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Parametric Empirical Bayes Estimators with Item-Censored Data

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Abstract

This paper is proposed the parametric empirical Bayes(EB) confidence intervals which corrects the deficiencies in the naive EB confidence intervals of the scale parameter in the Weibull distribution under item-censoring scheme. In this case, the bootstrap EB confidence intervals are obtained by the parametric bootstrap introduced by Laird and Louis(1987). The comparisons among the bootstrap and the naive EB confidence intervals through Monte Carlo study are also presented.

Key Words and Phrases: Parametric empirical Bayes estimators, Geometric mean estimator, Type III parametric bootstrap, Item-censored data, Weibull distribution.

1. Introduction

When data are collected from many units that are somehow similar, such as subjects, animals, cities *et al*, the statistcal problem is to combine the information from the various units to understand better the phenomenon under study. Usually there is substantial variability among units and a natural way to approach the problem is to build a two-stage 'hierarchical model', empirical Bayes(EB) model and then use it to make inference. Also EB methods effectively incorporate information from past data(or other components in simultaneous estimation) by means of analyzing the marginal density of all the data present and past given the prior parameters. We consider the familiar exchangeable Bayesian model. That is, we are simultaneously testing k populations. For the i-th population, $i = 1, \cdots, k$, we test n_i devices until the number of failures are r_i. At first stage, the independent lifetime t_{ij} for each device tested in the i-th population is assumed to be Weibull with known shape parameter β and unknown scale parameter θ_i, i.e.,

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$$f(t_{ij}|\theta_i,\beta)=\frac{\beta}{\theta_i}t_{ij}^{\beta-1}\exp\left(-\frac{t_{ij}^{\beta}}{\theta_i}\right) \tag{1}$$

Let $t_i = (t_{i1}, t_{i2}, \cdots, t_{ir_i})$ denote the ordered lifetimes of the r_i devices that failed in the i-th population, where $t_{i1} < \cdots < t_{ir_i}$. Then

$$T_i = (n_i - r_i)t_{ir_i}^{\beta} + \sum_{j=1}^{ri} t_{ij}^{\beta} \tag{2}$$

is the sufficient statistic for θ_i and has gamma distribution $G(r_i, \theta_i)$ by Sukhatme (1937), i.e.,

$$f(T_i|\theta_i)=\frac{1}{\Gamma(r_i)\theta_i^{r_i}}T_i^{r_i-1}\exp\left(-\frac{T_i}{\theta_i}\right),\ r_i,\ \theta_i>0, \tag{3}$$

At the second stage, the θ_i's are supposed independently and identically distributed (iid) the inverse gamma distribution $IG(u,\ 1/v)$, i.e.,

$$\pi(\theta_i)=\frac{1}{\Gamma(u)v^u\theta_i^{u+1}}\exp\left(-\frac{1}{\theta_i v}\right),\ u>0,\ v>0, \tag{4}$$

Then the posterior distribution of θ_i given T_i is $IG(u+r_i,\ (T_iv+1)/v)$, i.e.,

$$f(\theta_i|T_i,u,v)=\frac{1}{\Gamma(u+r_i)}\Big(\frac{T_iv+1}{v}\Big)^{u+r_i}\Big(\frac{1}{\theta_i}\Big)^{u+r_i+1}\exp\Big(-\frac{T_iv+1}{\theta_i v}\Big). \tag{5}$$

In the fully Bayesian setting, one chooses a value for the 'hyperparameter' $\delta = (u, v)$ based on subjective information or prior knowledge and then bases all inferences about θ on $f(\theta_i|T_i, \delta = (u, v))$. Therefore, the Bayes estimator for θ_i with respect to the squared error loss[$L(\theta,\ \mathbf{x}) = \sum_{i=1}^{p}(\theta_i - x_i)^2/k$, where x_i denotes an estimator for θ_i] is given by

$$\mu_B(i)=E(\theta_i|T_i)=\frac{T_i}{(u+r_i-1)}+\frac{1}{v(u+r_i-1)},\ u>1 \tag{6}$$

and the posterior variance for θ_i is given by

$$\sigma_B^2(i)=Var(\theta_i|T_i)=\frac{(T_iv+1)^2}{v^2(u+r_i-1)^2(u+r_i-2)},\ u>2. \tag{7}$$

