defined in terms of the MRL(e.g. increasing MRL, decreasing MRL) have been used as models for lifetimes for which such prior information is available. One such family of distributions is called as "an increasing initially then decreasing MRL (IDMRL)" distribution if there exists a turning point $t^*\geq 0$ such that

$$m(s)\leq m(t) \quad \text{for } 0\leq s\leq t< t^*, \quad m(s)\geq m(t) \quad \text{for } t^*\leq s\leq t. \tag{1.1}$$

The dual class of "decreasing initially, then increasing MRL (DIMRL)" distribution is obtained by reversing inequalities on the MRL function in (1.1). IDMRL distributions model life in which, in terms of residual life, aging initially is beneficial but eventually is detrimental. Such life times are exemplified. (i) Human lifetimes; High infant mortality causes the initially IMRL and deterioration with advancing age causes the subsequently DMRL. (ii) Employment time with a given company; The remaining employment time (residual life) of an employee with several years with a company is likely (due to time investment, career value, etc.) to exceed that of an employee with the company only several months. This results in increasing MRL with years of employment up to a certain point t^*, after which, due to retirement, MRL decreases. Also see Guess and Proschan(1988) and the references therein for further applications of the IDMRL family.

If F is an exponential distribution (i.e., $\overline{F}(t)=\exp(-\lambda t)$ for $t\geq 0$, $\lambda>0$), then F is life distribution. F is an exponential distribution if and only if $m(t)$ is constant for all $t\geq 0$. Due to this "no-aging" property, it is of practical interest to know whether a given life distribution F is constant MRL or IDMRL. Therefore, we consider the problem of testing

H_0 : F is constant MRL,

versus

H_1 : F is IDMRL (and not constant MRL),

based on a random censored data $X_1, \cdots, X_n$ from F (unknown). When the dual model is proposed, we test H_0 versus

H_1' : F is DIMRL (and not constant MRL).

In practice, censoring arises in engineering applications and in medical applications with animal studies or clinical trials. For examples, some components on test will still be functioning when the test ends or some patients will have dropped out of the study (by moving to another city, by refusing to continue the treatment, and so on) and other will still be alive at data analysis time.

For complete sample case, this problem was noted by Guess, et al.(GHP, 1986), who obtained tests assuming either (a) t^* is known, or (b) $p = F(t^*)$ is known. In practice, however, such informations are usually lacking, as was noted by GHP(1986). Hence Hawkins, et al.(HKL, 1992) proposed test which do not require these assumptions. Both of GHP(1986) and HKL(1992) tests are based an estimates of functional which distinguish that F is constant MRL against that F is IDMRL.

In the random censoring model, instead of observing a complete sample $X_1, \cdots, X_n$ from the life distribution F, we are able to observe only the pairs (Z_i, δ_i), $i = 1, \cdots, n$ where

$$Z_i = \min\{X_i, Y_i\}, \delta_i = \begin{cases} 1 & \text{if } X_i \le Y_i \text{ (the ith observation is uncensored)} \\ 0 & \text{if } X_i > Y_i \text{ (the ith observation is censored)}. \end{cases}$$

We assume that the censoring random variable $Y_1, \cdots, Y_n$ are independent and identically distributed(iid) according to the continuous distribution G and that the X's and Y's are mutually independent. Therefore, $Z_1, \cdots, Z_n$ are iid according to the distribution L, where $1 - L = \overline{L} = \overline{F}\,\overline{G} = (1-F)(1-G)$.

In the type I censoring model, $Z_i = \min\{X_i, t_c\}$, where t_c is some (preassigned) fixed number which we call the fixed censoring time. In this paper we will consider random censoring model only. Note that this includes type I censoring by simply setting $Y_i = t_c$.

In the random censored data, we can estimate the survival function using

Kaplan-Meier's product limit(PL) estimator or Susarla-Van Ryzin(SV) estimator. Kim and Na(1993) considered the test using PL estimator.

