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Application of Matched Asymptotic Expansion for Designing a Leading Edge of Super-cavitating Foil

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Abstract

The leading edge of a low-drag super-cavitating foil has been made to be thick enough by using a point drag which is supposed to be a linear model of the Kirchhoff lamina. In the present paper, the relation between the point drag and the Kirchhoff lamina is made clear by analyzing the cavity drag of both models and the leading edge radius of the point drag model and the lamina thickness of Kirchhoff's profile K. The matched asymptotic expansion is effectively made use of in designing a practical super-cavitating foil which is not only of low drag but also structurally sound. Also it has a distinct leading edge cavity separation point.

The cavity foil shapes of trans-cavitating propeller blade sections designed by present method are shown.
Keywords : Super- and Trans- cavitating propeller, Kirchhoff's lamina, Kirchhoff's profile K, Matched asymptotic expansion.

1 Introduction

In an inviscid irrotational flow, a good super-cavitating foil has high lift drag ratio and yet has a thick enough leading edge. A linear design method of such foil has been studied extensively by the full use of the advantage of linear theory, and reasonably good results have been obtained [Yim et al, 1983]. The cavity usually starts from the leading edge and the shape of leading edge may be made accordingly. However, the cavity separation point and the leading edge shape affect the cavity drag and the cavity shape [Yim et al, 1983]. For the manufacturing ease, the simpler the shape , the better.

In general, two dimensional foil theory has been used for designing hydrofoil sections or propeller blade sections where a linear theory is most convenient. A low drag super-cavitating foil was designed by superposing three basic foils [Yim, 1977]; a flat plate which supplies the angle of attack, a low drag camber such as two , three, or five term camber [Johnson, 1958], and a point drag which supplies the leading edge cavity thickness. The point drag has been considered as a linear substitute of the Kirchhoff profile K [Birkhoff, 1957], but so far the relations between the two have not been fully explored. Although the point drag model is mathematically very simple for supplying the leading edge, there are many disadvantages. First, it can not give a clear cut cavity separation point at the leading edge. Secondly in the basically nonlinear flow field of the leading edge, the boundary conditions are too much violated near the leading edge by the point drag representation. Indeed, for the better and more accurate representation of the leading edges of super-cavitating foils, the linear theory is not adequate .

For a simple yet exact model with the blunt leading edge, the Kirchhoff Profile K may be appropriate because its minimal cavity drag for the given cavity thickness at a given point [Birkhoff, 1957] as well as its simplest leading edge shape. Such foils were tested in a cavitation tunel and the good features were confirmed [Ogata et al, 1989]. It seems proper to use a matched asymptotic expansion method [Van Dyke, 1975] to accommodate the Kirchhoff profile K at the nose of the low drag super-cavitating foil.

2 A Point Drag

A point drag in a super-cavitating section is a concentrated pressure point located at the leading edge of the section. It does not induce any lift but cavity drag and cavity thickness. In a uniform flow U_1 of the infinite medium, the complex velocity is represented by [Yim, 1975]

$$u - iv = 1 + \frac{k_0}{\pi i}\frac{1}{\zeta} \tag{1}$$

nondimensionalized by U_1 and

$$\zeta = z^{1/2}$$

where k_0 is the strength of the point drag. The cavity drag coefficient is

$$C_D = \frac{2k_0^2}{\pi} \tag{2}$$

In a cascade [Yim, 1975],

$$u - iv = \frac{k_0}{K\pi i}\frac{1}{\zeta} + 1 + u_\infty - iv_\infty, \tag{3}$$

and

$$x = \frac{d}{2\pi}[cos\gamma log(1 - 2a\xi sin\gamma + a^2\xi^2)$$
$$+2sin\gamma tan^{-1}\frac{a\xi cos\gamma}{1 - a\xi sin\gamma}]$$

where γ is the stagger angle and d is the solidity of the cascade; a is the transformation constant for the chord length equal to one; and

$$K = \frac{a^2 d}{2\pi}cos\gamma$$

In the infinite medium, $K = 1$. The leading edge radius of curvature $|\frac{(1+(\frac{dy}{dx})^2)^{3/2}}{\frac{d^2y}{dx^2}}|$ can be readily calculated as