Also, the marginal density of T_i is given by

$$h(T_i|u,v)=\frac{\Gamma(u+r_i)S_i^{r_i-1}v^{r_i}}{\Gamma(u)\Gamma(r_i)(T_iv+1)^{u+r_i}}. \tag{8}$$

Thus the joint marginal density of $\mathbf{T}$ is given by

$$h(\mathbf{T}|u,v) = \prod_{i=1}^{k} \frac{\Gamma(u+r_i)}{\Gamma(u)\Gamma(r_i)} \frac{T_i^{r_i-1} v^{r_i}}{(T_i v+1)^{u+r_i}}, \tag{9}$$

where $\mathbf{T} = (T_1, T_2, \cdots, T_k)$.

2. Marginal(Hyperparameter) estimators

Dey and Kuo(1991) proposed a different EB estimator when u is known. This estimator expands the uaual estimator by a multiple of the geometric mean of the component estimators. To construct the EB estimator, they assumed that u is known and v is estimated from the joint marginal density $h(\mathbf{T}|u,v)$ as follows. From the marginal density of T_i in equation (8), it follows that $T_i \sim (r_i/vu)F_{2r_i,2u}$, where $F_{2r_i,2u}$ is the usual F distribution with degrees of freedom $2r_i$ and $2u$. Thus by equation (7.8.13) of Wilks (1962) it follows that

$$\mathrm{E}(T_i^t) = \frac{\Gamma(u-t)}{\Gamma(u)} \frac{\Gamma(r_i+1)}{\Gamma(r_i)} \frac{1}{v^t}. \tag{10}$$

Also

$$\mathrm{E}\Big(\prod_{i=1}^{k} T_i^{1/k}\Big) = \frac{1}{v}\Big[\frac{\Gamma(u-\frac{1}{k})}{\Gamma(u)}\Big]^k \prod_{i=1}^{k} \frac{\Gamma(r_i+\frac{1}{k})}{\Gamma(r_i)}. \tag{11}$$

where $t = 1/k$. Therefore, an unbiasded estimator of $1/v$ is obtained as

$$\widehat{\Big(\frac{1}{v}\Big)} = \Big[\frac{\Gamma(u)}{\Gamma(u-\frac{1}{k})}\Big]^k \prod_{i=1}^{k} \Big[\frac{\Gamma(r_i)}{\Gamma(r_i+\frac{1}{k})} T_i^{1/k}\Big]. \tag{12}$$

The EB geometric mean estimator by Dey and Kuo(1991) is obtained from equation (6) with $\widehat{v}_{D/K} = \max(\widehat{v},\ 0)$, where $\widehat{v}$ is estimated from equation (12) with known u. However, the assumption of known u is too restrictive in practice. Kuo and Yiannoutsos(1993) proposed the modified EB marginal estimator which estimates u by the moment method and estimates v as in Dey and Kuo(1991). That is, from equation (2.19) and equation (2.20) of Choi(1996),

$$\frac{\mu_m^2}{\sigma_m^2} = \frac{r(u-2)}{u-1+r}. \tag{13}$$

Then an estimator of u is obtained from equation (13) using $\widehat{\mu}_m = \sum T_i \ / \ k$, $\widehat{\sigma}_m^2 = \sum (T_i - \widehat{\mu}_m)^2 \ / \ (k-1)$ instead of μ_m and σ_m^2. It follows that the marginal estimates of u and v are

$$\widehat{u}_{\mathrm{K/Y}} = \max\Big[\frac{\widehat{\mu}_m^2(1-r) - 2r\widehat{\sigma}_m^2}{\widehat{\mu}_m^2 - r\widehat{\sigma}_m^2},\ \ 2\Big]$$

and

$$\widehat{v}_{\mathrm{K/Y}} = \widehat{v}_{\mathrm{D/K}}.$$

3. Naive empirical Bayes confidence interval

Let $\widehat{\delta} = (\ \widehat{u},\ \widehat{v}\ )$ be the marginal estimator of $\delta = (u, v)$ computed from the marginal distribution of T_i. Then the estimated posterior distribution of θ_i given T_i is $IG(\widehat{u} + r_i,\ (S_i\widehat{v} + 1)/\ \widehat{v}\ )$, that is,

$$f(\theta_i|T_i, \widehat{u},\ \widehat{v}\ ) = \frac{1}{\Gamma(\widehat{u} + r_i)}\Big(\frac{T_i\widehat{v} + 1}{\widehat{v}}\Big)^{\widehat{u}+r_i}\Big(\frac{1}{\theta_i}\Big)^{\widehat{u}+r_i+1}\exp\Big(-\frac{T_i\widehat{v} + 1}{\theta_i\widehat{v}}\Big). \tag{14}$$