In section 2, we generalize the HKL(1992) test using SV estimator to the random censored data, when the changing point is unknown. The SV estimator $\widehat{F_n}$ of F, introduced by Susarla and Van Ryzin (1978), is;

$$1-\widehat{F_n}(t)=\overline{\widehat{F}}_n(t)=\frac{\sum_{j=1}^{n} I(Z_{(j)}>t)}{n}\prod_{\{t Z_{(i)}\}}\left(\frac{n-i+2}{n-i+1}\right)^{I(\delta_{(i)}=0,\, Z_{(i)}\le t)}$$

where $Z_{(1)},\cdots,Z_{(n)}$ are ordered observations and $\delta_{(i)}$ is the censoring status corresponding to $Z_{(i)}$, $i=1,\cdots,n$. In section 3, Monte Carlo simulations are conducted to compare the power of the our generalized test using SV estimator with that of test using PL estimator for various values of sample size n, α and γ when the MRL function is $m_{\alpha,\beta,\gamma}(t)=\beta+\gamma\exp(-\alpha t)(1-\exp(-\alpha t))$.

2. THE IDMRL TEST PROCEDURE

In this section, we review the IDMRL test procedure in details and propose IDMRL test statistic. IDMRL test procedure is testing H_0 : F is constant MRL versus H_1 : F is IDMRL (not constant MRL). We do not assume knowledge of the proportion p of the population that dies at or before the turning point.

HKL(1992) suggested the functional $\phi(F)$ via the

$$\phi(F)=\sup\{\psi_t(F): t\ge 0\},$$

where $\psi_t(F)=\int_0^t[m(s)f(s)-\overline{F}(s)]\overline{F}(s)\,ds-\int_t^\infty[m(s)f(s)-\overline{F}(s)]\overline{F}(s)\,ds$ and $f(s)=F'(s)$. The functional $\phi(F)$ has properties such that

(1) If F is an exponential distribution, then $\phi(F)=0$.

(2) If F is IDMRL distribution, then $\psi_t(F)$ is strictly increasing (decreasing) for $t<t^*$ ($t>t^*$) and $\phi(F)=\psi_{t^*}(F)>0$, and distinguishes that F is constant MRL against that F is IDMRL.

Using integration by parts, we can rewrite $\psi_t(F)$ as

$$\begin{aligned}\psi_t(F) &= \int_0^t [\bar{F}(x) - 2\bar{F}^2(x)]dx + \int_t^{\infty}[(-1+2p)\bar{F}(x) + 2\bar{F}^2(x)]dx \\ &\equiv \int_0^{\infty} A(F(t))dt,\end{aligned} \tag{2.1}$$

where $p = F(t)$ and

$$A(s) = \begin{cases} 1 - s - 2(1-s)^2, & \text{for } 0 \le s \le p \\ (-1+2p)(1-s) + 2(1-s)^2, & \text{for } p < s \le 1. \end{cases}$$

HKL(1992) formed their statistic by replacing F by the empirical distribution function in (2.1). In our random censored model, we replace F by SV estimator $\widehat{F_n}$ in (2.1). In order to obtain test statistic, we need the following theorem.

THEOREM 2.1 Let $\psi_t(\hat{F}_n) = \int_0^{\infty} A(\widehat{F_n}(s))ds$ and $\psi_t(F) = \int_0^{\infty} A(F(s))ds$. Suppose following conditions (2.2) and (2.3) hold;

$$\int_0^{F^{-1}(1)} [\bar{G}(t)]^{-1} dF(t) < \infty \tag{2.2}$$

$$\int_0^{\infty} [\bar{F}^2(t) \int_0^t (\bar{F}^2\bar{G})^{-1} dF]^{1/2} dt < \infty. \tag{2.3}$$

