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The Multidimensional Subsampling of Reverse Jacket Matrix of Weighted Hadamard Transform for IMT2000

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IMT2000을 위한 하중 Hadamard 변환의 다차원 Reverse Jacket 메트릭스의 서브샘플링

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ABSTRACT

The classes of Reverse Jacket matrix $[RJ]_N$ and the corresponding Restclass Reverse Jacket matrix ($[RRJ]_N$) are defined; the main property of $[RJ]_N$ is that the inverse matrices of them can be obtained very easily and have a special structure.

$[RJ]_N$ is derived from the weighted Hadamard Transform corresponding to Hadamard matrix $[H]_N$ and a basic symmetric matrix D. The classes of $[RJ]_2$ can be used as a generalized Quincunx subsampling matrix and several polygonal subsampling matrices. In this paper, we will present in particular the systematical block-wise extending-method for $[RJ]_N$. We have deduced a new orthogonal matrix $M_1 \in [RRJ]_N$ from a nonorthogonal matrix $M_0 \in [RJ]_N$. These matrices can be used to develop efficient algorithms in IMT2000 signal processing,, multidimensional subsampling, spectrum analyzers, and signal scramblers, as well as in speech and image signal processing.

요 약

Reverse Jacket 메트릭스($[RJ]_N$)와 Restclass Reverse Jacket 메트릭스($[RRJ]_N$)를 정의 한다. $[RJ]_N$은 특별한 구조 때문에 그의 inverse가 쉽게 구해질 수 있으며, 기본 대칭 메트릭스 D와 하중 Hadamard메트릭스로 부터 유도 된다. $[RJ]_2$의 각 class는 일반적인 Quincunx 서브 샘플링 및 여러 가지의 다각형 서브샘플링 메트릭스로 사용될 수 있다. 본 논문에서는 nonor-thogonal 메트릭스인 $[RJ]_N$으로 부터 orthogonal 메트릭스 $[RRJ]_N$을 유도하고, $[RJ]_2$가 다차원서 브 샘플링에 이용될 수 있음을 보여 준다. 또 이 메트릭스는 IMT2000 신호 처리, 다차원서브 샘플링, 스펙트럼 분석기 및 음성 및 영상신호 처리 등에 이용될 수 있다.

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Ⅰ. Introduction

In fields of communications a large number of time functions can be represented by superpositions of sine and cosine functions. So Fourier analysis is often used as a mathematical tool to analysis a time function. The transition from the system of sine-cosine functions to general systems of orthogonal functions brings simplifications as well as complications. Hadamard matrices are named after their discoverer, J. Hadamard in 1893. They have been applied in a number of fields. A Hadamard matrix of order n is an $(n\times n)$ matrix of +1s and −1s such that any pair of distinct rows is orthogonal(i.e., their product is zero).

The arithmatic operations for Hadamard transform consist of only additions and subtractions instead of multiplications because of its matrix elements(+1s and −1s)[1]~[4]. The several algorithms have been developed for computing N-length Hadamard transforms and the number of operations, N^2, are lessened to $N\log_2 N$. These are usually generalizations of the Cooley-Tukey FFT algorithm. The application of Hadamard transforms for signal and image compression is well known [1]~[4]. A much investigated method, due to the ease and efficiency of its implementations, is based on Hadamard transform [2]. It has been presented in paper [1] that Walsh-Hadamard transform is the most well known of the nonsinusoidal orthogonal transforms. Walsh-Hadamard matrix is used for the Walsh representation of the data sequences in image coding and for PN(Pseudo-noise) generator in CDMA mobile communication. Their basis functions are sampled according to Walsh functions which can be expressed in terms of the Hadamard matrices $[H]_N$. Using the orthogonality of Hadamard matrices we construct a generalized Weighted Hadamard matrices [2], [5], [6], called Reverse Jacket matrices($[RJ]_N$), and they have reverse geometric structures. In this paper, $[RJ]_N$ and its subsampling examples are described. $[RJ]_N$ is nonorthogonal but the restclass of Reverse Jacket($[RRJ]_N$), which is subset of $[RJ]_N$, is orthogonal. These matrices can be used to develop efficient algorithms in IMT2000 signal processing,, multidimensional subsampling, spectrum analyzers, signal scramblers, and information theory as well as in speech and image signal processing.