Now, we construct the equal-tailed $100(1 - 2\alpha)\%$ EB naive confidence interval for θ_i based upon $f(\theta_i|T_i, \widehat{u},\ \widehat{v}\ )$ as follows:

$$\Big(\frac{2(T_i\widehat{v} + 1)}{\widehat{v}\ \mathcal{X}^{-1}_{2(\hat{u}+r_i)}(1 - \alpha)},\ \frac{2(S_i\widehat{v} + 1)}{\widehat{v}\ \mathcal{X}^{-1}_{2(\hat{u}+r_i)}(\alpha)}\Big), \tag{15}$$

where $\mathcal{X}_k$ denotes the cumulative distribution function $(c.d.f.)$ of chi-square distribution with k (not necessarily integer) degrees of freedom. This interval is called "naive" because they ignore randomness in $\widehat{\delta} = (\ \widehat{u},\ \widehat{v}\ )$. Though relatively easy to compute, they are often too short, inappropriately centered, or both, and hence fail to attain the nominal coverage probability. The explanation for this problem from a parametric EB point of view is that we are ignoring the variability in $\widehat{\delta} = (\ \widehat{u},\ \widehat{v}\ )$; from a Bayesian point of view, we are ignoring the posterior uncertainty about $\delta = (u, v)$. More precisely, we have

$$\mathrm{Var}(\theta_i|\mathbf{T}) = E_{\delta|\mathbf{T}}\{\mathrm{Var}(\theta_i|T_i, \delta)\} + \mathrm{Var}_{\delta|\mathbf{T}}\{E(\theta_i|T_i, \delta)\}, \tag{16}$$

and so the variance estimate based on $f(\theta_i|T_i, \widehat{u},\ \widehat{v}\ )$, $\mathrm{Var}(\theta_i|T_i, \widehat{u},\ \widehat{v}\ )$, will only approximate the first term in equation (16).

4. Bootstrapped empirical Bayes confidence intervals

We will suggest several bootstrap methods in order to correct the bias in the naive confidence interval based on $f(\theta_i|T_i, \widehat{u},\ \widehat{v}\ )$ and show how they may be used to compute confidence intervals.

4.1 Marginal posterior method

Laird and Louis(1987) suggested approximating the marginal posterior of θ_i given T_i by type III parametric bootstrap. That is, given $\widehat{\delta} = (\ \widehat{u},\ \widehat{v}\ )$, drew θ_i^* from $\pi(\theta|\ \widehat{\delta}\ )$. Then drew t_{ij}^* from $f(t_i|\theta_i^*)$, and finally calculate $\delta^* = (u^*,\ v^*)$ using t_{ij}^*. Repeating this process N times, they obtained $\delta_j^* = (u_j^*, v_j^*)$, $j = 1, \cdots, N$ distributed as $g(\cdot|\ \widehat{\delta}\ )$. By type III parametric bootstrap, we obtain the discrete mixture distribution mimicking the hyperprior calculation given by

$$H_{\mathrm{T}}^*(\theta_i|T_i) = \frac{1}{N}\sum_{j=1}^{N} \mathcal{X}_{2(\alpha_j^*+r_i)}\Big(\frac{2(T_i v_j^*+1)}{\theta_i\ v_j^*}\Big). \tag{17}$$

As defined, $H_{\mathrm{T}}^*(\theta_i|T_i)$ is at most an N-point mixture of $f(\theta_i|T_i,u,v)$ distributions, the mixing distribution having mass $1/N$ at $\delta_j^* = (u_j^*, v_j^*)$, $j = 1, \cdots, N$ and can be used directly to produce equal-tailed confidence intervals for θ_i by solving

$$\frac{\alpha}{2} = \int_{-\infty}^{C_L} dH_{\mathrm{T}}^*(\theta_i|T_i) = \int_{C_U}^{\infty} dH_{\mathrm{T}}^*(\theta_i|T_i). \tag{18}$$

Therefore, the $100(1-2\alpha)\%$ marginal (Larid and Louis) EB confidence interval for θ_i is given by $(C_L,\ C_U)$.

