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A Study of Curriculum Development for Mathematically Gifted Students[1]

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Even though there are extracurricular mathematics classes for gifted students in all levels of schools in Korea, teachers cannot conduct the classes properly because the contents of the textbook are not adequate for the purpose of the classes. So, what they tend to do in the classes is just drilling with many problems which have already shown up at university entrance examinations and various mathematics competitions.
The purpose of this paper is to give an example of what the content should be in "Mathematics III" (an elective subject for the science high school students according to the fifth and sixth amendment of national curriculum) and to suggest how to design the extracurricular classes for gifted students. Extracurricular classes of the ordinary secondary school as well as the elective course for the science high school can be suitably designed with choices of topics in the contents of Mathematics III.

0. MOTIVATION

At present, the Ministry of Education of the Korean Government is planning to change the national curriculum of primary(grades 1–6), middle(grades 7–9) and high(grades 10–12) schools in preparation for the future society of information and "Segyehwa" (globalization) for the 21st century (Koo 1996). The Ministry has changed the curriculum six times already since 1952. Hence this will be the seventh amendment.

Even though it is still in the draft stage, the Government has had an idea of the *curriculum revolution* long before the announcement of amendment of curriculum (President ial Commission for Educational Reform 1987).

The Ministry called the seventh revision of curriculum a "differentiated curriculum", because it emphasis more on individual difference of students. Here is the layout of the plan:

[1] The paper presented at TG 2: Mathematics education for the gifted students, the Seventh Southeast Asian Conference on Mathematics Education (SEACME-7), Hanoi, Vietnam, June 3–7, 1996.

Table 1. The Layout of Mathematics Courses

<table>
<tr><th>School</th><th>Grade</th><th colspan="2">Mathematics Courses</th></tr>
<tr><td rowspan="2">High School</td><td rowspan="2">12
11
10</td><td colspan="2">completely elective courses</td></tr>
<tr><td rowspan="3">10 steps of requisite courses</td><td rowspan="3">5 or 6 steps of elective courses</td></tr>
<tr><td>Middle School</td><td>9
8
7</td></tr>
<tr><td>Primary School</td><td>6
5
4
3
2
1</td></tr>
</table>

The author has participated in writing the textbook for "Mathematics III" which is an elective course for the science high school students according to the fifth amendment of the national curriculum (cf. Section 3). The author also has participated in survey and research for the "Mathematics III" curriculum (Choe 1992) of the sixth amendment. However, the result of survey and research, and the author's recommendations have not been accepted for various reason. Mathematics educators in Korea adhere stubbornly to the traditions and the government guidelines.

We still do not know the exact title[2] of the elective course for science high school students in the seventh amendment of national curriculum. Let us call it "Mathematics III" meanwhile.

The aim of this study in to find what contents should be in Mathematics III to maximize the efficiency of teaching and learning according to students interests, aptitudes and abilities as well as their needs.

1. HISTORICAL BACKGROUND

Korea has a written history of 5,000 years. However, its door was opened to westerners around 1880 and a new school system was adopted afterward. It was not long ago that mathematics became a subject of formal schools. "Wonsan School" and

[2] Added in Proof: The Ministry of Education named finally the elective course for science high school students "Advanced Mathematics" (**高級 數學**).

"Dongmun-Hag" opened in 1883 (Park 1991). They were the first schools of the new education system that had taken mathematics as a requisite. In 1895, Emperor Gojong of Chosun Kingdom (1392–1910 AD) issued several edicts for the new education system. Probably Hanseong Normal School (1895–?) was the first national school to take mathematics as a required subject. The next year all three types of elementary schools (national, public and private) were established. All of them took mathematics as a required subject.

In 1900, the first secondary school of the new education system was established. The contents of mathematics of the school were arithmetic, algebra, geometry, abacus and bookkeeping.

Even though the teaching of mathematics had started as a regular school subject, the curricula were introduced by foreigners (especially by Americans, Englanders, Germans, Canadians, Chinese and Japanese).

In 1910, the Korean Peninsula was annexed to the Japanese Empire and mathematics education in Korea was controlled by the Japanese government. For 35 years from 1910 to 1945, Japan changed mathematics curriculum several times, but Korean mathematics educators did not know the reason for the changes. One of the changes influenced by Progressivism of the western world around 1920 was started in the Peninsula around 1935. However, the influence of this change lasted in Korea very long even after the liberation from Japan.

2. Current Mathematics Curriculum of the Republic of Korea

Professor Chi-Young Kim (Yonsei University) and his associates undertook extensive research on curriculum of school mathematics, and as a result, they proposed a new curriculum reflecting the "new math" (Kim et al. 1969; Kim et al. 1971). The "new math" curriculum (the third curriculum change) became effective in 1974.

The "new math" textbooks based on the third curriculum change were in many respects patterned after the American experimental textbooks like S. M. S. G. and U. I. C. S. M., but the new movement was not implemented well in Korea because the contents of the textbook were too diverse. Another reason for the unsuccessful implementation was that many qualified mathematics teachers were leaving schools and the supply of qualified teachers was somewhat limited.

