Kangweon-Kyungki Math. Jour. **5** (1997), No. 2, pp. 135–139

THE WEAKLY SEMI-PRIME IDEALS OF *po*-Γ-SEMIGROUPS

*Young In Kwon and **Sang Keun Lee

Abstract. We introduce the concepts of weakly prime and weakly semi-prime ideals in *po*-Γ-semigroup and give some characterizations of weakly prime and weakly semi-prime ideals of *po*-Γ-semigroups analogous to the characterizations of weakly prime and weakly semi-prime ideals of *po*-semigroups considered by N. Kehayopulu.

M. K. Sen([2]) have introduced Γ-semigroups in 1981. M. K. Sen and N. K. Saha([3]) have introduced Γ-semigroups different from the first definition of Γ-semigroups in the sense of Sen(1981). From Sen([2]) we recall the following definition of Γ-semigroup.

Let M and Γ be any two non-empty sets. M is called a Γ-semigroup if

(1) $M\Gamma M \subseteq M, \Gamma M\Gamma \subseteq \Gamma$.

(2) $(axb)yc = a(xby)c = ax(byc)$

for all $a, b, c \in M$ and $x, y \in \Gamma$.

In 1996, authors([5]) have introduced *po*-Γ-semigroups(: partially ordered Γ-semigroups).

A *po*-Γ-semigroup is an ordered set M at the same time a Γ-semigroup such that:

$$a \leq b \Longrightarrow a\gamma x \leq b\gamma x \text{ and } x\mu a \leq x\mu b$$

$\forall a, b, x \in M$ and $\forall \gamma, \mu \in \Gamma$.

In 1990, Kehayopulu([1]) obtained the four equivalent conditions to be weakly semiprime for an ideal T of a *po*-semigroup S.

Received May 30, 1997.

1991 Mathematics Subject Classification: 03G25, 06F35.

Key words and phrases: *po*-Γ-semigroup, weakly prime, weakly prime ideal, weakly semi-prime, weakly semi-prime ideal.

The second author was supported in part by the Basic Science Research Institute Program, Ministry of Education, Korea, 1966, Project No. BSRI-96-1411.

THEOREM. *An ideal T of a po-semigroup S is weakly semiprime if and only if one of the following four equivalent conditions hold in S:*
(1) *For every $a \in S$ such that $(aSa] \subseteq T$, we have $a \in T$.*
(2) *For every $a \in S$ such that $(I(a))^2 \subseteq T$, we have $a \in T$.*
(3) *For every right ideal A of S such that $A^2 \subseteq T$, we have $A \subseteq T$.*
(4) *For every left ideal B of S such that $B^2 \subseteq T$, we have $B \subseteq T$.*

In this paper we obtain the similar results in *po*-Γ-semigroup and give the characterization of weakly prime ideals in *po*-Γ-semigroups.
Now we recall the definitions and notations.

NOTATION 1. For subsets A, B of M, let

$$A\Gamma B := \{a\gamma b | a \in A, b \in B \text{ and } \gamma \in \Gamma\}.$$

DEFINITION 1. Let M be a *po*-Γ-semigroup and A a nonempty subset of M. A is called a *right*(resp. *left*) *ideal* of M if
(1) $A\Gamma M \subseteq A$(resp. $M\Gamma A \subseteq A$).
(2) $a \in A, b \leq a$ for $b \in M \Longrightarrow b \in A$.
A subset A of M is called an *ideal* of M if it is a right and left ideal of M.

See to [4] for the definitions of the left(right) ideals and ideals in Γ-semigroups.

NOTATION 2[1]. For $H \subseteq M$, we denote

$$(H] = \{a \in M : a \leq h \text{ for some } h \in H\}.$$

We write $(a]$ instead of $(\{a\}](a \in M)$. We denote by $I(a)$(resp. $R(a)$, $L(a)$) the ideal(resp. right ideal, left ideal) of M generated by $a(a \in M)$, respectively.

