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Stochastically Dependent Sequential Acceptance Sampling Plans

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Abstract

In a traditional sequential acceptance sampling plan, it is assumed that the sampled items are independent each other. In this paper, stochastically dependent sequential acceptance sampling plans are dealt when there exists dependency between sampled items. Monte-Calro algorithm is used to find the acceptance and rejection probabilities of a lot. The number of defectives for the test to be accepted and rejected in probability ratio sequential test can be found by using these probabilities. The formula for measures of performance of these sampling plans is developed. Type I and II error probabilities are estimated by simulation. This research can be applied to sequential sampling procedures in place of control charts where there is a recognized and necessary dependency during the production processes. Also, dependent multiple acceptance sampling plans can be derived by extending this sequential sampling procedure. As a numerical example, a Markov dependent process model is given, and the characteristics of the sampling plans are examined according to the change of the dependency factor.

1. Introduction

Sequential sampling plans are of special interest when the formation of inspection for lot-by-lot acceptance may be impracticable or artificial as in conveyor-line production, or when there is an important need for rectifying the quality of a product as it is manufactured. This is especially true where observations or outcomes, such as runs, naturally occur in succession rather than in parallel. The assumption of the independence between the observations has been

used and the probability is one that the sequential process will eventually terminate. Practically, however, it might be difficult to attain a state of statistical control with this strict sense; dependency or correlation between items and other systematic time related effects are sometimes substantial. Until recently the general methodology of the correlated sequential sampling plans has not been studied much because of the varieties of the dependent process model and the difficulties of finding the necessary probabilities.

There have been several studies about the dependent sampling plans. Bhat, Lal and Karunaratne(1990) approached the single acceptance sampling problem with an augmented Markov chain matrix. These analytical studies are limited to special cases only, mainly Markov processes, due to the mathematical complexities of the probability structure. Moreover, most efforts have been concentrated on single sampling plans. Even for such a single sampling plan, the analysis is extremely complicated. Nelson(1993) has developed a method for estimating single acceptance sampling plans for general production process models that can be simulated.

Let us examine an example of a dependent production process. Suppose there is a process where a product is going to be produced on a machine. Consider a production process having a lot of size N. We assume that the production has the stochastic output process $\{X_1, X_2, \cdots, X_N\}$. If the state of the process producing the ith item is good, then $X_i=0$, and if the process is bad, then $X_i=1$. The inspector tests each item sequentially until a decision is made according to the constructed sampling plan. Assume the probability of producing a defective part varies according to the previous item, i.e., the process is specified completely by the following Markov transition probability matrix[Bhat and Lal, 1988, Bhat et al. 1990] :

With the admissible range $1-\min\{1/p_i, 1/(1-p_i)\}<\rho<1$, the matrix is

$$P=\begin{bmatrix} 1-p_i(1-\rho) & p_i(1-\rho) \\ (1-p_i)(1-\rho) & p_i+\rho(1-p_i) \end{bmatrix},$$

where p_i, i=1, 2 are Acceptable Quality Level(AQL) and Lot Tolerance Percent Defectives(LTPD) respectively. The dependency factor ρ is lag 1 serial correlation of the process. If ρ=0, successive items are independent. As ρ increases to 1 a succeeding item is more likely to be the same as the previous item, and as ρ

decreases to -1 a succeeding item is more likely to be different from the previous item. Such a process can be quite common in manufacturing. Broadbent(1958) proposed a work estimating the lag 1 serial correlation ρ. The process normally produces good items, but if a defect is caused, the same cause produces a run of bad items.

Now suppose a new tool is used at the beginning of production. We want to produce a lot of 400 items and sampled items are examined successively. Assume the AQL is 0.01 with the acceptable tool wear with type I error probability α of 0.1. A more rapid tool wear makes the product quality level poorer. Also assume the prespecified LTPD is 0.1 with type II error probability β of 0.1. In this case, traditional sequential sampling plans can not be used because of the lack of independence, i.e., varying defective ratios. The purpose of this paper is the development of the methodology of dependent sequential sampling plans in this situation.