In 1981, the fourth curriculum change (the first revision of the "new math" curriculum) was made. Some contents of the "new math" were deleted and basic skills were emphasized, which resulted in a considerable retreat from the "new math". The

forth curriculum change also emphasized on integrative approach to problem-solving.

In 1987, the fifth curriculum change (the second revision of "new math" curriculum) was made with a further emphasis on basic knowledge and skills, problem-solving, and mathematical activity.

Even though 'new math' curricula (the fourth and fifth amendments) emphasized problem-solving, they asked all the same level of problems to every students. Another serious problem arising in 1974 and again in 1981 was that the textbook publishers monopolized the market with the help of the government. They published only one kind of textbook for the middle school and only five for the high school. This lowered the quality of the textbooks and eventually the quality of mathematics education.

Furthermore, the Ministry of Education of the government monopolized the selection of university students. The Ministry administered a national examination to decide which students were qualified to enter universities. The examination was consisted of multiple choice questions, so it could not evaluate performance of students properly. The national exam had a great influence on the teaching of mathematics of high schools and finally on mathematics education of the whole nation. Application and creativity of mathematics were completely ignored.

The sixth mathematics curriculum change was enacted in 1992 (KEDI 1992) and the new textbooks of mathematics except Mathematics III have come out in 1995. However, the new textbook for Mathematics III has not come out yet. The new curriculum has placed more emphasis on problem-solving and use of computers in mathematics. However, mathematics educators in Korea do not want to diversify the contents of school mathematics.

3. HISTORY OF GIFTED EDUCATION IN KOREA

The totalitarian government (1961–1987) leveled down the scholarly achievement of the individual students. After a long period of "leveling" of school, they finally opened a few high schools for gifted students in science and mathematics.

1983 Gyeonggi Science High School opened.
1984 Three more Science High Schools opened.
1985 The fifth amendment of national curriculum for elementary (grade 1–6), middle (7–9) and high (10–12) schools.
"Mathematics III" for the elective course of science high school students established.
1986 Korea Institute of Technology (college for gifted students) opened.
1987 The first Korean Mathematical Olympiad.

1988 The first Korean Mathematical Olympiad Winter School (4 weeks).
The Korean Team attended the 29th International Mathematical Olympiad.

1989 The Ministry of Education established the National Competition of Mathematics and Science for High School Students.

1991 The sixth amendment of national curriculum. "Mathematics III" reformation failed.

1993 The first text book of "Mathematics III" (according to the fifth amendment of national curriculum) published.

1996 The Ministry of Education is planning the seventh amendment of national curriculum.

4. Problems of Mathematics Education in the Republic of Korea

Education in Korea made remarkable advances in quantity as well as quality over the last five decades. The desire of Koreans for higher education was the major driving force for national development. The best quality education requires the best quality teachers. However, the supply of qualified teachers was somewhat limited because the best students from high schools did not want to be future mathematics teachers.

Pursuing education with a narrow mind and regarding learning as a mere vehicle to enter the next level of school are raising another problem. Recent statistics showed that over 93% of Korean parents expect their children to go to universities. School mathematics is just a tool to get into universities.

High schools are turning into institutions to prepare for university entrance examination, instead of pursuing the inherent goals of education. High school mathematics teachers are looking for strategies to increase the number of students who pass the entrance exam. One strategy is to abandon very difficult questions and very easy ones. They do not respect individual differences in the ability to solve mathematics problems.

The abolition of entrance examinations for middle schools and high schools contributed to uniformity of elementary and middle school education. The uniformity of education programs is, however, another problem. Educational programs which are presently offered by schools neglect individual differences in interest, ability, aptitude and other personal attributes. The problem has been further complicated by teachers stubbornly adhering to conventional instruction. One-third of students is abandoned to accumulate deficiencies beyond the point of remedy.

5. CONTENTS OF THE TEXTBOOK "MATHEMATICS III" (1993 version)

Chapter I Matrices and Determinants

§1. Matrices and Operations on Matrices
§2. Determinants
§3. Systems of Linear Equations and Matrices
(1) The Inverse Matrix
(2) Systems of Linear Equations

Chapter II Functions

§1. Real Numbers
§2. Functions
§3. Limits of Functions
§4. Continuity of Functions

Chapter III Transcendental Functions and Differentiation

§1. Logarithmic Functions and Differentiation
§2. Exponential Functions and Differentiation
§3. Inverse Trigonometric Functions and Differentiation
§4. Hyperbolic Functions and Differentiation
§5. Inverse Hyperbolic Functions and Differentiation

Chapter IV Applications for Derivatives

§1. Mean Value Theorem and its Extensions
§2. Increasing and Decreasing of Function Values and Extreme Values
§3. l' Hôpital' s rule