We can easily prove that:

$$I(a) = (a \cup M\Gamma a \cup a\Gamma M \cup M\Gamma a\Gamma M],$$

$$R(a) = (a \cup a\Gamma M], \ \ L(a) = (a \cup M\Gamma a]$$

for all $a \in M$.

DEFINITION 2. Let M be a *po*-Γ-semigroup and T a nonempty subset of M. T is called *weakly prime* if for all ideals A, B of M such that $A\Gamma B \subseteq T$, then $A \subseteq T$ or $B \subseteq T$. T is called a *weakly prime ideal* if T is an ideal which is weakly prime.

We can easily prove the following lemma.

LEMMA 1. *Let M be a po-Γ-semigroup. Then we have*
(1) $A \subseteq (A]$ *for any* $A \subseteq M$.
(2) $(A] \subseteq (B]$ *for* $A \subseteq B \subseteq M$.
(3) $(A]\Gamma(B] \subseteq (A\Gamma B]$ *for all* $A, B \subseteq M$.
(4) $((A]] \subseteq (A]$ *for all* $A \subseteq M$.
(5) *For every left (right, two-sided) ideal T of M,* $(T] = T$.
(6) *If A and B are ideals of M, then $(A\Gamma B]$ and $A \cup B$ are also ideals of M.*
(7) *For every $a \in M$, $(M\Gamma a\Gamma M]$ is an ideal of M.*

In [5; Theorem 5], we gave characterizations of weakly prime ideal elements of *poe*-Γ-semigroups (*po*-Γ-semigroup with the greatest element e.

Now we give characterizations of weakly prime ideals of *po*-Γ-semigroups(not necessarily having the greatest element e).

THEOREM 1. *Let M be a po-Γ-semigroup and T an ideal of M. Then T is prime if and only if for a left ideal A and a right ideal B of M such that $A\Gamma B \subseteq T$, we have $A \subseteq T$ or $B \subseteq T$.*

Proof. $\Longrightarrow$: It is obvious.
$\Longleftarrow$: Let A be a left ideal and B be a right ideal of M. Then by hypothesis, $a\Gamma b \subseteq T$ for any $a \in A$ and for any $b \in B$. Then

$$\begin{aligned} L(a)\Gamma R(b) &= (a \cup M\Gamma a]\Gamma(b \cup b\Gamma M] \\ &\subseteq (a\Gamma b \cup a\Gamma b\Gamma M \cup M\Gamma a\Gamma b \cup M\Gamma a\Gamma b\Gamma M] \\ &\subseteq (T \cup T\Gamma M \cup M\Gamma T \cup M\Gamma T\Gamma M] \\ &= (T] = T \end{aligned}$$

By hypothesis, $a \in L(a) \subseteq T$ or $b \in R(b) \subseteq T$, and so $A \subseteq T$ or $B \subseteq T$. Therefore T is weakly-prime. □ □

Definition 3. Let M be a *po*-Γ-semigroup and T a subset of M. Then T is called *weakly semi-prime* if every ideal A of M such that $A\Gamma A \subseteq T$, we have $A \subseteq T$.

Theorem 2. *Let M be a po-Γ-semigroup and T an ideal of M. Then the following are equivalent:*
(1) *T is weakly semi-prime.*
(2) *For every $a \in M$ such that $(a\Gamma M\Gamma a] \subseteq T$, we have $a \in T$.*
(3) *For every $a \in M$ such that $I(a)\Gamma I(a) \subseteq T$, we have $a \in T$.*
(4) *For every right ideal A of M such that $A\Gamma A \subseteq T$, we have $A \subseteq T$.*
(5) *For every left ideal A of M such that $A\Gamma A \subseteq T$, we have $A \subseteq T$.*

Proof. (1) $\Longrightarrow$ (2). Let $a \in M$ and $(a\Gamma M\Gamma a] \subseteq T$. Then we have

$$\begin{aligned}(M\Gamma a\Gamma M]\Gamma(M\Gamma a\Gamma M] &\subseteq (M\Gamma a\Gamma M\Gamma M\Gamma a\Gamma M]\\ &\subseteq (M\Gamma(a\Gamma M\Gamma a]\Gamma M]\\ &\subseteq (M\Gamma T\Gamma M]\\ &\subseteq (T] = T.\end{aligned}$$

