Journal of Statistical Theory & Methods
1997, Vol. 8, No. 1, pp. 99 ~109

The Asymptotic Properties of Mean Residual Life Function on Left Truncated and Right Censoring Model

Kyoung Ae Moon[1], Im Hee Shin[2]

Abstract The estimation procedure of mean residual life function has been placed an important role in the study of survival analysis. In this paper, the product limit estimator on left truncated and right censoring model is proposed with asymptotic properties. Also, the small sample properties are investigated through the Monte Carlo study and the proposed product limit type estimator is compared with ordinary Kaplan-Meier type estimator.

Keywords : mean residual life function, product limit estimator, left and right censoring model.

1. Introduction

For a long time, the estimation procedure of the mean residual life function(MRLF) has been studied since Cox(1961) and Swartz(1973) represented the mean residual life function at age t , $e(t)$ in terms of $E(X-t|X>t)$.

Both nonparametric model and parametric model, many important asymptotic results for the estimator of MRLF have been shown by Yang(1977, 1978) and Ghorai and Rejto(1987). Especially, Wang, Jewell and Tsai(1986) considered the left truncated and right censoring(LTRC) model and proposed the product limit estimator of survival function. And also, they compared with Kaplan-Meier estimator and showed the better points of product limit estimator.

In this paper, we propose the estimator for MRLF using the product limit estimator on the LTRC model and show the proposed estimator is better than the estimator using Kaplan-Meier estimator in the expect of the M.S.E.'s through

[1] Kyung Ae Moon is instructor of Donghae Junior College
[2] Im Hee Shin is lecturer of Kyungpook National University and Catholic University of Taegu-Hyosung

Monte Carlo study.

2. Preliminaries and Proposed Estimator.

Some notations and useful lemmas are given for the estimation of the survival function.

Let U_i, $i = 1,2,\cdots$ be the lifetime following the distribution F and T_i, C_i be the random truncation time and the censoring time with distribution function G and H, respectively to be independent of U_i, $i = 1,2,\cdots$. Suppose that (X_i, δ_i, T_i) is observable only when $X_i \geq T_i$, where $X_i = \min(U_i, C_i)$ and

$$\delta_i = \begin{cases} 1 & \text{if } U_i \leq C_i \\ 0 & \text{if } U_i \geq C_i \end{cases}.$$

Thus, the observed data is given by a set of n independent identically distributed observations (X_i, δ_i, T_i), $i = 1,2,\cdots,n$.

Tsai, Jewell and Wang(1987) suggested the product-limit estimator for the survival function $S(t)$ given by

$$\hat{S}(t) = \sum_{x_j \leq t} \frac{n_j - d_j}{n_j},$$

with d_j is the number of failures at time X_j and $n_j = \sum I(T_i \leq X_j \leq Y_i)$ is the number in the risk set at time X_j, where I is the usual indicator function. Note that $\hat{S}(t)$ reduces to the Kaplan and Meier estimator(1958) for right censored data if $T_1 = T_2 = \cdots = T_n = 0$.

Though, it is not possible over the support of lifetime distribution F without further restriction on the underlying distribution functions. For any distribution R on $[0,\infty)$, let $a_R = \inf\{t > 0: R(t) > 0\} \geq 0$ and $b_R = \sup\{t > 0: R(t) < 1\} \leq \infty$. If $a_F < a_G$, then we can not expect to estimate $S(t)$ without parametric assumption of $S(t)$. Similarly, if $b_H < b_F$, then we can not estimate $S(t)$ for $t > b_H$ without parametric assumption.

We consider the estimation $S(t)$ for $t \in [0,K]$. Let $X_{(1)}, X_{(2)}, \cdots X_{(n)}$ be the order statistics of $X_1, X_2, \cdots, X_n$ and $s_{(1)}, s_{(2)}, \cdots, s_{(r)}$ among $X_1, X_2, \cdots, X_n$ be the distinct observed lifetimes.

It can be easily seen that the estimator of $S\ (t)$ is the step function with possible jumps at $s_{(1)}, s_{(2)}, \cdots, s_{(r)}$. Then Tsai, Jewell and Wang(1987) suggested the nonparametric conditional maximum likelihood estimator for the survival function $S(t)$ given by

$$\hat{S}_{PL}(t) = \prod_{i:T^* \le s_{(i)} \le t} \left(1 - \frac{\Delta N(s_{(i)})}{Y(s_{(i)})}\right),$$

given $N(t) = \sum_{i=1}^{n} I(X_i \le t, \delta_i = 1)$ and $Y(t) = \sum_{i=1}^{n} I(T_i \le t \le X_i)$.