$$r_c = \frac{2k_0^2}{K\pi^2} \tag{4}$$

The cavity drag coefficient is

$$C_D = \frac{2k_0^2}{\pi K} = \pi r_c \tag{5}$$

3 Basic Foils

In designing a practical low drag super-cavitating foil, a linear superposition of three basic foils has been used [Yim, 1977]. The basic foils are a flat plate, low drag foils such as two term, three term or five term camber[Johnsn, 1958], and the point drag. The flat plate supplies a part of lift and the angle of attack which prevents face cavitation. However if we want to have a shock free entry foil, the flat plate may be totally omitted. For the camber of the foil, the low drag foils with positive cavity thickness have been investigated [Johnsn, 1958] and widely used. However, these foils have very thin leading edges. To supply the leading edge thickness a point drag has been used. This not only supplies the low drag thick leading edge but also is convenient because it does not change the lift of the superposed foil. With the linear theory the superposed cavity thickness or the cavity shape can be obtained without difficulty [Yim et al, 1977, 1975].

In general the superposed foil looks good and has been used with reasonably good results [Yim et al, 1983]. However the leading edge shape may not supply too accurate shape because of its singular behavior as in the linear airfoil theory. Especially although the point drag is supposed to be a linear model of Kirchhoff's profile K, both have very different leading edge shapes. The Kirchhoff profile is particularly attractive because it gives not only low cavity drag [Birkhoff, 1957] but also a simple leading edge shape with a distinct cavity separation point.

4 Matched Asymptotic Expansion

When we consider a problem like the small neighbourhood of the nonlinear leading edge flow, it has been customary to consider the matched asymptotic expansion method [Van Dyke, 1975]. We take the small area separately as an expanded inner two dimensional region in a uniform flow with the stretched coordinates. This method has been often used to inbed a nonlinear two dimensional solution in the appropriate local linear flow. In the present problem, a simple use of matched asymptotic expansion seems to be very appropriate whether the superposed foil is in the infinite medium or in a cascade.

Velocities due to any shapes, whether they are linear or nonlinear, are linearly superposable because the flow is inviscid and irrotational although the exact pressure may not be superposable. In fact, in the design problem, one or more of basic foils which compose a superposed hydrofoil may be linear or nonlinear by the designer's choice. However in the present paper the maximal usage of advantage of the linear and the nonlinear models and their simplicity are aimed.

4.1 Kirchhoff's Profile K

The Helmholtz motions for a direct impact of a stream U_2 on a lamina is presented in fluid dynamics text books [Milne Thomson, 1955]. A brief review of main equations is shown here. A lamina of breadth l is placed parallel to the y-axis that makes right angle to the x-axis which is parallel to the ambient stream. Using a combination of the Schwarz-Christoffel transformation, and the hodograph method, we obtain the complex velocity, $u - iv$ represented in the tranformed $\zeta = \xi + i\eta$ plane (see Figure 1),which maps whole potential field onto the upper half plane.

$$u - iv = -i[\frac{1}{\zeta} - (\frac{1}{\zeta^2} - 1)^{1/2}] \quad (6)$$

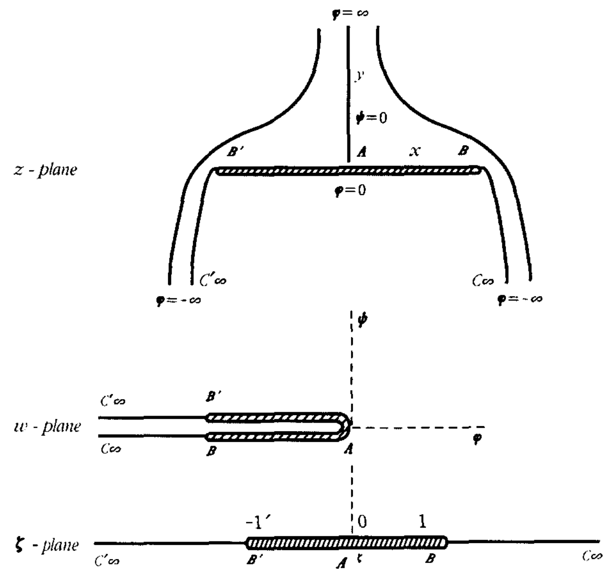

Figure 1. Conformal Mapping

The relation between the physical plane (z) and the transformed potential plane (ζ) is shown as