Ⅱ. The Weighted Hadamard Transform(WHT)

Let Hadamard and Weighted Hadamard matrices of order $N=2^k$ be denoted by $[H]_N$ and $[WH]_N$, respectively. The WHT of an $N\times 1$ vector $[f]$ and an $N\times N$ (image) matrix $[g]$ are given by [8] as follows

$$[F]=[WH]_N[f] \tag{1}$$

and

$$[G]=[WH]_N[g][WH]_N^T \tag{2}$$

The lowest order Weighted Hadamard matrix is of size (4×4) and is defined as follows [5];

$$[WH]_4 \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & -1 \\ 1 & 2 & -2 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}. \tag{3}$$

The inverse of (3) is

$$[WH]_4^{-1}=\frac{1}{8}\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & -1 & 1 & -2 \\ 2 & 1 & -1 & -2 \\ 2 & -2 & -2 & 2 \end{bmatrix}. \tag{4}$$

This choice of weighting was dictated, to a large extent, by the requirement of digital hardware simplicity [2]. As with the Hadamard matrix, a recursive relation governs the generation of higher order Weighted Hadamard matrices, i.e.,

$$[WH]_N=[WH]_{N/2}\otimes[H]_2, \tag{5}$$

where $\otimes$ *is the kronecker product.* $[H]_2$ is the lowest order Hadamard matrix given by [1], [4]:

$$[H]_2=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{6}$$

The fast algorithm of WHT[2] is related to the fast Hadamard Transform(FHT) algorithm [2], [4], [8]. The FHT can be derived by decomposing $[H]_N$ into a product of k sparse matrices, each having rows/columns with only two nonzero elements. In order to develop a similar algorithm for the WHT, define a coefficient matrix $[RC]_N$ by

$$[RC]_N=[H]_N[WH]_N. \tag{7}$$

Since $[H]_N^{-1}=1/N[H]_N$, we have from (7) that

$$[WH]_N^{-1}=1/N[H]_N[RC]_N. \tag{8}$$

Ⅲ. Reverse Jacket Matrix

The $[RJ]_N$ is a generalized form of $[WH]_N$. As our two side jacket is an inside and outside compatible, at least two positions of a Reverse Jacket matrix $[RJ]_N$ are replaced by their inverse; these elements are changed their positions from inside of the middle circle to outside or from outside to inside without loss of signs; which are very interesting phenomena. This is the reason why we call it Reverse Jacket matrix.

If we regard the upper left (2×2) block matrix D of $[WH]_N$ then we can find some regular and recursive structures. Therefore we define several sip matrices for $[RJ]_N$ as follows:

$$[Z]_2\triangleq\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\ [S]_2\triangleq\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \text{and}\ [J]_2\triangleq\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}. \tag{9}$$

Let I_{2^k} and 0_{2^k}, $k\in N$ be the $(2^k\times 2^k)$ unit matrix and zero matrix respectively. Then for $k\geq 1$ we get

$$Z_{2^{k+1}}=\begin{bmatrix}I_{2^k} & 0\\ 0_{2^k} & -I_{2^k}\end{bmatrix},\quad S_{2^{k+1}}=\begin{bmatrix}0_{2^k} & I_{2^k}\\ I_{2^k} & 0_{2^k}\end{bmatrix}\ \text{and}$$

$$J_{2^{k+1}}=\begin{bmatrix}0_{2^k} & I_{2^k}\\ -I_{2^k} & 0_{2^k}\end{bmatrix}. \tag{10}$$

Further we define

$$D=\begin{bmatrix}a & b\\ b & -c\end{bmatrix}=M_1,\ a,\ \mathrm{b},\ \mathrm{c}\neq 0,$$

$$M_2=Z_2DS_2\ \text{and}\ M_4=J_2^{-1}DJ_2. \tag{11}$$

Finally, we define the $[RJ]_4$ as

$$[RJ]_4\triangleq\begin{bmatrix}M_1 & M_2\\ M_2^T & M_4\end{bmatrix}=\begin{bmatrix}a & b & b & a\\ b & -c & c & -b\\ b & c & -c & -b\\ a & -b & -b & a\end{bmatrix} \tag{12}$$