Then

$$\sqrt{n}[\psi_t(\hat{F}_n) - \psi_t(F)] \xrightarrow{d} N(0, \sigma^2(F, G)) \quad \text{as } n \to \infty$$

where

$$\sigma^2(F, G) = \int_0^{\infty}\int_0^{\infty} A'(F(x))A'(F(y))\bar{F}(x)\bar{F}(y) \int_0^{\min\{x,y\}} [\bar{F}^2\bar{G}]^{-1} dF dx dy. \tag{2.4}$$

Proof

$$\begin{aligned}\sqrt{n}[\psi_t(\widehat{F_n}) - \psi_t(F)] &= \sqrt{n}\int_0^{\infty} A(\bar{F}_n(t) - A(F(t))]dt \\ &= \sqrt{n}\int_{I_1}[A(\widehat{F_n}(t)) - A(F(t))]dt + \sqrt{n}\int_{I_2}[A(\hat{F}_n(t)) - A(F(t))]dt \\ &= \int_0^{\infty} A'(F(t))\Lambda_n(t)dt + R_{1n} + R_{2n}\end{aligned}$$

where

$$R_{1n}=\int_{I_1}[A'\{b_n(t)\ \hat{F}_n(t)+(1-b_n(t))F(t)\}-A'(F(t))]\Lambda_n(t)dt,$$
$$R_{2n}=\int_{I_2}[A'\{b_n(t)\ \hat{F}_n(t)+(1-b_n(t))F(t)\}-A'(F(t))]\Lambda_n(t)dt,$$

$\Lambda_n(t)=\sqrt{n}(\hat{F}_n(t)-F(t))$, $I_1=[0,F^{-1}(p)]$, $I_2=(F^{-1}(p),\infty)$ and $0\le b_n(t)\le 1$ for $t\ge 0$. Note that b_n exists by the mean value theorem. We will show that R_{1n} and R_{2n} converge in probability to 0 as $n\to\infty$.

$$|R_{1n}|\le \sup_t|A'[b_n(t)\ \hat{F}_n(t)+(1-b_n(t))F(t)]-A'(F(t))|\int_0^\infty|\Lambda_n(t)|dt.$$

Susarla and Van Ryzin (1978) show that $\hat{F}_n(t)$ converges almost sure(a.s.) to $F(t)$ for all $0\le t<t^0=\min\{F^{-1}(1),G^{-1}(1)\}=F^{-1}(1)$ under condition (2.2). Since F is continuous, it follows by a standard argument(pp. 132-134 in Chung, 1974) that $\hat{F}_n(t)\to F(t)$ uniformly in t a.s.. Since A' is bounded and continuous at $t\in I_1$,

$$\sup_t|A'[b_n(t)\ \hat{F}_n(t)+(1-b_n(t))F(t)]-A'(F(t))|\to 0 \text{ a.s..}$$

We also use the fact that Susarla and Van Ryzin(1978) show that $\{\Lambda_n(t),0\le t\le T\}$ converges weakly to $\{\Lambda(t),0\le t\le T\}$ if $F(T)<1$, where $\{\Lambda(t),0\le t\le F^{-1}(1)\}$ is a Gaussian process with mean zero and covariance $\mathrm{Cov}(\Lambda(x),\Lambda(y))=\overline{F}(x)\overline{F}(y)\int_0^{\min\{x,y\}}(\overline{F}^2\overline{G})^{-1}dF$. Next note that $\int_0^\infty\Lambda(t)dt$ is proper random variable since

$$E[\int_0^\infty|\Lambda(t)|dt]=\int_0^\infty E|\Lambda(t)|dt\le\int_0^\infty[E(\Lambda^2(t)]^{1/2}dt$$
$$=\int_0^\infty[\overline{F}^2(t)\int_0^t(\overline{F}^2\overline{G})^{-1}dF]^{1/2}dt<\infty$$

under condition (2.3). By the continuous mapping theorem (see, p. 30 in Billingsly, 1968), $\int_0^\infty |\Lambda_n(t)|\,dt \xrightarrow{d} \int_0^\infty |\Lambda(t)|\,dt$ and hence $\int_0^\infty |\Lambda_n(t)|\,dt$ is bounded in probability. Thus R_{1n} converges in probability to 0 as $n \to \infty$. A similar argument holds for R_{2n}. Therefore, by the continuous mapping theorem and Slusky's theorem, we get

$$\sqrt{n}[T(\hat{F}_n(t) - T(F(t))] \xrightarrow{d} \int_0^\infty A'(F(t))\Lambda(t)dt \text{ as } n \to \infty.$$