$$\frac{dz}{d\zeta} = \frac{2li}{4+\pi}[1 + (1-\zeta^2)^{1/2}]. \tag{7}$$

Integrating this along the ξ axis or $\eta = 0$ from $\xi = 0$ to ξ, where $0 < \xi < 1$ represents the lamina and $\xi > 1$, the cavity surface, we obtain

$$z = \frac{2li}{4+\pi}[\xi + 0.5[\xi(1-\xi^2)^{1/2} + sin^{-1}(\xi)]] \tag{8}$$

That is, on the lamina, $0 < \xi < 1$

$$y = \frac{2l}{4+\pi}[\xi + 0.5[\xi(1-\xi^2)^{1/2} + sin^{-1}(\xi)]] \tag{9}$$

The cavity surface $\eta = 0, |\xi| > 1$ can be written as

$$y = \frac{2l}{4+\pi}(\xi + \frac{\pi}{4}), \tag{10}$$

$$x = \frac{l}{4+\pi}[\xi(\xi^2-1)^{1/2} - log(\xi + \sqrt{\xi^2 - 1})] \tag{11}$$

The cavity drag is

$$D = \frac{\pi \rho l U_2^2}{4+\pi} \tag{12}$$

4.2 Matching

The linear model, a point drag flow field may be considered as the outer field of a hydrofoil with a blunt leading edge represented by a singularity of its type. Then the flow field due to the Kirchhoff profile K may become the inner field of the point drag flow by a proper matching. In Equation 11 we consider with a small number ϵ

$$\epsilon = \frac{l}{4+\pi} \tag{13}$$

ξ in the transformed plane is practically a stretched coordinate $\xi^2 = \frac{q}{\epsilon}$ we obtain

$$x = q[(1-\frac{\epsilon}{q})^{1/2} - \epsilon\frac{log(\frac{q}{\epsilon})^{1/2} + \sqrt{\frac{q}{\epsilon}-1}}{q}] \tag{14}$$

and for the first order

$$x = \frac{l}{4+\pi}\xi^2$$

Now inserting this into Equation 6 and comparing Equations 1 and 6 considering Equation 13, we obtain

$$\frac{l}{4+\pi} = \frac{k_0^2}{\pi^2} = r_0/2 \tag{15}$$

and

$$U_1 = U_2 \tag{16}$$

where r_0 is the leading edge radius of curvature in the infinite medium of the point drag model since in Equation 4, $K = 1$ in the infinite medium.

In the cascade, by the similar procedure we obtain

$$\frac{l}{4+\pi} = \frac{k_0^2}{\pi^2 K} = r_c/2 \tag{17}$$

and

$$U_1(1 + u_\infty - iv_\infty) = U_2 \tag{18}$$

With these relations between l and k_0, it is very interesting to note that the cavity drags for the point drag model and the Kirchhoff profile are exactly the same. The far field of the inner field is the same as the outer flow with the negligible difference.

The composite solution can be obtained adding to the final linear solution, the exact inner solution of a special basic foil (the Kirchhoff profile) minus the outer solution for the same basic foil (the point drag model in the infinite medium). By this way only the small region near the leading edge is made nonlinear smoothly connecting to the surrounding regions without affecting other area too much.

5 Application

The foil and cavity shapes may be obtained by superposing flow fields. The present method of matched asymptotic expansion is applied to only the point drag, as a part of basic foils that compose the superposed final super-cavitating foil. That is, in the inner field only the point drag part is smoothly replaced by the Kirchhoff lamina. Therefore the outer bounary which influenced before this process does not affect the inner flow any more. In this way, the Kirchhoff lamina may be only perpendicular to the oncoming flow when the the foil is shock-free without the flat plate component, where the cavity behavior may be best. When the flat plate component is large the stagnation point may be on the pressure side of the foil a little off the leading edge.

Because of the flat lamina of the Kirchhoff profile, there may be two distinct discontinuities in the foil shape near the leading edge. The discontinuity in the upper part of the leading edge where the cavity should start is very advantageous. However the discontinuity in the lower part of leading edge connecting to the pressure side of the foil may not be appreciated. To improve the present status of hybrid design method only the upper part of the super-cavitating foil may be replaced by the matched asympotic expansion leaving the pressure side as was. Because the existing linear design method has been reasonably good, and the only hope was to improve the leading edge cavity separation, the present attitute seems appropriate as a nonlinear correction to the leading edge of the linear solution.