Tsai, Jewell and Wang(1987) showed the consistency and the weak convergency of the product-limit estimator $\hat{S}_{PL}(t)$.

Lemma 2.1. (Tsai, Jewell and Wang(1987)) Let F, G and H be continuous. Then for $0 \le t \le K$,

a) $\Pr(|\hat{S}_{PL}(t) - S\ (t)|) \to 0$ with probability 1 as $n \to \infty$.

b) $\sqrt{n}(\hat{S}_{PL}(t) - S^*(t)) \to W(t)$ weakly in $D[0, K]$, where $W(t)$ is a mean zero Gaussian process with covariance structure given by

$$Cov(W(s), W(t)) = S(s)S(t)\int_{T^*}^{\min(s,t)} \frac{dF(u)du}{(1 - H\ (u))^2},$$

where $-H\ (t) = \Pr(T \le t \le X | X \ge T)$ and $F\ (t) = \Pr(X \le t, \delta = 1 | X \ge T)$.

In this section, we consider the estimation of MRLF based on the product-limit estimator.

The MRLF, $e_K(t)$ is defined to be the expected value remaining lifetime given reliability to age t as follows:

$$e_K(t) = E(X - t | X > t) = (S\ (t))^{-1}\int_t^K S\ (u)du.$$

Then the estimator $\hat{e}_K{}^{PL}(t)$ is proposed using the product-limit estimator $\hat{S}_{PL}(t)$ as follows ;

$$\hat{e}_K{}^{PL}(t) = (\hat{S}_{PL}(t))^{-1}\int_t^K \hat{S}_{PL}(u)du.$$

3. Asymptotic Properties of the Proposed Estimators.

$\hat{e}_K^{PL}(t)$ converges weakly to a Gaussian process as $n \to \infty$ is shown using the Lemma 2.1 and 3.1.

Lemma 3.1 (Yang(1977)). Let $D[0,K]$ be the space of functions on the interval $[0,K]$ that are right continuous and have left limits. Let d be the Skorohod metric on $D[0,K]$. Define a map $H: D[0,K] \to D[0,K]$ by having

$$H(W)(x) = S\ (x)\int_x^K W(u)du - W(x)\int_K^x S\ (u)du,$$

for $\in D[0,K]$. Then H is a continuous map and with respect to d.

Theorem 3.1. Let F, G and H be continuous . Then for $0 \le t \le K$,

$$\Pr(|\hat{e}_K^{PL}(t) - e_K(t)|) \to 0$$

with probability 1 as $n \to \infty$.

Proof. For fixed $t \in [0,K]$ and $s \le t$,

$$|\hat{e}_K^{PL}(t) - e\ (t)| = \left| \frac{\int_t^K \hat{S}_{PL}(u)du}{\hat{S}_{PL}(t)} - \frac{\int_t^K S\ (u)du}{S\ (t)} \right|$$

$$= (\hat{S}_{PL}(t)S(t))^{-1} | S\ (t)\int_t^K (\hat{S}_{PL}(u)du - S\ (u)du)$$
$$+(S(t) - \hat{S}_{PL}(t))\int_t^K S\ (u)du |$$
$$\le (\hat{S}_{PL}(t)S\ (t))^{-1}(S\ (t)\int_t^K |\hat{S}_{PL}(u) - S\ (u)|du$$
$$+|S\ (t) - \hat{S}_{PL}(t)|\int_t^K S\ (u)du)$$
$$\le (\hat{S}_{PL}(t)S\ (t))^{-1}(S\ (t)K \| \hat{S}_{PL}(t) - S\ (t)\|_K$$
$$+|S\ (t) - \hat{S}_{PL}(t)|\int_t^K S\ (u)du),$$

where $\|f\|_K = \sup\{|f(u)| \,|\, t \le u \le K\}$. So the result follows by lemma 2.1 a).