By the theory of stochastic integration (see chapter 5 in Cramer and Leadbetter, 1967), we can obtain that the limiting random variable is normal with mean zero and variance given by (2.4). □

Corollary 2.2 If F is an exponential distribution with mean θ, then

$$\sqrt{n}(T(\hat{F}_n)) \xrightarrow{d} N(0, \theta^2\sigma_0^2(F,G)) \text{ as } n \to \infty,$$

where

$$\sigma_0^2(F,G) = \int_0^1 g(z)\{\overline{L}(-\theta\ln(z))\}^{-1}dz \text{ with } g(z) = \frac{A^2(1-z)}{z}. \quad (2.5)$$

Result holds by straightforward calculation from (2.4).

Since the null asymptotic variance $\theta^2\sigma_0^2(F,G)$ depends on the nuisance parameter θ and G, we need consistent estimator of θ and $\sigma_0^2(F,G)$ in order to make our scale-invariant test. Under the assumption that the mean $\int_0^\infty \overline{F}(x)\,dx$ is finite and suitable regularity on the amount of censoring, $\theta_n = \int_0^\infty \overline{\hat{F}}_n(x)dx$ is a consistent estimator of θ. Also we can obtain a consistent estimator of $\sigma_0^2(F,G)$, $\hat{\sigma}_0{}^2 = \int_0^1 g(z)\{\overline{L}_n(-\theta_n\ln(z))\}^{-1}dz$, by replacing $\overline{L}$ with $\overline{L}_n$, the empirical survival function of $Z_1,\cdots,Z_n$, and θ with θ_n in (2.5).

Now, we propose the following scale invariant test statistic using random censored data:

$$\phi^*(\hat{F}_n) = \frac{\sqrt{n}\max_{\{1\le k\le n\}}\psi_k(\hat{F}_n)}{\theta_n\widehat{\sigma_0}}.$$

The IDMRL test procedure rejects the null hypothesis of exponentiality in favor of the alternative H_1 : F is IDMRL (not constant MRL) at the approximate significant level α if $\phi^*(\hat{F}_n) \ge z_\alpha$.

Analogously, the approximate significant level α test of H_0 versus H_1' : F is DIMRL (not constant MRL) reject H_0 if $\phi^*(\hat{F}_n) \le -z_\alpha$.

3. POWER COMPARISON

In order to compare the power of the generalized HKL test using SV estimator with that using PL estimator, a Monte Carlo simulation is performed. For Monte Carlo study we used the subroutine IMSL of the package FORTRAN on IBM SP2 super computer at Seoul National University. The random numbers are generated from

$$\overline{F}_{\alpha,\beta,\gamma}(x) = \left\{\frac{\beta}{\beta+\gamma}\exp(-\alpha x)(1-\exp(-\alpha x))\right\}\left\{\frac{[1+d]^2-c^2}{[\exp(\alpha x)+d]^2-c^2}\right\}^{1/2\alpha\beta}$$
$$\times\left\{\frac{\exp(\alpha x)+d-c}{\exp(\alpha x)+d+c}\,\frac{1+d+c}{1+d-c}\right\}^{\gamma/4\alpha\beta^2 c}, \quad x\ge 0$$