Throughout the whole paper we assume that D is invertible. We define the inverse Reverse Jacket matrix as follows:

$$[RJ]_{2^k}^{-1}=c_kL_k,\ \ k\in\{1,2\},\ \ c_k\in R \tag{13}$$

where

$$c_1=\frac{-signum(a\cdot c)}{det([RJ]_2)},\quad c_2=\frac{1}{4lcm(a,b,c)}$$

(lcm denotes least common multiple).

$$L_2=signum(a\cdot c)\begin{bmatrix}c & b\\ b & -a\end{bmatrix}, \tag{14}$$

$$L_2=lcm(a,b,c)\begin{bmatrix}1/a & 1/b & 1/b & 1/a\\ 1/b & -1/c & 1/c & -1/b\\ 1/b & 1/c & -1/c & -1/b\\ 1/a & -1/b & -1/b & 1/a\end{bmatrix}. \tag{15}$$

The matrices $[RJ]_{2^k}$, $k\geq 3$ will be defined subsequently. The fast algorithm for the Reverse Jacket transform(FRJT) is similar fashion as in [5]. The fast Hadamard transform(FHT) can be derived by decomposing $[H]_{2^k}$ into a product of sparse matrices, each having rows/columns with only two nonzero elements. The Sylvester construction for Reverse Jacket matrices can be expressed recursively in terms of Kronecker product.

$$[RJ]_{2^{k+1}} = [H]_2 \otimes [RJ]_{2^k} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes [RJ]_{2^k}, \quad k \geq 2. \tag{16}$$

If $[RJ]_{2^k}$, $(k \geq 2)$, is a Reverse Jacket matrix, then following equation is possible.

$$[RJ]_{2^{k+1}} = \begin{bmatrix} [RJ]_{2^k} & [RJ]_{2^k} \\ [RJ]_{2^k} & -[RJ]_{2^k} \end{bmatrix}, \quad k \geq 2. \tag{17}$$

Now, we define a coefficient matrix $[RC]_{2^k}$ by

$$[RC]_{2^k} = \frac{1}{2^k} [H]_{2^k} [RJ]_{2^k}, \quad k \geq 2. \tag{18}$$

Then, the following equation is also possible.

$$\begin{aligned} [RC]_{2^{k+1}} &= [H]_{2^{k+1}} [RJ]_{2^{k+1}} \\ &= [H]_{2^{k+1}} ([H]_2 \otimes [RJ]_{2^k}) \\ &= ([H]_2 \otimes [H]_{2^k})([H]_2 \otimes [RJ]_{2^k}) \\ &= 2I_2 \otimes ([H]_{2^k} [RJ]_{2^k}) \\ &= 2I_2 \otimes [RC]_{2^k}. \end{aligned} \tag{19}$$

Ⅳ. Restclass Reverse Jacket Matrix

Analogously to (9)-(12) we construct the higher dimensional Reverse Jacket matrices as follows:

$$[RJ]_{2^{k+1}} \triangleq \begin{pmatrix} \widetilde{M}_1 & \widetilde{M}_2 \\ \widetilde{M}_2^T & \widetilde{M}_4 \end{pmatrix}, \quad k \geq 2, \tag{20}$$

where $\widetilde{M}_1 = [RJ]_{2^k}$, $\widetilde{M}_2 = Z_{2^k}[RJ]_{2^k}$, and $\widetilde{M}_4 = J_{2^k}^{-1[RJ]_{2^k}} J_{2^k}$.

Let

$$c_{2m+1} = \frac{1}{4} c_{2m-1}, \; c_{2m+2} = \frac{1}{4} c_{2m}, \quad m \geq 1, \tag{21}$$

and

$$L_{2m+1} = signum(a, c) \begin{bmatrix} -D_{2m-1} & B^T_{2m-1} & B^T_{2m-1} & -D_{2m-1} \\ B_{2m-1} & -A_{2m-1} & A_{2m-1} & -B_{2m-1} \\ B_{2m-1} & A_{2m-1} & -A_{2m-1} & -B_{2m-1} \\ -D_{2m-1} & -B^T_{2m-1} & -B^T_{2m-1} & -D_{2m-1} \end{bmatrix} \tag{22}$$

where

$A_{2m-1} = [RJ]_{2^m}$, $B_{2m-1} = Z_{2^m}[RJ]_{2^m} S_{2^m}$, $D_{2m-1} = -J_{2^m}[RJ]_{2^m} J_{2^m}$.