Now, we have the weak convergency of $\hat{e}_K^{PL}(t)$ of MRLF. The following theorem implies the asymptotic normality of the estimator $\hat{e}_K^{PL}(t)$ for each $t \in [0,K]$

Theorem 3.2. Let F, G and H be continuous . Then for $0 \le t \le K$,

$$\sqrt{n}(\hat{e}_K^{\ PL}(t) - e\ (t)) \to Z(t)$$

weakly in $D[0,K]$, where $Z(t)$ is a mean zero Gaussian process given by

$$Z(t) = (S(t))^{-2}\left(S(t)\int_t^K W(u)du - W(t)\int_t^K S\ (u)du \right)$$

and the covariance structure of Z is given by

$$\begin{aligned} Cov(Z(s),Z(t)) = (S\ (s)S\ (t))^{-2}\Big(& S\ (s)S\ (t)E(\int_s^K \int_t^K W(u)W(v)dudv) \\ & + E(W(s)W(t))\int_s^K S\ (v)dv\int_t^K S\ (u)du \\ & - S(s)\int_t^K S\ (u)duE(W(t)\int_s^K W(v)dv) \\ & - S\ (t)\int_s^K S\ (v)dvE(W(s)\int_t^K W(u)du)\Big), \end{aligned}$$

for $0 \le s \le t \le K$.

Proof. $\sqrt{n}(\hat{e}_K^{\ PL}(t) - e\ (t)) = \sqrt{n}(\hat{S}_{PL}(t)S(t))^{-1}\Big(S\ (t)\int_t^K (\hat{S}_{PL}(u)du - S\ (u))du$

$$-(\hat{S}_{PL}(t) - S\ (t))\int_t^K S\ (u)du\Big).$$

By lemma 2.1 a), the asymptotic distribution of $\sqrt{n}(\hat{e}_K^{PL}(t) - e(t))$ is the same as that of

$$\sqrt{n}(S\ (t))^{-2}\left(S\ (t)\int_t^K (\hat{S}_{PL}(u)du - S\ (u))du - (\hat{S}_{PL}(t) - S\ (t))\int_t^K S\ (u)du \right).$$

Thus, the result follows from lemma 2.1.b) and continuity theorem in Billingsley(1968) that can be applied by lemma 3.1. And the covariance function is evaluated by the interchange of expectation with integral signs.

4. Simulation Study

We shall investigate the small sample performances of the proposed estimator $\hat{e}_K^{PL}(t)$ for MRLF, $e_K(t)$ based on the biases and mean squared errors (MSE's) via Monte Carlo simulation.

The simulation is performed to compare the small sample properties of $\hat{e}_K^{PL}(t)$ with those of the Kaplan-Meier type estimator $\hat{e}_K^{KM}(t)$ based on the Kaplan-Meier estimator

$\hat{S}_{KM}(t)$ for the survival function $S(t)$ obtained by ignoring the truncation effects with Weibull distribution with constant failure rate, decreasing failure rate and increasing failure rate according to the scale and shape parameters. The censoring distributions are supposed to be exponential and uniform with the censoring rate about 10% and 30%. For sample size n = 20, 50, the procedure is repeated 500 times to get estimates of biases and MSE's of the estimators $\hat{e}_K^{PL}(t)$ and $\hat{e}_K^{KM}(t)$ such that $t = S^{-1}(p), \quad p = 0.9, 0.8, \cdots, 0.1$.

The results are listed in Table 1 through Table 8. From these tables, we can draw the following facts.

(1) For the case of $Weib(1.0, 1.0)$ and the censoring distribution is exponential, the product-limit type estimator $\hat{e}_K^{PL}(t)$ tends to have smaller bias and MSE than the Kaplan-Meier type estimator $\hat{e}_K^{KM}(t)$ except in the area of upper tail of the distribution with the censoring rate 10% and $n = 20$. But, $\hat{e}_K^{PL}(t)$ is more efficient than $\hat{e}_K^{KM}(t)$ with the censoring rate 30% regardless of sample size. In the case of censoring distribution is given as uniform, $\hat{e}_K^{PL}(t)$ always seems to be more efficient than $\hat{e}_K^{KM}(t)$ with respect to bias and MSE.

(2) For the case of $Weib(1.0, 1.5)$ and the censoring distribution is exponential or uniform with $n = 20$ and the censoring rate 10%, the product-limit type estimator $\hat{e}_K^{PL}(t)$ has smaller bias than the Kaplan-Meier type estimator $\hat{e}_K^{KM}(t)$. But with respect to MSE, $\hat{e}_K^{KM}(t)$ seems to be a little more efficient than $\hat{e}_K^{PL}(t)$ in the area of upper tail of the distribution. In the case of $n = 50$, $\hat{e}_K^{PL}(t)$ has tendency to be more efficient than $\hat{e}_K^{KM}(t)$ regardless of the censoring rate.