The linear or nonlinear design method of a good super-cavitating section may be widely used in designing super- or trans- cavitating propellers, or super-cavitating hydrofoils. In the preliminary design of super-cavitating propeller such sections in the appropriate cascade must be used for the correct neighboring cavity-foil interference.

6 Examples and Discussion

For a rough feeling of the application in the propeller design, super-cavitating foil sections in a design of four bladed trans-cavitating propeller for 34 knot ship [Yim, 1981], [Yim et al, 1995], with 88.19 cm (34.72inch) diameter are shown in Figures 2. In the figures, r indicates the section radius devided by the radius. The lower parts of the figures are pressure sides of sections; the upper parts are the cavity shapes. Therefore the actual foil should be beneath the cavity surface that starts from the edge of the Kirchhoff lamina. The angles of attack and lift coefficients of the sections are represented by (section radius, angle of attack, lift coefficient) $= (0.6, 3.76^o, 0.151), (0.7, 4.47^o, 0.1485), (0.8, 4.335^o, 0.1462)$.

These sections are designed by the present method in the cascade in the preliminary design [Yim et al,1977,1983]. According to the location of the sections in the propeller blade the corresponding cascade parameters are different. Then in the final design the three dimensional effect is taken into account. But the leading edge shape is not considered to be affected by the three dimensional effect.

These foils have little flat plate component. That is, while the lift is shared by the flat plate and the five term camber, only 10 per cent is assigned to the flat plate. The cavity thickness near the leading edge is given as an input at the 10 per cent of the chord from the leading edge. The shape of super-cavitating section is a function of the lift, the leading edge cavity thickness and cascade parameters. When the flat plate component is too small with the five term camber for a large lift,

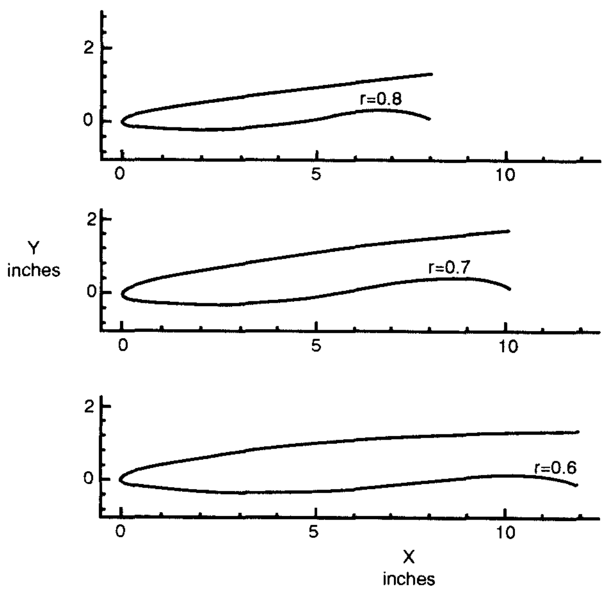

Figure 2. Foil-cavity Shapes of Super-cavitating Sections

the camber looks to be so large that it might induce face cavitation although propeller efficiency is better.

If an accurate three dimensional lifting surface theory is available, the present method may not be necessary. However, since the design problem is in general relying upon the method of trial and error, a simple and quick calculation is necessary. Thus a quick preliminary design by several seconds computing helps greatly.

The cavity separation point at the leading edge influences not only cavity shape but also the cavity drag [Yim et al, 1983]. A point drag model helped the design of super-cavitating sections with reasonably thick yet low drag leading edge. Only the fault of the point drag model was that difficulty to locate the cavity separation point. This problem is solved by the present approach.

As in Equation 16 and 18 the Kirchhoff lamina is perpendicular to the velocity at far front. In the propeller section design, it may not be in too much error when we make the Kirchhoff lamina perpendicular to the blade nose tail line.

The breadth of lamina is very small compared to the chord length. However, for the large propeller, the actual length may be quite large. In any case, this lamina may give a strong influence to the whole flow field.

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