Then,

$$L_{2m+2} = \begin{bmatrix} -D_{2m} & B^T_{2m} & B^T_{2m} & -D_{2m} \\ B_{2m} & -A_{2m} & A_{2m} & -B_{2m} \\ B_{2m} & A_{2m} & -A_{2m} & -B_{2m} \\ -D_{2m} & -B^T_{2m} & -B^T_{2m} & -D_{2m} \end{bmatrix} \tag{23}$$

and

$$A_{2m} = L_{2m}, \quad B_{2m} = Z_{2^{2m}} L_{2m} S_{2^{2m}}, \quad D_{2m} = -J_{2^{2m}} L_{2m} J_{2^{2m}}. \tag{24}$$

Let

$$[RJ]_{2^k} \textit{ is element of } [RJ]_N \cap R^{2^k \times 2^k}. \tag{25}$$

A $2^k \times 2^k$ matrix $[RRJ]_{2^k} = [RJ]_{2^k} - L_k$, $k \geq 2$ is said to belong to the Restclass of Reverse Jacket matrix respect to Hadamard matrix H_{2^k} if

$$[RRJ]_{2^k} = [RRJ]_{2k}^{-1} [RRJ]_{2^k} [H]_{2^k}. \tag{26}$$

where

$[RJ]_{2^k}^{-1} = c_k L_k,\ c_k \in IR \setminus \{0\}$
and $[RRJ]_{2^k},\ L_k \in C^{2_k \times 2_k},\ k \geq 2.$

The equation (26) is also equivalent to:

$$[RRJ]^*_{2^k} = [H]^{-1}_{2k} [RRJ]^*_{2^k} [H]_{2^k} = [RRJ]^{[*]}_{2^k} \quad (27)$$

and

$$[RRJ]^{-1}_{2^k} = \frac{1}{det([RRJ]_{2^k})} [RRJ]^{[*]}_{2^k},\ k \geq 2. \quad (28)$$

We can construct 4×4 H-orthogonal matrices associated with Hadamard matrix $[H]_{2^k}$ starting from a 2×2 nonorthogonal matrix D, and each matrix $[RJ]_{2^k}$ for k=2m+1, m≥1 is Restless of Reverse Jacket matrix.

V. The Examples of Reverse Jacket matrices

There are five cases in the elements decision of basic symmetric matrix, D_k(k=1, ⋯, 5), which is $[RJ]_2$. The matrix Dk consists of three elements(a, b, and c), which are all not zero and take the values of $\pm 2^i$ (i=0, 1, ⋯). Their conditions are $a \leq b \leq c$ and each element takes the minimun integer value under the upper conditions. It can be used for multidimensional subsampling of signals.

1) Case-I

If the elements are all same, a=b=c then they are all ones, a=b=c=1, and the basic symmetric matrix D1=[RJ]2 is same as Hadamard matrix $[H]_2$ and as following.

$$D_1 = [RJ]_2 = \begin{bmatrix} a & b \\ b & -c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (29)$$

From equation (12), the $[RJ]_4$ is obtained.

$$[RJ]_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}. \quad (30)$$

Since the determinant of $[RJ]_2$ is 2, the sampling ratio and the number of subchannels are 2. The $[RJ]_4$ is same as Hadamard matrix $[H]_4$, which is orthogonal symmetric matrix as well as special case of $[RJ]_4$ and $[RJ]_2$ is a Quincunx subsampling matrix in this case. The inverse of $[RJ]_2$ is $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, and $[RJ]_4$ and its inverse matrix $[RJ]_4^{-1}$ consist of same elements also.

When this is used for data coding, the transmit and receive units are same.

$$[RJ]_4^{-1} = c_2 L_2 = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (31)$$

2) Case-II

If the two elements of a and b are same but c is different from them, a=b≠c then they are a=b=1 and c=2. In this case, $D_2=[RJ]_2$ is same as upper left 2×2 elements of $[WH]_4$ and nonorth-ogonal symmetric matrix.