2. Dependency and Estimation of a Rejection Probability of a Lot

To design sampling plans we need the following three probabilities at two quality levels AQL and LTPD. Let us define the following variables. For i=1, 2, ···, N and j=1, 2, ···, i,

$C_i = \sum_{k=1}^{i} X_k$: cumulative number of defective items discovered through item i,

$f_{ij} = \Pr(C_i = j)$: probability that there are exactly j defectives among the inspected i items,

$\gamma_{ij} = \Pr(C_i \geq j)$: rejection probability of a lot such that there are j or more defectives among i items,

D_j : order of the inspected item on which the jth defective item is found, and

$v_{ij} = \Pr(D_j = i)$: probability that the jth defective item occurs at the ith item.

Since the two events $\{C_i \geq j\}$ and $\{D_j \leq i\}$ are equivalent, the relationship between three probabilitie f_{ij}, γ_{ij}, and v_{ij} can be found. Note that $\gamma_{i0} = 1$ and $\gamma_{i,i+1} = 0$. For all i=1, 2, ···, N and j=1, 2, ···, i,

$$\gamma_{ij} = \Pr(C \geq j) = \Pr(D_j \leq i) = \sum_{k=1}^{i} \Pr(D_j = k)$$
$$= \sum_{k=1}^{j-1} \Pr(D_j = k) + \sum_{k=j}^{i} \Pr(D_j = k)$$
$$= \sum_{k=j}^{i} \Pr(D_j = k) = \sum_{k=j}^{i} v_{kj}.$$

Thus, we know that $v_{ij} = \gamma_{ij} - \gamma_{i-1,j}$, and $f_{ij} = \gamma_{ij} - \gamma_{i,j+1}$.

To find the rejection probability γ_{ij}, the concept of Monte-Carlo integration can be applied[Ripley, 1987]. Suppose we wish to evaluate the parameter θ such that

$$\theta = E[\Phi(X)] = \int \Phi(x) f(x) dx,$$

where $f(x)$ is a probability density function of a continuous random variable X. To estimate θ, sample $X_1, X_2, \cdots, X_m$ independently from the distribution having $f(x)$ and form $\hat{\theta} = \sum_{i=1}^{m} \Phi(x_i)/m$, where m is the number of independent replication number in simulation. If we want to find $\theta = \Pr(X \geq c)$, the obvious way to estimate θ is

$$\theta = \int_{-\infty}^{\infty} I(X \geq c) f(x)\, dx,$$

where $I(X \geq c)$ is 1 if $X \geq c$ and 0 otherwise. We get the estimator of θ by substituting $\Phi(X)$ to $I(X \geq c)$:

$$\hat{\theta} = \sum_{i=1}^{m} \Phi(x_i)/m = \sum_{i=1}^{m} I(X_i \geq c)/m.$$

When X is a discrete random variable, only the integral is changed to summation and the estimation procedure is the same as the continuous case. In our simulation, we want to estimate $\gamma_{ij} = \Pr(C_i \geq j)$. By applying Monte-Carlo integration to the discrete case, we get

$\hat{\gamma}_{ij} = \sum_{k=1}^{m} I(C_i^k \geq j)/m$, where C_i^k is the kth replication of C_i.

Note that $\hat{\gamma}_{ij}$ forms a lower triangular array since i=1, 2, ···, N and range of C_i is from 0 to i. To form the array [$\hat{\gamma}_{ij}$], m independent replications are

required for each i=1, 2, ···, N and j=1, 2, ···, i. So Maximum $mN(N+1)/2$ computations are required for making a whole rejection probability array [$\widehat{\gamma_{ij}}$]. We need a procedure to form the array more efficiently.

Let $S_{ij}=\sum_{k=1}^{m} I(C_i^k \geq j)$. Then S_{ij} represents the total number of simulation replications that j or more defective items are found among i items. The estimator $\widehat{\gamma_{ij}}$ is obtained from $\widehat{\gamma_{ij}}=S_{ij}/m=\overline{S_{ij}}$. But instead of storing the real number array [$\overline{S_{ij}}$], storing the integer valued array [S_{ij}] is preferred, which reduces the numerical truncation error and enables fast accessing. The following algorithm shows a pseudo code for the generation of a rejection probability array [S_{ij}].