where $d=\gamma/2\beta$, $c^2=[4(\beta/\gamma)+1]/[4(\beta/\gamma)^2]$.
This distribution has MRL function $m_{\alpha,\beta,\gamma}(x)=\beta+\gamma\exp(-\alpha x)(1-\exp(-\alpha x))$, $x\ge 0$. The motivation(see HKL, 1992) for choosing $\overline{F}_{\alpha,\beta,\gamma}$ is that $\overline{F}_{\alpha,\beta,\gamma}$ has IDMRL structure with turning point $t^*=\frac{1}{\alpha}\ln 2$ for any choice of (α,β,γ) and $\overline{F}_{\alpha,\beta,\gamma}$ is exponential distribution if $\gamma=0$. The censoring random numbers are generated from $\overline{G}_{\alpha,\beta,\gamma,\lambda}(x)=[\overline{F}_{\alpha,\beta,\gamma}(x)]^\lambda$ for $\lambda=\frac{1}{4}$ and $\frac{1}{9}$, here λ is viewed as a censoring parameter since the probability that an observation will be censored is $\Pr(\delta_i=0)=\frac{\lambda}{\lambda+1}$.

<Table 1> Monte Carlo power comparison from 1000 replications with $\alpha=1$ and $\beta=1$

		λ=1/9 10% censoring						λ=1/4 20% censoring					
γ		1		2		3		1		2		3	
sample size	α	PL	SV	PL	SV	PL	SV	PL	SV	PL	SV	PL	SV
10	0.10	0.876	0.878	0.908	0.915	0.918	0.926	0.847	0.850	0.876	0.875	0.897	0.901
	0.05	0.756	0.762	0.789	0.811	0.846	0.853	0.701	0.709	0.765	0.767	0.793	0.813
	0.01	0.358	0.365	0.497	0.504	0.601	0.617	0.355	0.373	0.449	0.468	0.522	0.547
20	0.10	0.881	0.887	0.944	0.948	0.990	0.990	0.855	0.856	0.947	0.944	0.985	0.984
	0.05	0.761	0.761	0.884	0.884	0.973	0.977	0.702	0.711	0.874	0.888	0.951	0.955
	0.01	0.444	0.445	0.693	0.696	0.874	0.878	0.369	0.377	0.645	0.656	0.790	0.808
30	0.10	0.900	0.903	0.984	0.983	0.997	0.997	0.865	0.863	0.978	0.975	0.997	0.995
	0.05	0.799	0.795	0.957	0.960	0.993	0.993	0.748	0.753	0.939	0.938	0.989	0.989
	0.01	0.493	0.495	0.846	0.846	0.963	0.967	0.447	0.452	0.761	0.782	0.927	0.937
40	0.10	0.927	0.930	0.990	0.989	0.998	0.999	0.903	0.907	0.990	0.990	0.999	0.999
	0.05	0.849	0.851	0.983	0.983	0.998	0.998	0.814	0.806	0.974	0.975	0.999	0.999
	0.01	0.576	0.572	0.900	0.901	0.989	0.990	0.501	0.511	0.882	0.894	0.979	0.983
50	0.10	0.947	0.948	0.998	0.997	0.998	0.999	0.937	0.932	0.994	0.994	0.999	0.999
	0.05	0.880	0.884	0.993	0.992	0.998	0.998	0.868	0.853	0.986	0.986	0.998	0.999
	0.01	0.651	0.650	0.955	0.954	0.998	0.998	0.585	0.589	0.913	0.927	0.989	0.996
75	0.10	0.970	0.972	0.999	0.999	0.999	0.999	0.963	0.957	0.999	0.999	0.999	0.999
	0.05	0.935	0.930	0.998	0.998	0.999	0.999	0.917	0.914	0.999	0.999	0.999	0.999
	0.01	0.772	0.765	0.989	0.990	0.999	0.999	0.724	0.722	0.983	0.993	0.996	0.999
100	0.10	0.987	0.986	0.999	0.999	0.999	0.999	0.975	0.976	0.999	0.999	0.999	0.999
	0.05	0.962	0.959	0.998	0.998	0.999	0.999	0.954	0.953	0.999	0.999	0.999	0.999
	0.01	0.879	0.867	0.988	0.988	0.999	0.999	0.851	0.835	0.993	0.996	0.999	0.999