The $[RJ]_2$ is $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ and $[RJ]_2^{-1}$ is $\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$. This means that the sampling ratio and subchannels are all 3. The coset vectors are $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The 4×4 Reverse Jacket matrix and its inverse one have a fast algorithm as shown in Fig. 1 and are following.

$$[RJ]_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & -1 \\ 1 & 2 & -2 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}, \text{ and}$$

$$[RJ]_4^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & -1 & 1 & -2 \\ 2 & 1 & -1 & -2 \\ 2 & -2 & -2 & 2 \end{bmatrix}. \quad (32)$$

3) Case-Ⅲ

In this case, the elements of a and c is same but the only c is different, $a = c \neq b$ then they are $a = c = 1$ and b = 2. The basic matrix $D_3 = [RJ]_2$ is $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and the $[RJ]_2^{-1}$ is $\frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ which is $\frac{1}{5}[RJ]_2$. The $[RJ]_4$ and $[RJ]_4^{-1}$ are symmetric non-orthogonal matrix as follows.

$$[RJ]_4 = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & -1 & 1 & -2 \\ 2 & 1 & -1 & -2 \\ 1 & -2 & -2 & 1 \end{bmatrix}, \text{ and}$$

$$[RJ]_4^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & -2 & 2 & -1 \\ 1 & 2 & -2 & -1 \\ 2 & -1 & -1 & 2 \end{bmatrix}. \quad (33)$$

The two positions of $[RJ]_4$ can be replaced by $[RJ]_4^{-1}$ also.

4) Case-Ⅳ

If the only two elements of b and c are same but a is different then they are $a = 1$ and $b = c = 2$. The $[RJ]_2$ is $\begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$ and $[RJ]_2^{-1}$ is $\frac{1}{5}\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$. The 4×4 forward and inverse Reverse Jacket matrix is nonorthogonal symmetric matrix and is following.

$$[RJ]_4 = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & -2 & 2 & -2 \\ 2 & 2 & -2 & -2 \\ 1 & -2 & -2 & 1 \end{bmatrix}, \text{ and}$$

$$[RJ]_4^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 2 & -1 & -1 & 2 \end{bmatrix}. \quad (34)$$

5) Case-Ⅴ

If all elements are not equal each other, $a \neq b \neq c$ and $a \neq c$, then they are $a = 1$, $b = 2$ and $c = 4$. The basic symetric matrix $D_4 = [RJ]_2$ is $\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$ and $[RJ]_2^{-1}$ is $\frac{1}{8}\begin{bmatrix} 4 & 2 \\ 2 & -1 \end{bmatrix}$. This is polygonal subsampling case of which sampling ratio and subchannels are 8. It has eight coset vectors.

$$[RJ]_4 = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & -4 & 4 & -2 \\ 2 & 4 & -4 & -2 \\ 1 & -2 & -2 & 1 \end{bmatrix}, \text{ and}$$

$$[RJ]_4^{-1} = \frac{1}{16}\begin{bmatrix} 1 & 4 & 4 & 1 \\ 4 & -2 & 2 & -4 \\ 4 & 2 & -2 & -4 \\ 1 & -4 & -4 & 1 \end{bmatrix}. \quad (35)$$

In this case, each matrix has three zones which are ±1's, ±2's and ±4's areas. The ±1's, ±2's and ±4's areas of $[RJ]_4$ are able to be replaced by ±4's, ±2's and ±1's areas of $[RJ]_4^{-1}$ respectively.

The two positions of $[RJ]_4$ can be replaced by $[RJ]_4^{-1}$ also.

Ⅵ. The Multidimensional Subsampling

The subsampled version of a D_1-Dimensional signal $x_a(t)$ is defined as

$$x(n) = x_a(V_n), \quad (36)$$

where V is a integer $D_k \times D_k$ nonsingular matrix.

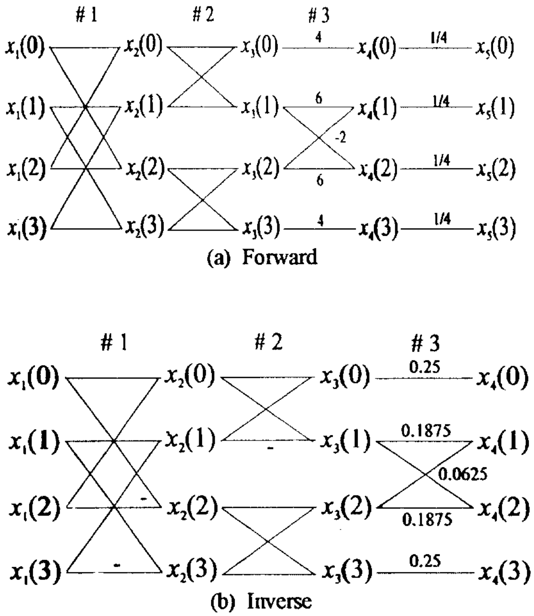


Fig. 1 The fast algorithm for $[RJ]_4$ and $[RJ]_4^{-1}$ flow graph.