Algorithm : Generation of [S_{ij}] by Monte-Carlo integration :

0. Initialize S_{ij}=0 for i=1, 2, ···, N and j=1, 2, ···, i.
1. Repeat for k = 1 to m.
2. Set c=0.
3. Repeat for i=1 to N.
4. Generate process value X_i for a specified dependent process model.
5. If the ith item is rejected(i.e., X_i=1) then increase c by 1.
6. Repeat for j=1 to i.
7. If $c \geq j$ then increase S_{ij} by 1.
8. Next j
9. Next i
10. Next k

In the above algorithm, c represents the number of defective items found by the inspection procedure. The step numbers 6 through 8 are just an adaptation of the Monte-Carlo integration procedure for estimating. But we can make a more efficient algorithm by replacing the step number 6 through 8 as follows :

6. Repeat for j=1 to c.
7. Increase S_{ij} by 1.
8. Next j

This modification is very simple, but reduces the computations required by step

6 remarkably. Most practical lots have a small fraction of defectives, e.g., 1%, 2%, ··· usually below 20%. The number of defective items c among a sample of size i is far less than the sample size i when i is large, i.e., $c<<i$ and there is no need of IF comparisons. The smaller the fraction of defective, the more loop reduction is achieved. Generation of $[S_{ij}]$ requires a great deal of simulation time. Therefore it is important to make $[S_{ij}]$ efficiently. We can also apply the variance reduction techniques to reduce the replication number. See Nelson(1987) and Wilson(1984) for other general variance reduction techniques.

3. Determination of Acceptance and Rejection Boundaries

The implementation of SPRT(Sequential Probability Ratio Test) starts from finding the acceptance and rejection numbers at each sample point. For i=1, 2, ···, N, define the probability ratio,

$$l(x_1, \cdots, x_i) = p_1(x_1, \cdots, x_i)/p_0(x_1, \cdots, x_i),$$

where $p_1(x_1, \cdots, x_i)$ is the joint probability density of the first i items when the fraction of defectives is LTPD, and $p_0(x_1, \cdots, x_i)$ is the joint probability density of the first i items when the fraction of defectives is AQL. The probability ratio test is rejected if $l(x_1, \cdots, x_i) \geq (1-\beta)/\alpha$and accepted if $l(x_1, \cdots, x_i) \leq \beta/(1-\alpha)$. Wald (1947) proved that the above fundamental inequalities remain valid for a test procedure in spite of the dependency of the successive observations, provided that the probability is one, e.g., the procedure will eventually terminate.

In general, it is difficult to find the joint probability densities $p_0(x_1, \cdots, x_i)$ and $p_1(x_1, \cdots, x_i)$ analytically if dependency exists between the processes. However, by using simulation we can find the dependent joint probability densities as in the previous section. Let us define

f_{ij}^{LTPD} : the joint probability that there are exactly j defectives among the i items when the fraction of defectives is LTPD,

f_{ij}^{AQL} : the joint probability that there are exactly j defectives among the i items when the fraction of defectives is AQL,

r_i : the number of defectives for the test to be rejected at the sample number i, and

a_i : the number of defectives for the test to be accepted at the sample number i.

The rejection number r_i and acceptance number a_i at the ith sample point can be found from the following equations. For i=1, 2, ···, N, we get

$$r_i = \text{minimum } j \text{ satisfying } f_{ij}^{LTPD} / f_{ij}^{AQL} \geq (1-\beta)/\alpha,$$
$$a_i = \text{maximum } j \text{ satisfying } f_{ij}^{LTPD} / f_{ij}^{AQL} \leq \beta/(1-\alpha).$$

The minimum j value in determining r_i and the maximum j value in determining a_i give the narrowest boundary width which reduces the difference minimally between the actual and estimated parameters α and β. The paths of r_i and a_i are decision boundaries of test procedures and generally form step function shapes. The inspection size can be quite large in SPRT theoretically, although the probability is very low. We will only consider up to the curtailed inspection point which is the lot size N for practical use. The probability that the test exceeds over this point is almost zero. Once the acceptance and rejection numbers are found, we can compute the acceptance and rejection probabilities at each sample point. These probabilities are necessary for computing the measures of performance. Let us define the following events :

A_i={The SPRT is accepted at the ith sample point},

R_i={The SPRT is rejected at the ith sample point},

E_i={The SPRT is finished, i.e., either accepted or rejected at the ith sample point}.