Since $(M\Gamma a\Gamma M]$ is an ideal of M and T is weakly semiprime, we have $(M\Gamma a\Gamma M] \subseteq T$. Then we get

$$(I(a)\Gamma I(a)]\Gamma(I(a)\Gamma I(a)] \subseteq (T] = T.$$

Since T is weakly semiprime and $(I(a)\Gamma I(a)]$ is an ideal of M, we have $(I(a)\Gamma I(a)] \subseteq T$, and so $I(a)\Gamma I(a) \subseteq T$. And since T is weakly semiprime and $I(a)$ is an ideal of M, we have $I(a) \subseteq T$, and so $a \in T$.

(2) $\Longrightarrow$ (3). Let $a \in M$ and let $I(a)\Gamma I(a) \subseteq T$. Now

$$\begin{aligned}(a]\Gamma(M\Gamma a] &\subseteq (I(a)\Gamma M\Gamma a]\\ &\subseteq (I(a)(\Gamma M\Gamma)I(a)]\\ &\subseteq (I(a)\Gamma I(a)]\\ &\subseteq (T] = T.\end{aligned}$$

and so

$$((a]\Gamma(M\Gamma a]] \subseteq (T] = T.$$

Since $(a\Gamma M\Gamma a] \subseteq ((a]\Gamma(M\Gamma a]] \subseteq ((a\Gamma M\Gamma a]] \subseteq (a\Gamma M\Gamma a]$, we have $(a\Gamma M\Gamma a] = ((a]\Gamma(M\Gamma a]]$. Therefore $(a\Gamma M\Gamma a] \subseteq T$. By (2), we get $a \in T$.

(3) $\Longrightarrow$ (4). Let A be a right ideal of M such that $A\Gamma A \subseteq T$ and a any element of A. Then we have

$$\begin{aligned} I(a) &= (a \cup M\Gamma a \cup a\Gamma M \cup M\Gamma a\Gamma M] \\ &\subseteq (A \cup M\Gamma A \cup A\Gamma M \cup M\Gamma A\Gamma M] \\ &= (A \cup M\Gamma A]. \end{aligned}$$

Thus we get

$$\begin{aligned} I(a)\Gamma I(a) &\subseteq (A \cup M\Gamma A]\Gamma(A \cup M\Gamma A] \\ &\subseteq ((A \cup M\Gamma A)\Gamma(A \cup M\Gamma A)] \\ &= (A\Gamma A \cup A\Gamma M\Gamma A \cup M\Gamma A\Gamma A \cup M\Gamma A\Gamma M\Gamma A] \\ &\subseteq (T \cup M\Gamma T] \\ &= (T] = T. \end{aligned}$$

By (3), a is contained in T. Therefore $A \subseteq T$.

(3) $\Longrightarrow$ (5). The proof is similar to the one of (3) $\Longrightarrow$ (4).

(4), (5) $\Longrightarrow$ (1). They are obvious. □ □

References

1. N. Kehayopulu, *On weakly prime ideals of ordered semigroups*, Math. Japon. **35** (1990), 1051-1056.
2. M. K. Sen, *On* Γ*-semigroups*, Proc. of the Int. Conf. on Algebra and it's Appl., Decker Publication, New York 301.
3. M. K. Sen and N. K. Saha, *On* Γ*-semigroup I*, Bull. Cal. Math. Soc. **78** (1986), 180-186.
4. N. K. Saha, *On* Γ*-semigroup II*, Bull. Cal. Math. Soc. **79** (1987), 331-335.
5. Y. I. Kwon and S. K. Lee, *Some special elements in ordered* Γ*-semigroups*, Kyungpook Math. J. Vol 35 (1996), 679-685.

Department of Mathematics
College of Education
Gyeongsang National University, Chinju 660-701, Korea
E-mail: *yikwon@ nongae.gsnu.ac.kr, **sklee@ nongae.gsnu.ac.kr