$$V=[v_0\ v_1\ \cdots\ v_{D_k}-1], \tag{37}$$

and $n\in N$. The set of all sample points is the set

$$t=Vn, \quad n\in N \tag{38}$$

that is, the set of vectors $\sum_{i=0}^{D_k-1} n_i v_i$. This is the set of all integer linear combinations of the columns v_0, v_1, $\cdots$, $v_{D_k}-1$ *of* V which is called subsampling matrix $[RJ]_2$.

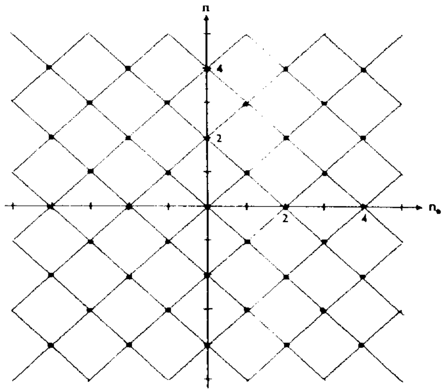
The kth element of $Z(M)$ is given by

$$z_0^{M_{0,k}} z_0^{M_{1,k}}, \quad k=0,\ 1, \tag{39}$$

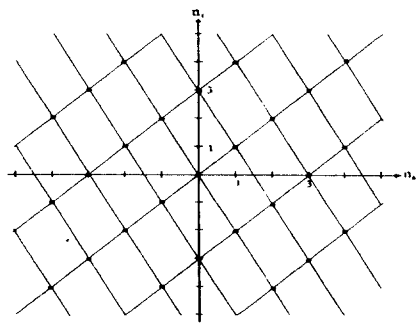
so that we have

$$Z^{(2)}=\begin{bmatrix} z_0 & z_1 \\ z_0 & z_1^{-1} \end{bmatrix} \ (Quincunx\ 2). \tag{40}$$

So for the Quincunx expander, the output can be rewritten as



(a) Subsampling lattice, $D_1=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.



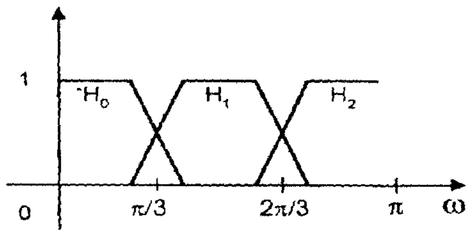
(b) Subsampling lattice, $D_2=\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$.

Fig. 2 The subsampling lattice of $[RJ]_2$.

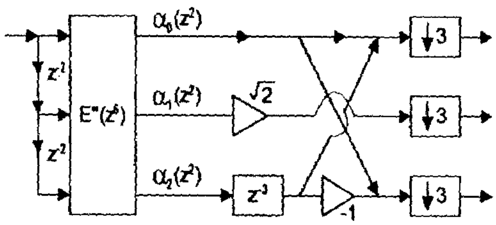
$$Y_2(z_0,\ z_1)=X_2(z_0 z_1,\ z_0 z_1^{-1}). \tag{41}$$

Fig. 2 shows the examples of subsampling lattice with D_1 and D_2 of $[RJ]_2$.

In Fig. 3, the filter banks with D_1, D_2, and D_3 subsampling and ideal spectrum splitting are shown. Fig. 3(a) demonstrates a typical set of desired magnitude responses for the three-channel analysis filters. The passband of the filters are nonoverlapping. In the frequency region designated as passband for the filter $H_k(z)$, all other filters have their stopbands. The paraunitary property of $E(z)$ ensures that the analysis filters power complementary.



(a) Typical magnitude responses for an analsys bank with three channels.



(b) Three-channel analysis bank, with $H_k(z) = H_{2-k}(z)$.

Fig. 3 The magnitude responses and analysis bank of three channels.