4.2 Bias-corrected naive method

In the exchangeable case, Carlin and Gelfand(1991) developed a direct conditional bias-corrected naive method as follows. Let $q_\alpha(T_i, \delta = (u,v))$ is such that

$$\Pr(\theta_i \le q_\alpha(T_i,\delta) \mid \theta_i \sim f(\theta_i \mid T_i,\delta)) = \alpha. \tag{19}$$

Define

$$r(\widehat{\delta},\delta,T_i,\alpha) = \Pr(\theta_i \le q_\alpha(T_i,\widehat{\delta}) \mid \theta_i \sim f(\theta_i \mid T_i,\delta)) \tag{20}$$

and

$$R(\delta,T_i,\alpha) = E_{\widehat{\delta}|T_i,\delta}\{r(\widehat{\delta},\delta,T_i,\alpha)\}, \tag{21}$$

where the expectation is taken over $g(\widehat{\delta}|T_i,\delta)$ which is a density with respect to Lebesgue measure. Since equation (21) need not be close to α, let $R(\delta, T_i, \alpha') = \alpha$ for α'. To estimate expectations under the sampling density of $\widehat{\delta}$, $g(\ \widehat{\delta}\ |T_i,\delta)$, they obtained δ^* from the distribution $g(\cdot|T_i,\ \widehat{\delta}\ )$ as follows. Drew $\theta_1^*, \cdots, \theta_k^*$ *iid* from $\pi(\theta|\ \widehat{\delta}\ )$, then drew t_{ij}^*, $j = 1, \cdots, n_i$, independently from $f(t_i|\theta_i^*)$, $i = 1, \cdots, k$, and constructed T_i^* using t_{ij}^*, and finally computed δ^* from the pseudodata T_i^* in the same way that $\widehat{\delta}$ was computed from the data T_i. Concisely we have

$$\widehat{\delta} \longrightarrow \{\theta_i^*\} \longrightarrow \{T_i^*\} \longrightarrow \delta^*. \tag{22}$$

By unconditional EB correction method, we obtaine the type III parametric bootstrap estimate of $R(\delta, T_i, \alpha'_{(1)})$ by scheme (22) given by

$$\mathcal{A}_1^* = \frac{1}{N}\sum_{j=1}^{N} \mathcal{X}_{2(\widehat{u}+r_i)}\Big(\frac{v_j^*(S_i\widehat{v}+1)}{\widehat{v}\,(T_i v_j^*+1)}\mathcal{X}^{-1}_{2(u_j^*+r_i)}(\alpha'_{(1)})\Big) = \alpha \tag{23}$$

where we equate to α and solve for $\alpha'_{(1)}$. Then the $100(1-2\alpha)\%$ unconditional bias-corrected(I) naive EB interval for θ_i is given by

$$\Big(\frac{2(T_i\widehat{v}+1)}{\widehat{v}\,\mathcal{X}^{-1}_{2(\widehat{u}+r_i)}(1-\alpha'_{(1)})},\ \frac{2(S_i\widehat{v}+1)}{\widehat{v}\,\mathcal{X}^{-1}_{2(\widehat{u}+r_i)}(\alpha'_{(1)})}\Big). \tag{24}$$

If we desire interval corrected only for unconditional EB coverage, the bootstrap equation becomes

$$\mathcal{A}_2^* = \frac{1}{N}\sum_{j=1}^{N} \mathcal{X}_{2(\widehat{u}+r_i)}\Big(\frac{v_j^*(T_{ij}^*\widehat{v}+1)}{\widehat{v}\,(T_{ij}^* v_j^*+1)}\mathcal{X}^{-1}_{2(u_j^*+r_i)}(\alpha'_{(2)})\Big) = \alpha \tag{25}$$

where we equate to α and solve for $\alpha'_{(2)}$. Analogous to expression (24), we obtaine the $100(1-2\alpha)\%$ unconditional bias-corrected(II) naive EB intervals for θ_i given by

$$\Big(\frac{2(T_i\widehat{v}+1)}{\widehat{v}\,\mathcal{X}^{-1}_{2(\hat{u}+r_i)}(1-\alpha'_{(2)})},\ \frac{2(T_i\widehat{v}+1)}{\widehat{v}\,\mathcal{X}^{-1}_{2(\hat{u}+r_i)}(\alpha'_{(2)})}\Big). \tag{26}$$