Then we have

$$\Pr[E_i] = \Pr[A_i \cup R_i] = \Pr[A_i] + \Pr[R_i].$$

The probability of acceptance $\Pr[A_i]$ and the probability of rejection $\Pr[R_i]$ at the ith sample point can be found by simulation or analytical methods.

4. Measures of Performance

We will consider three measures of performance : Average Sample Number(ASN),

Average Total Inspection(ATI), and Average Outgoing Quality(AOQ). As we have known the acceptance and rejection probabilities at each sample point, ASN can be computed as follows.

$$\text{ASN} = \sum_{i=1}^{N} iPr[E_i] = \sum_{i=1}^{N} i\{\Pr[A_i] + \Pr[R_i]\}.$$

Since SPRT is an open sequential test, i.e., the sample size can be infinite in theory, we might expect the distribution of the sample size to skew. Some asymptotic properties of the sample size distribution were obtained by Wald. The ASN has been the point of interest in sequential sampling plans by many researchers[Johnson, 1961]. Corneliussen and Ladd(1970) developed a recursive method for exact calculation of the sample size distribution and ASN. They found that Wald's formula for the ASN is beneath the true ASN at maximum by about 20%. Thus, the saving in observations resulting from use of an SPRT is very much an average property, and in particular cases, an SPRT may require many more observations than a nonsequential plan having the same probabilities of error. The work of Baker(1950), Page(1954), and Kemp(1958) regarding the ASN all indicates that the approximation of Wald's ASN formula can sometimes substantially underestimate the true ASN, especially if the starting point of the SPRT is close to one of the boundaries. Wald and Wolfowitz(1948) proved that SPRT produces the lowest possible ASN.

If the SPRT is accepted, then the total inspection is simply i. If the SPRT is rejected, then the whole lot will be inspected, hence the total inspection is the lot size N. Therefore, the ATI is

$$\text{ATI} = \sum_{i=1}^{N} i\Pr[A_i] + \sum_{i=1}^{N} N\Pr[R_i].$$

In the above formula, we have used the assumption that the SPRT is finished at the curtailed inspection point. If the test is not finished at this point, then truncation occurs to prevent an excessive test size, and the lot is assumed to be accepted.

To find the AOQ we need to know the average number of defective items remaining in a lot and the average number of items actually shipped after inspection. We can obtain the expectation of C_n as follows :

$$E[C_n] = \sum_{j=0}^{\infty} \Pr[C_n > j] = \sum_{j=1}^{\infty} \Pr[C_n \geq j] = \sum_{j=1}^{n} \gamma_{nj}.$$

Let $[N_D]_i^j$ represent the number of defective items among the item number i through j, where $j > i$. The expected value of can be found as follows :

$$E\{[N_D]_i^j\} = E\{[N_D]_1^j\} - E\{[N_D]_1^{i-1}\}$$
$$= E[C_j] - E[C_{i-1}] = \sum_{k=1}^{j} \gamma_{jk} - \sum_{k=1}^{i-1} \gamma_{(i-1),k}.$$

Let N_D be the total number of defective items remaining in a lot of size N after inspection. Also let $N_D|R_i$ and $N_D|A_i$ be the number of defective items remaining in a lot of size N under the event R_i and A_i for i=1, ···, N. Then the expected value of N_D is

$$E[N_D] = \sum_{i=1}^{N} E[N_D|R_i]\Pr[R_i] + \sum_{i=1}^{N} E[N_D|A_i]\Pr[A_i].$$

The first two terms of the right hand side become 0 since there are no defective items in a rejected lot. Therefore, we have

$$E[N_D] = \sum_{i=1}^{N} E[N_D|A_i]\Pr[A_i],$$

where

$$E[N_D|A_i] = E\{[N_D]_{i+1}^N\} = \sum_{j=1}^{N} \gamma_{Nj} - \sum_{j=1}^{i} \gamma_{ij}.$$

Now let N_S represent the number of items actually shipped after inspection. If all defective items found are replaced with good ones, then $N_S = N$. When the defective items found are discarded and not replaced with good ones, $E[N_S]$ is calculated as follows.