For the three channel real coefficient case, the symmetry condition is able to be incorporated by the following constraint.

$$H_2(z) = H_0(-z), \quad H_1(z) = \alpha_1(z^2) \tag{42}$$

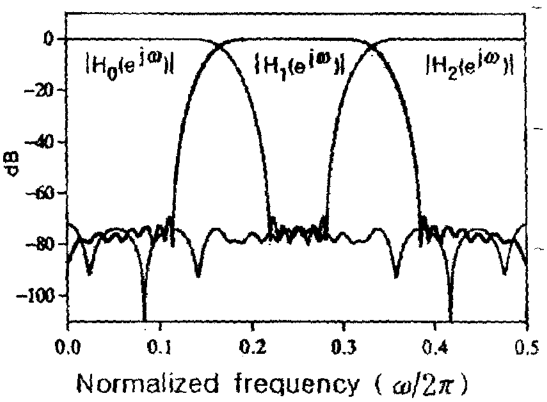
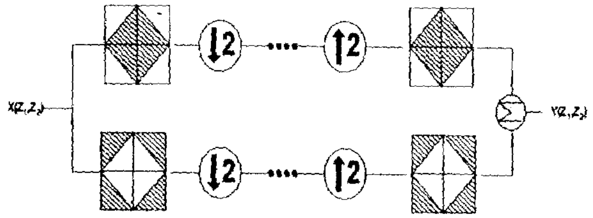
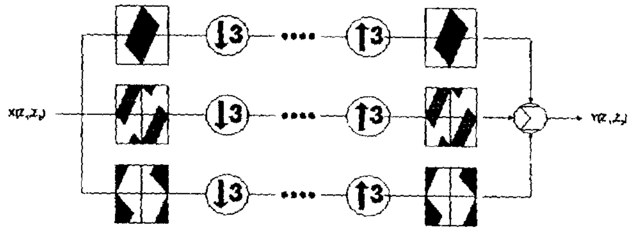


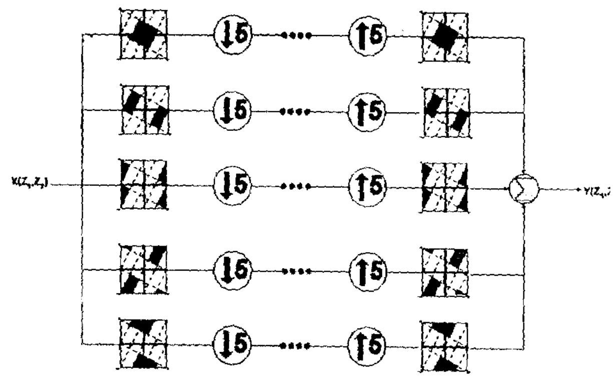
Fig. 4 Magnitude responses of analysis filters for a three channel FIR perfect reconstruction system, and filter order N = 55.



(a) Subsamp $D_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, and frequency spectrum.



(b) Subsampling matirx is $D_2 = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$, and frequency spectrum.



(c) Subsampling matirx is $D_3 = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, and frequency spectrum.

Fig. 5 Filter banks with D_k sampling and ideal spectrum splitting.

The frequency response of an FIR perfect reconstruction system based on Fig. 3(b) which shows a structure for imposing equation (42) is shown in Fig. 4.

Fig. 5(a) shows the two-band splittings and Quincunx frequency spectrum. In Fig. 5(b), since $|\det(D_2)|$ is 3, the subbands are 3 and the shapes frequency spectrum are polygonals. Fig. 5(c) shows the structures of band splittings with 5 subbands and each passband spectrums are polygonals also.

Ⅶ. Conclusions

Hadamard matrix is an orthogonal symmetric matrix but the weighted Hadamard matrix is a nonorthogonal symmetric matrix and slight modification of a Hadamard matrix. The Reverse Jacket matrix is a generalized weighted Hadamard matrix form and has recursive structure and symmetric characteristics. The elements positions of the matrix can be replaced by its inverse matrix and the signs of them are not changed between the matrix and its inverse. The $[RJ]_N$ matrices have five cases of basic symmetric matrces according to the elements conditions. Hadamard matrix is a special case of Reverse Jacket matrces also. In this paper, the subsampling examples of $[RJ]_2$ have shown and the subchannels are equal to the determinants of the subsampling matrces. These matrices can be used to develop efficient algorithms in signal processing, for IMT2000, multidimensional subsampling, spectrum analyzers, and signal scramblers, as well as in speech and image signal processing.

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