For correction conditional on T_i, Carlin and Gelfand(1990) modified the Laird and Louis(1987) procedure to draw observations from g^* rather than g by changing scheme (22) to

$$\widehat{\delta} \longrightarrow \{\theta_p^*,\ p \neq i\} \longrightarrow \{T_p^*,\ p \neq i\} \longrightarrow \delta^* = \delta^*(T_i,\ \{T_p^*,\ p \neq i\}). \tag{27}$$

Repeating this process N times, they obtained

$$\delta_j^* \sim g^*(\cdot|S_i, \widehat{\delta}\,),\ \ j = 1, \cdots, N.$$

By EB bias-corrected method conditional on $T_i = t_i$, we obtain the type III parametric bootstrap estimate of $R(\delta, T_i, \alpha')$ by scheme (27) given by

$$\mathcal{A}_3^* = \frac{1}{N}\sum_{j=1}^{N} \mathcal{X}_{2(\widehat{a}+r_i)}\Big(\frac{b_j^*(S_i\widehat{b}+1)}{\widehat{b}\,(T_i b_j^*+1)}\mathcal{X}^{-1}_{2(a_j^*+r_i)}(\alpha')\Big) = \alpha \tag{28}$$

where we equate to α and solve for α'. Therefore, the $100(1-2\alpha)\%$ conditional bias-corrected naive EB interval for θ_i is given by

$$\left(\frac{2(T_i\widehat{v}+1)}{\widehat{v}\,\mathcal{X}^{-1}_{2(\widehat{u}+r_i)}(1-\alpha')},\ \frac{2(T_i\widehat{v}+1)}{\widehat{v}\,\mathcal{X}^{-1}_{2(\widehat{u}+r_i)}(\alpha')} \right). \tag{29}$$

4.3 Morris-delta bootstrap method

Morris(1983, 1987) used only the improved approximation of the estimated posterior mean and variance to compute EB confidence intervals for Gaussian-Gaussian setting under a flat prior. We obtain the bootstrap estimators of μ_{T_i} and $\sigma^2_{T_i}$ given by,

$$\mu^*_{T_i} = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{T_i}{(u^*_j+r_i-1)} + \frac{1}{v^*_j(u^*_j+r_i-1)}\right)$$

and

$$\begin{aligned}\sigma^{2*}_{T_i} &= \frac{1}{N}\sum_{j=1}^{N}\left(\frac{(T_iv^*_j+1)^2}{v^{2*}_j(u^*_j+r_i-1)^2(u^*_j+r_i-2)}\right)\\ &+ \frac{1}{N-1}\sum_{j=1}^{N}\left(\left(\frac{T_i}{(u^*_j+r_i-1)} + \frac{1}{v^*_j(u^*_j+r_i-1)}\right) - \mu^*_{T_i}\right)^2,\end{aligned}$$

where (u^*_j, v^*_j) is the bootstrap estimate of the hyperparameter (u,v) from the j-th type III parametric bootstrap samples by scheme (22). Therefore, the $100(1-2\alpha)\%$ Morris-Delta type EB bootstrap interval for θ_i is approximated by

$$\left(\mu^*_{T_i} - z_\alpha\sqrt{\sigma^{2*}_{T_i}},\ \ \mu^*_{T_i} + z_\alpha\sqrt{\sigma^{2*}_{T_i}} \right),$$

where z_α denotes the upper 100α percentile of standard normal distribution.