If the procedure is rejected at sample number i, then the actual number of shipped items is $N - r_i$ because the r_i defective items are discarded, and the probability of this event is $\Pr[R_i]$. Similarly, if the procedure is accepted at sample number i, then the actual number of shipped items is $N - a_i$ because the a_i defective items are discarded, and the probability of this event is $\Pr[A_i]$. Therefore, we get the AOQ from the definition AOQ= $E[N_D]/E[N_S]$, where $E[N_S] = \sum_{i=1}^{N}$ $\{(N - r_i)\Pr[R_i] + (N - a_i)\Pr[A_i]\}$.

Type I error probability is $\alpha = P_{AQL}\{l(x_1, x_2, \cdots, x_N) \geq A\}$, where $A = (1-\beta)/\alpha$ and the index AQL means that the experiment is conducted under the condition that the fraction of defectives is AQL. The obvious way of estimation is to calculate the following estimator value under the condition that the fraction of defectives is AQL. Applying the Monte-Carlo intergration, the estimator is

$$\hat{\alpha} = \sum_{k=1}^{m} I\{l(x_1, x_2, \cdots, x_{N_k}) \geq A\}/m,$$

where N_k is a random variable called stopping time having values {1, 2, ···, N} in the kth independent experiment. The event $\{ l(x_1, x_2, \cdots, x_{N_k}) \geq A\}$ is equivalent to $\{ C_{N_k} \geq r_{N_k}\}$ for some N_k at which the test is finished. Similarly, type II error probability is $\beta = P_{LTPD}\{l(x_1, x_2, \cdots, x_N) \leq B\}$, where $B = \beta/(1-\alpha)$. Under the condition that the observations are conducted with LTPD, the estimator of β is

$$\hat{\beta} = \sum_{k=1}^{m} I\{l(x_1, x_2, \cdots, x_{N_k}) \leq B\}/m.$$

The event $\{l(x_1, x_2, \cdots, x_{N_k}) \leq B\}$ is equivalent to $C_{N_k} \leq a_{N_k}$ for some N_k at which the test is finished.

5. Numerical Examples : Markov Dependent Process Model

We will examine the Markov dependent process model with the dependency factor ρ varying from -0.8 to 0.8 with stepsize 0.2. Assume that the following parameters are set.

N	m	AQL	LTPD	α	β
400	400000	0.01	0.1	0.1	0.1

<Table 1> shows the first sample point i for the test to be rejected when the number of defectives is r_i. For example, when ρ=-0.8, if 2 defective items are found between the sample number 3 and 17, or 3 defective items are found between the sample number 18 and 32, etc., then the test is rejected. Similarly, <Table 2> shows the first sample point i for the test to be accepted when the

number of defectives is α_i. From <Table 1>, we see that the first sample point becomes smaller as the absolute value of dependency factor ρ deviates from 0 farther. This means that if there exists more negative or positive dependency between the process data then the more defectives or samples are required for the test to be rejected. However, <Table 2> shows this situation is different in case that the test is accepted. The first sample point point becomes larger as the dependency factor ρ approaches to +1. This means that for the test to be accepted the fewer samples are required as ρ approaches to -1 at the fixed number of defectives. In other words, the more defectives are required if the process is more negatively dependent at the fixed sample point. On the contrary, the fewer defectives are required if the process is more positively dependent at the fixed sample point.