<Tables 1> and 2 contain Monte Carlo estimated powers based on 1000 replications of sample size n=10, 20, 30, 40, 50, 75 and 100 from $\overline{F}_{\alpha,\beta,\gamma}$ and $\overline{G}_{\alpha,\beta,\gamma,\lambda}$ for $\beta=1$ and a selection of (α,γ,λ). Looking at tables, first note that the test using SV estimator generally dominates the test using PL estimator. Second feature of tables is that for fixed α, the power of all the tests increase rapidly as γ increases. This is generally to be expected since the width of $m(x)$

increases as γ increases. Another feature of tables is that for fixed γ, the power of tests increases as α increases (i.e., the turning point t^* decreases).

From the simulation results, we may conclude that speed of convergence to normality of the generalized HKL test using SV estimator is faster than that of the generalized HKL test using PL estimator.

<Table 2> Monte Carlo power comparison from 1000 replications with $\alpha=2$ and $\beta=1$

		$\lambda=1/9$ 10% ensoring						$\lambda=1/4$ 20% nsoring					
γ		1		2		3		1		2		3	
sample size	α	PL	SV	PL	SV	PL	SV	PL	SV	PL	SV	PL	SV
10	0.10	0.927	0.930	0.975	0.978	0.981	0.985	0.924	0.929	0.958	0.962	0.981	0.979
	0.05	0.840	0.844	0.940	0.946	0.960	0.962	0.825	0.831	0.909	0.912	0.939	0.943
	0.01	0.563	0.569	0.751	0.767	0.871	0.881	0.528	0.540	0.708	0.706	0.821	0.840
20	0.10	0.963	0.963	0.997	0.997	0.999	0.999	0.957	0.956	0.995	0.994	0.998	0.997
	0.05	0.908	0.914	0.991	0.990	0.999	0.999	0.891	0.896	0.982	0.985	0.995	0.997
	0.01	0.650	0.656	0.942	0.944	0.991	0.991	0.627	0.638	0.903	0.913	0.984	0.988
30	0.10	0.979	0.980	0.999	0.999	0.999	0.999	0.976	0.975	0.997	0.998	0.999	0.999
	0.05	0.938	0.938	0.998	0.998	0.999	0.999	0.920	0.932	0.997	0.998	0.999	0.999
	0.01	0.709	0.722	0.981	0.984	0.998	0.998	0.702	0.711	0.971	0.979	0.995	0.997
40	0.10	0.981	0.986	0.999	0.999	0.999	0.999	0.976	0.980	0.999	0.999	0.999	0.999
	0.05	0.960	0.965	0.999	0.999	0.999	0.999	0.946	0.947	0.998	0.999	0.999	0.999
	0.01	0.832	0.838	0.994	0.994	0.999	0.999	0.779	0.792	0.991	0.994	0.999	0.999
50	0.10	0.989	0.988	0.999	0.999	0.999	0.999	0.988	0.988	0.998	0.999	0.999	0.999
	0.05	0.967	0.968	0.999	0.999	0.999	0.999	0.962	0.968	0.998	0.999	0.999	0.999
	0.01	0.869	0.869	0.999	0.999	0.999	0.999	0.836	0.848	0.996	0.997	0.999	0.999
75	0.10	0.995	0.995	0.999	0.999	0.999	0.999	0.994	0.997	0.999	0.999	0.999	0.999
	0.05	0.991	0.994	0.999	0.999	0.999	0.999	0.990	0.993	0.999	0.999	0.999	0.999
	0.01	0.957	0.958	0.999	0.999	0.999	0.999	0.936	0.942	0.999	0.999	0.999	0.999
100	0.10	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
	0.05	0.996	0.996	0.999	0.999	0.999	0.999	0.995	0.997	0.999	0.999	0.999	0.999
	0.01	0.983	0.987	0.999	0.999	0.999	0.999	0.970	0.972	0.999	0.999	0.999	0.999

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