5. Comparisons and conclusions

The EB confidence intervals are approximated by Monte Carlo method. We consider the censoring rate (CR) defined by $100(1-r/n)\%$ of 0%(=complete case), 30%, 50%. For given independent random samples a EB confidence intervals are computed by each methods with bootstrap replications $B = 400$ times. And the Monte Carlo samplings are repeated 400 times. Let C_θ denotes the coverage probability for θ. We define L_θ by

$$L_\theta = \frac{1}{R}\sum_{j}^{R}(\widehat{\theta_{j,up}} - \widehat{\theta_{j,lo}})$$

where R is the number of Monte Carlo simulation replication. First, the comparisons of the naive and the bootstrap intervals against the estimator proposed by Kuo and Yiannoutsos(1993) when the hyperparameters u, v are unknown are presented in Table 1. We can observe the following results:

1. The C_θ's of bootstrap intervals obtained using given marginal estimator are better than that of the naive interval except for the Morris-Delta interval.
2. The L_θ's of bootstrap intervals obtained using given marginal estimator are longer than that of the naive interval.
3. Bias-corrected naive(BCN) bootstrap intervals obtained using the marginal estimator have more accurate than those of the other bootstrap intervals in the desired nominal coverage.
4. The C_θ's of all the intervals are linearly down as CR increases.

Secondly, the comparisons of the naive and the bootstrap intervals against the estimator proposed by Dey and Kuo(1991) when the hyperparameter u is fixed are presented in Table 2. We can observe the following results:

1. As a whole, the C_θ's and the L_θ's of the naive and all bootstrap intervals are obtained using given marginal estimator are nearly same.
2. The C_θ's and the L_θ's of all intervals are insensitive as the CR's changes
3. From Table 2.1(n=10), we have to similar results as Table 2.2(n=20).

Table 1. Comparison of Interval Methods When u, v are Unknown.

1.1 Sample size: $n = 10$, $\alpha = 0.05$, $\rho = 2$

Interval method	0 %	(CR)	30 %	(CR)	50 %	(CR)
	Coverage	Length	Coverage	Length	Coverage	Length
Naive	0.719	0.492	0.645	0.540	0.628	0.553
Cond-BCN	0.828	0.666	0.700	0.817	0.700	0.874
Unc-BCN(I)	0.755	1.019	0.653	1.140	0.665	1.175
Unc-BCN(II)	0.788	0.965	0.685	1.363	0.680	1.374
Laird & Louis	0.703	0.595	0.728	0.718	0.743	0.761
Morris-Delta	0.655	0.610	0.658	0.722	0.685	0.779

1.2 Sample size: $n = 20$, $\alpha = 0.05$, $\rho = 2$

Interval method	0 %	(CR)	30 %	(CR)	50 %	(CR)
	Coverage	Length	Coverage	Length	Coverage	Length
Naive	0.860	0.422	0.745	0.446	0.725	0.493
Cond-BCN	0.938	0.466	0.908	0.600	0.848	0.674
Unc-BCN(I)	0.935	0.640	0.853	1.040	0.835	1.075
Unc-BCN(II)	0.828	0.445	0.705	0.563	0.540	0.614
Laird & Louis	0.793	0.495	0.658	0.558	0.653	0.619
Morris-Delta	0.655	0.610	0.658	0.722	0.685	0.779

Table 2. Comparison of Interval Methods When u is Known.

2.1 Sample size: $n = 10$, $\alpha = 0.05$, $\rho = 2$

Interval method	0 %	(CR)	30 %	(CR)	50 %	(CR)
	Coverage	Length	Coverage	Length	Coverage	Length
Naive	0.779	0.492	0.745	0.540	0.688	0.553
Cond-BCN	0.778	0.496	0.750	0.547	0.709	0.554
Unc-BCN(I)	0.775	0.499	0.753	0.550	0.720	0.575
Unc-BCN(II)	0.798	0.515	0.785	0.583	0.750	0.604
Laird & Louis	0.783	0.495	0.758	0.558	0.713	0.561
Morris-Delta	0.745	0.510	0.718	0.582	0.685	0.609

2.2 Sample size: $n = 20$, $\alpha = 0.05$, $\rho = 2$

Interval method	0 %	(CR)	30 %	(CR)	50 %	(CR)
	Coverage	Length	Coverage	Length	Coverage	Length
Naive	0.859	0.412	0.825	0.480	0.828	0.513
Cond-BCN	0.858	0.426	0.820	0.477	0.830	0.524
Unc-BCN(I)	0.855	0.419	0.846	0.480	0.835	0.529
Unc-BCN(II)	0.868	0.425	0.855	0.493	0.850	0.537
Laird & Louis	0.856	0.415	0.836	0.486	0.823	0.521
Morris-Delta	0.845	0.420	0.798	0.492	0.825	0.539

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