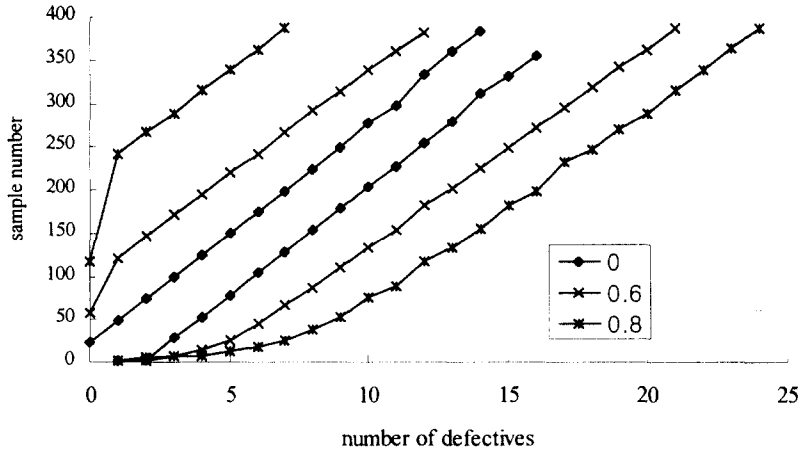
< Table 1 > The first sample point i for the test to be rejected when the number of defectives is r_i and the dependency factor is ρ

r_i \ ρ	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0	13	15	17	20	24	29	39	58	117
1	28	31	36	42	49	61	81	121	242
2	43	48	55	63	74	86	105	146	266
3	57	64	73	85	99	110	130	171	289
4	72	81	92	107	124	134	154	195	315
5	87	97	111	129	150	158	178	220	339
6	102	115	130	151	175	181	201	241	362
7	116	131	149	173	199	206	224	267	388
8	132	148	166	195	223	229	250	291	
9	145	164	185	216	248	254	272	314	
10	158	179	203	240	278	277	294	338	
11	176	197	223	261	297	302	320	360	
12	187	213	242	280	333	328	341	382	
13	212	229	259	306	360	350	364		
14	221	239	267	330	384	371	390		
15	228	259	307	349		396			
16	252	274	315	353					
17	263	288	353	399					
18	303	325	382						
19	331	353							
20	347	370							
21	373								
22	382								
23	391								

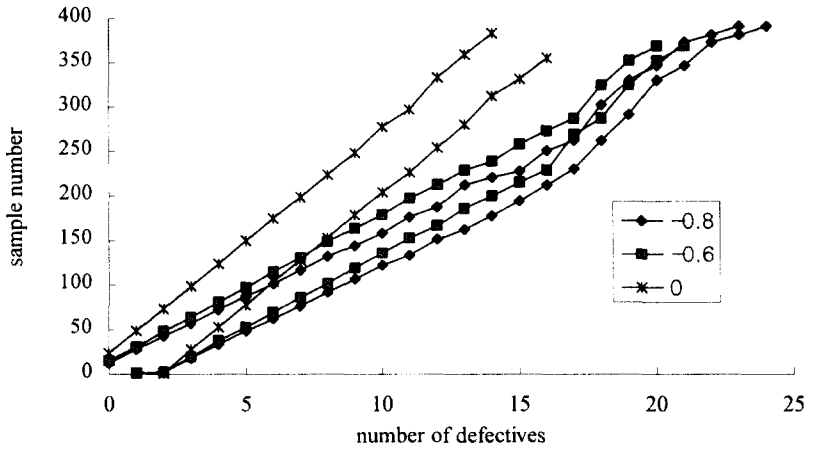
< Table 2 > The first sample point i for the test to be accepted when the number of defectives is a_i and the dependency factor is ρ

a_i \ ρ	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
1	1	1	1	1	1	1	1	1	1
2	3	3	3	3	2	2	3	2	5
3	18	20	22	25	28	18	10	7	7
4	33	37	41	47	53	41	27	15	8
5	48	53	60	69	78	65	48	25	12
6	63	70	79	91	104	89	72	45	18
7	77	86	98	113	128	113	96	66	26
8	92	102	116	135	153	136	119	86	38
9	107	119	135	157	179	160	142	110	52
10	122	136	154	179	204	184	165	134	76
11	134	153	174	199	227	206	187	154	88
12	151	167	190	220	254	231	211	182	118
13	163	186	208	241	280	257	236	202	133
14	178	200	230	262	312	279	259	226	155
15	195	215	248	284	332	302	282	248	182
16	212	229	260	310	355	321	306	272	199
17	230	269	312	317		347	329	295	233
18	263	288	353	346		376	354	319	247
19	292	325	382	388		399	374	342	271
20	331	353					395	362	289
21	347	370						387	315
22	373								339
23	382								364
24	391								388