Through the above results, we can conclude that with data having the truncation effect, the product-limit type estimator seems to be more reasonable than Kaplan-Meier type estimator for the mean residual life function in the aspect of biases and MSE's.

References

1. Billingsley, P. (1968). *Convergence of Probability Measures*, John Wiley & Sons,
2. Cox, D. R. (1961) *Renewal Theory*, Methuen, London.
3. Ghorai, J. K. and Rejto, L. (1987). Estimation of mean residual life with censored data under the proportional hazard model, *Communication and*

Statistics- Theory. and Method. 16. 2097- 2114.
4. Kaplan, E. and Meier, P.(1958). Nonparametric estimation from Incomplete observations, *Journal of American Statistical Association,* 53, 457-481.
5. Lai, T. Z. and Ying, Z. (1991). Estimating a distribution function with truncate and censored data, *The Annals of Statistics,* 19, 417-442.
6. Swartz, G. B. (1973). The Mean Residual Life function, *IEEE Transaction Reliability R22,* 108-109.
7. Wang, M. C., Jewell, N. P. and Tsai, W. Y. (1986). Asymptotic properties of the product limit estimate under random Truncation, *The Annals of Statistics,* 14, 1597-1605.
8. Woodroofe, M. (1985). Estimating a distribution function with truncated data. *The Annals of Statistics,* 13, 163-177.
9. Tsai, W. Y., Jewell, N. P. and Wang, M. C. (1987). A note on the product-limit estimator under right censoring and left truncation, *Biometrika.* 74, 883-886.
10. Yang, G. (1977). Life expectancy under reandom censorship, *Stochastic Processes and Their Applications*, 6, 33-38.
11. Yang, G. (1978). Estimation of a Biometric Function, *The Annals of Statistics*, 6, 112-116.

Table 1. The Estimated Biases and MSE's (n=20)

Lifetime : Weibull(1.0, 1.0) Truncation : Exp(2.0)

		Censoring : Exp(.111) Censoring rate : 10%		Censoring : Exp(.429) Censoring rate : 30%	
S(t)		K-M	P-L	K-M	P-L
0.1	Bias	-.0691	-.0691	-.6354	-.6354
	MSE	.9906	.9906	.8316	.8316
0.2	Bias	.1367	.1367	-.1885	-.1885
	MSE	.5076	.5076	.3914	.3914
0.3	Bias	.1597	.1592	-.0209	-.0209
	MSE	.3628	.3608	.1975	.1975
0.4	Bias	.1268	.1259	.0065	.0060
	MSE	.2154	.2121	.1256	.1253
0.5	Bias	.1366	.1353	.0424	.0417
	MSE	.1668	.1629	.1097	.1093
0.6	Bias	.1517	.1493	.0703	.0687
	MSE	.1519	.1488	.1017	.1010
0.7	Bias	.1784	.1749	.1134	.1108
	MSE	.1367	.1336	.1077	.1070
0.8	Bias	.2211	.2169	.1636	.1604
	MSE	.1413	.1376	.1118	.1108
0.9	Bias	.2898	.2848	.2362	.2323
	MSE	.1755	.1710	.1439	.1420

Table 2. The Estimated Biases and MSE's (n=50)

Lifetime : Weibull(1.0, 1.0) Truncation : Exp(2.0)

S(t)		K-M (Censoring : Exp(.111), Censoring rate : 10%)	P-L (Censoring : Exp(.111), Censoring rate : 10%)	K-M (Censoring : Exp(.429), Censoring rate : 30%)	P-L (Censoring : Exp(.429), Censoring rate : 30%)
0.1	Bias	.1166	.1166	-.2169	-.2169
	MSE	.3944	.3944	.5446	.5446
0.2	Bias	.0630	.0626	-.0169	-.0169
	MSE	.1262	.1254	.2030	.2030
0.3	Bias	.0538	.0527	.0097	.0097
	MSE	.0852	.0837	.1570	.1570
0.4	Bias	.0688	.0672	.0207	.0203
	MSE	.0703	.0685	.1067	.1066
0.5	Bias	.0881	.0858	.0471	.0461
	MSE	.0614	.0596	.0718	.0715
0.6	Bias	.1084	.1052	.0688	.0670
	MSE	.0508	.0489	.0549	.0541
0.7	Bias	.1464	.1420	.1096	.1070
	MSE	.0574	.0550	.0629	.0618
0.8	Bias	.1942	.1884	.1553	.1518
	MSE	.0728	.0694	.0722	.0707
0.9	Bias	.2614	.2547	.2250	.2207
	MSE	.0132	.0114	.0987	.0962