Chapter V Methods of Integration

§1. Indefinite Integral and Definite Integral
§2. Substitution Methods and Integration by Parts
§3. Integration of Trigonometric Functions
(1) Integrals which become Inverse Trigonometric Functions
(2) Integrals involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$

§4. Integration of Rational Functions
§5. Improper Integrals

Chapter VI Applications of Definite Integrals

§1. Areas and Volumes
§2. Lengths of Curves
§3. Centers of Mass and Moments

Chapter VII Coordinates in the Plane

§1. Polar Coordinates and Cartesian Coordinates
  (1) Directed Lines
  (2) Polar Coordinates
  (3) Relation Between Polar Coordinates and Cartesian Coordinates
  (4) Application of Polar Coordinates
§2. Polar Equations of Curves
  (1) Equations of Lines
  (2) Polar Equations of Conic Sections
§3. Parametric Equations of Curves
  (1) Parametric Equations of Lines
  (2) Parametric Equations of Conic Sections

Chapter VIII Coordinates of the Space

§1. Coordinates of the Space
  (1) Cartesian Coordinates
  (2) Cylinderical Coordinates
  (3) Spherical Coordinates
§2. Equations of Quadric Surfaces (Standard Forms)

Chapter IX Vectors

§1. n-dimensional Vectors
§2. Inner Products and Outer Products of Vectors
§3. Vector-Valued Functions and Differentiations

Chapter X Partial Derivatives

§1. Functions of Several Variables

§2. Partial Derivatives and High Order Partial Derivatives
§3. Total Differentials
§4. Methods of Partial Differentions
§5. Directional Derivatives
§6. Gradients and Tangent Planes to the Surfaces
§7. Maxima and Minima of the Two-variable Functions

Chapter XI Multiple Integrals and Applications

§1. Double Integrals
§2. Iterated Integrals

Chapter XII Sequences and Series

§1. Sequences and Limits
(1) Sequences
(2) Limits of Sequences
(3) Properties of Sequences
§2. Test for the Convergence of the Infinite Series
(1) Infinite Series
(2) Series without Negative terms
(3) Alternating Series
§3. Absolute Convergence and Conditional Convergence
(1) Absolute Convergence and Conditional Convergence
(2) Strategies for the Test of Convergence
§4. Power Series
§5. Taylor Expansions of Functions

Chapter XIII Differential Equations

§1. Definition of Differential Equations
§2. Separation of Variables
§3. Homogeneous First Order Differential Equations
§4. Exact Differential Equations
§5. Integral Factors
§6. First Order Linear Differential Equations
§7. Applications of Differential Equations

6. A Suggestion of Contents for the Mathematics III

We give a suggestion for the contents which are suitable for Mathematics III.

Unit I Theory of Numbers

§1. Number System
§2. Congruence and Residue Classes
§3. Euler's Theorem
§4. Fermat's Theorem
§5. Applications of Theory of Numbers
§6. Pigeon Hole Principle

Unit II Discrete Mathematics

§1. Counting
§2. Permutations and Combinations
§3. Graphic Theory and Algorithms
§4. Game Theory
§5. Discrete Probability
§6. Groups and Fields

Unit III Linear Algebra

§1. Matrices and Operations
§2. Inverse Matrices
§3. Determinants
§4. Systems of Linear Equations (Including Underdetermined System)
§5. Vector Spaces
§6. Linear Transformation

Unit IV Inequalities

§1. Absolute Inequalities
§2. Conditional Inequalities
§3. Cauchy-Schwartz Inequalities
§4. Word Problems
§5. Inequalities of Several Variables

§6. Geometric Inequalities

Unit V Geometry Revisited

§1. Reasoning and Proof
§2. Basic Theorem
§3. Drawing
§4. Traces
§5. Figures in Spaces (Lines, Planes, Spheres)

Unit VI Analytic Geometry

§1. Vector-Valued Functions
§2. Equations of Various Figures in Spaces
§3. Polar Coordinates
§4. Polar Equations of Curves
§5. Complex Numbers and Transformations in the Plane
§6. De Moivre Theorem and Primitive Roots

Unit VII Differentiation

§1. Differentiation of Various Functions
§2. Mean Value Theorem, l' Hôpital' s Rule
§3. Infinite Series and Their Convergence
§4. Expansion of Taylor Series
§5. Partial Derivatives
§6. Gradients and Tangential Planes
§7. Extreme Values of Two-Variable Functions
§8. Lagrange' s Multiplier Methods

Unit VIII Integration

§1. Integration of Various Functions
§2. Length of Curves
§3. Volumes and Surface Areas of Revolutions
§4. Double Integrals
§5. Applications of Double Integrals
§6. Differential Equations

7. Conclusion

Even though the government of Korea has changed the mathematics curriculum more than six times, the teachers in the front line never understood the reason fully.

The suggestion given in Section 6 is just an example of the topics that may be dealt with in the program for mathematically gifted students of the high school.

We need more extensive study of mathematics education for gifted students as well as for ordinary or retarded students. We have to open our minds widely and accept various opinions for betterment of the quality of mathematics education.

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