The graph of the decision boundaries, i.e., acceptance and rejection lines when $\rho \geq 0$ and $\rho \leq 0$ are in <Figure 1> and <Figure 2> respectively. In <Figure 1>, upper 3 lines are acceptance lines and lower 3 lines are rejection lines. Note that they are not overlapped each other. The figure shows that the gap of the boundary lines between acceptance and rejection lines becomes wider if ρ approaches to 1. This means that if the process is more positively dependent then the chance of continuation of the test at each sample point becomes higher. In other words, it is more difficult to make decision early if the process is more positively dependent. Note that the independent case($\rho=0$) has the narrowest gap among the other several cases. Consequently, the average sample size of the dependent process necessarily becomes larger as ρ approaches to 1(See <Table 4> and <Figure 3> also). In case of negative dependency, however, the gap



< Figure 1 > The boundary lines of acceptance and rejection numbers for the Markov dependent model when ρ=0, 0.6 and 0.8



< Figure 2 > The boundary lines of acceptance and rejection numbers for the Markov dependent model when ρ=0, -0.6 and -0.8

between the decision boundary lines does not become wider but become narrower as the sample number or defectives increases. <Figure 2> shows this. Among the paired two lines having the same symbol mark, the upper line is the acceptance line and the lower line is the rejection line. Due to the randomness of the simulation the boundary lines are rather bent with more negative ρ values. From <Figure 1> and <Figure 2>, we can see that there exists big difference of the

acceptance and rejection boundaries between the positive and negative dependence factor cases. If the dependency factor is positive, the quality of the product resembles the former. Once the former item is bad, the probability of defective item of the following item becomes higher. The variance of estimated rejection probability increases, which makes the gap between the accept and reject lines wider as the inspection procedure continues. In case of negative dependency, the quality of the product has a tendency not resembling the former items. The defective ratio of the product does not vary much, which makes the gap between the accept and reject lines not wider compared to the positive case.

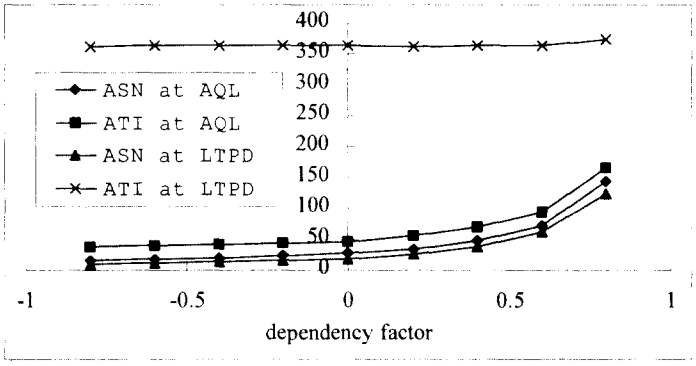
The estimated values of α and β are shown in <Table 3>. The estimated type I error probabilities are lower than α but the type II error probabilities are almost consistent to β. The inaccuracy of the estimated type I errors comes from the Walds boundary equations. The measures of performance of the sequential sampling plans are shown in <Table 4>. The graphs of ASN and ATI are shown in <Figure 3>, and the graphs of AOQ without replacement are shown in <Figure 4>. The ASN at AQL is rather higher than at LTPD. The ASN increases as the dependency factor ρ increases. When ρ is negative the ASN increases linearly but when ρ is positive the ASN increases nonlinearly, i.e., more rapidly. Especially the increment of ASN is fairly high as ρ approaches to 1. In the case of ATI, the increment tendency of ATI at AQL is similar to that of ASN. But the ATI at LTPD remains almost same and has much higher values than at AQL. As ρ increases, the AOQ which is represented as percent in <Table 4> decreases linearly when ρ is negative, but decreases rather slowly when ρ is positive. Unlikely in the case of ASN, the AOQ at AQL is not always higher than at LTPD. There is not much difference of AOQ whether the defective items are replaced with good ones or not.