Table 3. The Estimated Biases and MSE's (n=20)

Lifetime : Weibull(1.0, 1.5) Truncation : Exp(1.67)

S(t)		K-M (Censoring : Exp(.119), Censoring rate : 10%)	P-L (Censoring : Exp(.119), Censoring rate : 10%)	K-M (Censoring : Exp(.432), Censoring rate : 30%)	P-L (Censoring : Exp(.432), Censoring rate : 30%)
0.1	Bias	-.0192	-.0192	-.2133	-.2133
	MSE	.2303	.2303	.1902	.1902
0.2	Bias	.1350	.1350	.1055	.1055
	MSE	.1117	.1117	.1279	.1279
0.3	Bias	.1981	.1981	.1889	:1889
	MSE	.1007	.1007	.0929	.0929
0.4	Bias	.2356	.2354	.2090	.2090
	MSE	.1057	.1056	.0823	.0823
0.5	Bias	.2668	.2665	.2414	.2414
	MSE	.1082	.1080	.0866	.0866
0.6	Bias	.2895	.2892	.2758	.2757
	MSE	.1211	.1209	.1031	.1030
0.7	Bias	.3218	.3209	.3075	.3069
	MSE	.1397	.1390	.1206	.1201
0.8	Bias	.3524	.3509	.3439	.3430
	MSE	.1543	.1529	.1430	.1421
0.9	Bias	.3977	.3957	.3802	.3788
	MSE	.1881	.1861	.1694	.1682

Table 4. The Estimated Biases and MSE's (n=50)

Lifetime : Weibull(1.0, 1.5) Truncation : Exp(1.67)

S(t)		Censoring : Exp(.119) Censoring rate : 10% K-M	 P-L	Censoring : Exp(.432) Censoring rate : 30% K-M	 P-L
0.1	Bias	.0351	.0351	.0045	.0045
	MSE	.0668	.0668	.0820	.0820
0.2	Bias	.1093	.1093	.1223	.1223
	MSE	.0404	.0404	.0533	.0533
0.3	Bias	.1585	.1584	.1712	.1712
	MSE	.0458	.0458	.0550	.0550
0.4	Bias	.2014	.2012	.1991	.1991
	MSE	.0567	.0566	.0588	.0588
0.5	Bias	.2368	.2364	.2321	.2321
	MSE	.0684	.0682	.0676	.0676
0.6	Bias	.2667	.2662	.2587	.2585
	MSE	.0823	.0819	.0767	.0765
0.7	Bias	.3000	.2989	.2911	.2906
	MSE	.1008	.1001	.0942	.0939
0.8	Bias	.3341	.3325	.3202	.3193
	MSE	.1215	.1204	.1129	.1122
0.9	Bias	.3774	.3751	.3631	.3618
	MSE	.1518	.1500	.1424	.1414

Table 5. The Estimated Biases and MSE's (n=20)

Lifetime : Weibull(1.0, 1.0) Truncation : Exp(2.0)

S(t)		Censoring : Uni(10.0) Censoring rate : 10% K-M	 P-L	Censoring : Uni(3.197) Censoring rate : 30% K-M	 P-L
0.1	Bias	-.1350	-.1350	-.8465	-.8465
	MSE	.6934	.6934	.7815	.7815
0.2	Bias	.1225	.1225	-.3458	-.3458
	MSE	.3564	.3564	.2743	.2743
0.3	Bias	.0791	.0787	-.1320	-.1320
	MSE	.2048	.2041	.1427	.1427
0.4	Bias	.0811	.0799	-.0728	-.0728
	MSE	.1439	.1430	.1086	.1086
0.5	Bias	.0976	.0959	-.0318	-.0319
	MSE	.1070	.1059	.0798	.0799
0.6	Bias	.1211	.1183	.0125	.0121
	MSE	.1040	.1028	.0725	.0725
0.7	Bias	.1550	.1509	.0523	.0510
	MSE	.1020	.0999	.0622	.0622
0.8	Bias	.2046	.1991	.1090	.1070
	MSE	.1152	.1121	.0665	.0663
0.9	Bias	.2765	.2704	.1739	.1715
	MSE	.0345	.0330	.0202	.0202