< Table 3 > Estimated type I and II error probabilities

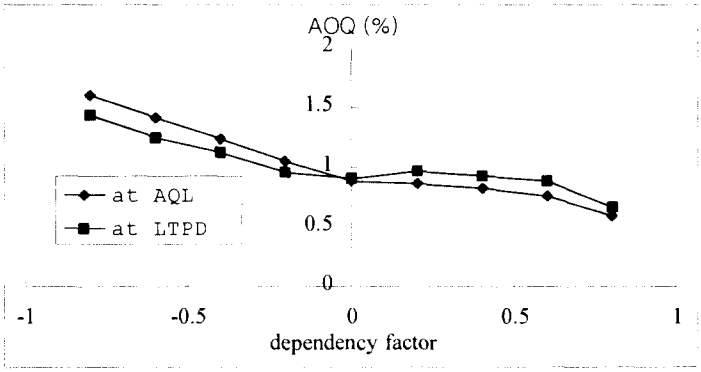
ρ	α	β
-0.8	0.0561	0.0978
-0.6	0.0556	0.0946
-0.4	0.0536	0.0964
-0.2	0.052	0.0948
0	0.0429	0.0974
0.2	0.0534	0.1057
0.4	0.0611	0.1054
0.6	0.0618	0.1095
0.8	0.0632	0.1014

< Table 4 > Measures of performance of the sequential sampling plan

	at AQL				at LTPD			
ρ	ASN	ATI	AOQ(%) without replacement	AOQ(%) with replacement	ASN	ATI	AOQ(%) without replacement	AOQ(%) with replacement
-0.8	15.4	37.3	1.6042	1.6032	10.5	362.4	1.4399	1.4339
-0.6	17.7	39.3	1.4213	1.4204	11.8	363.9	1.2505	1.2452
-0.4	20.2	41.1	1.2393	1.2384	13.5	363.4	1.1269	1.122
-0.2	23.9	44	1.056	1.0552	15.7	364.4	0.9577	0.9534
0	29	45.3	0.8875	0.8869	18.9	363.9	0.9067	0.9024
0.2	34.9	55.4	0.8625	0.8618	25.6	361.6	0.9648	0.9591
0.4	47.1	70.3	0.8257	0.8248	37.1	363.4	0.9219	0.9146
0.6	71.8	94.7	0.7653	0.7641	62	365	0.8855	0.8744
0.8	143.9	166	0.5896	0.588	124.3	374	0.6663	0.6497



< Figure 3 > Trends of the ASN and ATI according to the dependency factor ρ



< Figure 4 > Trends of the AOQ(%) without replacement according to the dependency factor ρ

6. Conclusions

Generally, if there exists a dependency factor in a statistical problem, it is very complicated or impracticable to handle the problem analytically. Consequently, we usually use simulation. In this paper, we have developed design procedures for sequential sampling plans for dependent process models. The algorithms developed in this paper can be verified by comparing the results with some known dependent process model like a Polya process in which the distribution of the number of defectives can be found by analytical method. In the previous numerical example, we can see that there are much differences of the resulting sampling plans according to the change of dependency factor. Also, the behavior of the sampling plans when the dependency factor is positive is very different from when the dependency factor is negative. Therefore, when the process shows correlation between observed items, we should not use standard independent sequential sampling plans. If independent sampling plans are applied to the dependent process model, the required type I and II error probabilities can not be met. The extra cost and time for identifying the dependent process model can be compensated by avoiding the risk of wrong decision.

One of the problems in applying the procedure to the real world is how to find the existence of dependency and what the process model is. The study for the procedure of identifying the process model and its parameters is not addressed in this paper. However this work must be done before the actual applications. This can be established by using some commercial statistical package and by simulating the production processes.

As a further study, we can develop the multiple acceptance sampling plans for any dependent process by extending the work of sequential sampling plans. Also this research can be used in place of control charts where there is dependency in the process. It should be emphasized that control charts in industry are used to check for changes in independent processes, and they should not be used where there is a recognized and necessary dependency.

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