Table 6. The Estimated Biases and MSE's (n=50)

Lifetime : Weibull(1.0, 1.0) Truncation : Exp(2.0)

S(t)		Censoring : Uni(10.0) Censoring rate : 10% K-M	P-L	Censoring : Uni(3.197) Censoring rate : 30% K-M	P-L
0.1	Bias	.0406	.0406	-.6583	-.6583
	MSE	.2120	.2120	.5133	.5133
0.2	Bias	.0412	.0406	-.2271	-.2271
	MSE	.0954	.0952	.1016	.1016
0.3	Bias	.0358	.0348	-.1389	-.1389
	MSE	.0644	.0639	.0646	.0646
0.4	Bias	.0610	.0592	-.0810	-.0813
	MSE	.0514	.0508	.0433	.0433
0.5	Bias	.0746	.0719	-.0386	-.0392
	MSE	.0453	.0446	.0357	.0358
0.6	Bias	.1040	.1004	-.0036	-.0024
	MSE	.0448	.0438	.0298	.0298
0.7	Bias	.1404	.1357	.0468	.0449
	MSE	.0490	.0473	.0263	.0261
0.8	Bias	.1886	.1825	.0979	.0951
	MSE	.0630	.0603	.0331	.0326
0.9	Bias	.2570	.2499	.1674	.1640
	MSE	.0086	.0080	.0496	.0484

Table 7. The Estimated Biases and MSE's (n=20)

Lifetime : Weibull(1.0, 1.5) Truncation : Exp(1.67)

S(t)		Censoring : Uni(9.024) Censoring rate : 10% K-M	P-L	Censoring : Uni(3.002) Censoring rate : 30% K-M	P-L
0.1	Bias	-.0742	-.0742	-.3012	-.3012
	MSE	.2004	.2004	.1567	.1567
0.2	Bias	.1643	.1643	.0323	.0323
	MSE	.1015	.1015	.0756	.0756
0.3	Bias	.1956	.1954	.1451	.1451
	MSE	.0869	.0866	.0596	.0596
0.4	Bias	.2339	.2335	.1924	.1924
	MSE	.0915	.0911	.0761	.0761
0.5	Bias	.2627	.2623	.2281	.2280
	MSE	.0985	.0980	.0868	.0867
0.6	Bias	.2885	.2878	.2675	.2671
	MSE	.1121	.1114	.1019	.1015
0.7	Bias	.3159	.3148	.2934	.2929
	MSE	.1274	.1264	.1125	.1120
0.8	Bias	.3520	.3505	.3317	.3306
	MSE	.1503	.1488	.1356	.1349
0.9	Bias	.3960	.3941	.3776	.3764
	MSE	.1843	.1822	.1690	.1680

Table 8. The Estimated Biases and MSE's (n=50)

Lifetime : Weibull(1.0, 1.5) Truncation : Exp(1.67)

S(t)		Censoring : Uni(9.024) Censoring rate : 10% K-M	P-L	Censoring : Uni(3.002) Censoring rate : 30% K-M	P-L
0.1	Bias	.0168	.0168	-.1385	-.1385
	MSE	.0443	.0443	.0728	.0728
0.2	Bias	.1154	.1154	.0673	.0673
	MSE	.0348	.0348	.0255	.0255
0.3	Bias	.1556	.1555	.1305	.1305
	MSE	.0396	.0395	.0333	.0333
0.4	Bias	.2049	.2046	.1745	.1743
	MSE	.0547	.0545	.0446	.0446
0.5	Bias	.2374	.2369	.2125	.2122
	MSE	.0685	.0682	.0580	.0579
0.6	Bias	.2700	.2693	.2489	.2484
	MSE	.0837	.0832	.0720	.0718
0.7	Bias	.2998	.2985	.2777	.2770
	MSE	.0995	.0986	.0855	.0850
0.8	Bias	.3378	.3360	.3138	.3127
	MSE	.1234	.1221	.1070	.1063
0.9	Bias	.3795	.3772	.3564	.3549
	MSE	.1530	.1512	.1358	.1347