# Robust Adaptive Sliding Mode Control of Robot Manipulators Using a Model Reference Approach

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#### Abstract

In this paper, a robust adaptive sliding mode control algorithm for accurate trajectory tracking of robot manipulators is proposed, with unknown parameters being estimated on-line. The controller is designed based on a Lyapunov method, which consists of a adaptive feed-forward compensation part and a discontinuous control part. It is shown that, in the presence of the uncertainty and the disturbances arising from the actuator or some other causes, the tracking errors is bound to converge to zero asymptotically. An illustrative example is given to demonstrate the results of the proposed method.

### I. Introduction

Mathematical modelling of a mechanical manipulator results in a set of coupled nonlinear differential equations. Physically, these nonlinearities arise from inertial loading, complex reaction forces among the various joints and gravitational loading of the links. Furthermore, in performing tasks, the characteristics of the manipulator payload, such as mass, can change from task to task. Thus, robotic manipulators are highly coupled and nonlinear multivariable system with unknown parameters.

The control of a manipulator will always be challenged by the uncertainty as mentioned before and the disturbances possibly arising from the actual running of the actuator or some other causes. Therefore, robustness is an important issue in robotic controller design. There are several control strategies which provide robust control for robotic manipulators. In recent years increasing attention has been given to controller designs of robot manipultors that utilize the theory of variable structure system.

Among developed algorithms using the theory of VSS, several approaches have been considered. Some choose a control that makes each surface attracting in order to guarantee the asymptotic stability of their intersection, which constrains the problem unduly, resulting in a control law

defined implicitly by a set of fairly complicated algebraic inequalities[1-4]. Others do exploit the known structure of system dynamics, resulting in a control law that ensures the stability of the intersection of the surface without necessarily stabilizing each individual one[5-6]. Another class of algorithms is based on the combination of the deterministic approach to the control of uncertain systems[7] and VSS to design manipulator control algorithms[8-10]. C.Y.Su and Y.Stepanenko[11] proposed for an adaptive sliding mode control of robot manipulators by using a general sliding surface, which can be nonlinear or time-varying. In [12], an adaptive sliding mode control scheme is developed for accurate tracking control of robotic manipulators, with unknown manipulator and payload parameters being estimated on-line.

In this paper, unlike usual adaptive sliding mode control schemes[11,12], a robust adaptive sliding mode control algorithm for robot manipulators is developed based on the model reference adaptive systems(MRAS) technique[13]. The reference model is composed of simple double integrator. The proposed controller is designed based on a Lyapunov method, which consists of a adaptive feed-forward compensation part and a discontinuous control part. The role of the compensation part acts to maintain the tracking errors on the sliding surfaces, and the control part overcomes the effects of the uncertainties and bends entire system trajectory to the sliding surfaces until sliding mode occurs. Moreover the algorithm is computationally simple, due to an effective exploitation of the particular structure of manipulator dynamics.

The organization of this paper is as follows. In Section II

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the robot dynamics and its structure properties are reviewed. Section III presents robust adaptive sliding mode control algorithm using the model reference adaptive system. Section IV discusses how to eliminate undesirable chattering. Simulation results are presented in Section V to demonstrate the performance of the proposed controller. Section VI presents brief concluding remarks.

# II. Dynamic model of robot manipulators

For a general open-chain n-link rigid manipulator, the dynamic model can be derived either using Lagrangian-Euler or Newtonian-Euler method and can be expressed in a symbolic form

$$M(q)\dot{q} + B(q, \dot{q})\dot{q} + G(q) = \tau + \tau_d$$
 (1)

where  $q, \dot{q}, \dot{q} \in R^n$  is the joint position, velocity and acceleration vectors;  $\tau \in R^n$  is the actuator torque vector acting on the joints of the robot;  $M(q) \in R^{n \times n}$  is the symmetric positive definite inertia matrix;  $B(q, \dot{q})\dot{q} \in R^n$  is the vector of centrifugal and Coriolis torques,  $G(q) \in R^n$  is the vector of gravitational torques,  $\tau_d \in R^n$  is the vector of uncertainties presenting friction, torque disturbances, etc.

As remarked by several authors([11, 12, 14-16]), the robot model (1) is characterized by the following structural properties, which are of importance to our stability analysis.

**Property 1.** There exists a vector  $\theta \in \mathbb{R}^m$  with components depending on manipulators parameters (masses, moments of inertia, etc.), such that

$$M(q)\dot{q} + B(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \dot{q}) \theta$$
 (2)

where  $Y(q, \dot{q}, \dot{q}) \in R^{nxm}$  is called the regressor and  $\theta \in R^m$  is the vector containing the unknown manipulator and payload parameters.

**Property 2.** The two n x n matrices M(q) and B(q, q) are not independent. Specially, given a proper definition of B(q,  $\dot{q}$ ), the matrix ( $\dot{M}$ (q) - 2B(q,  $\dot{q}$ )) is skew-symmetric [12, 13].

# III. Robust adaptive sliding mode controller

In this section, an adaptive sliding mode control scheme for robot manipulators is developed based on the model reference adaptive systems(MRAS) technique. We will use a reference model described by

$$\dot{q}_{m} = d/dt(q_{m}) \tag{3}$$

$$u = d/dt(\dot{q}_m)$$
 (4)

where  $u \in R^n$  is the acceleration input which will be determined in the following and  $q_m$ ,  $\dot{q}_m \in R^n$  are the angular position and velocity vectors of the reference model.

First we chose u according to the following:

$$u = \dot{q}_d + K_V(\dot{q}_d - \dot{q}) + K_P(\dot{q}_d - \dot{q})$$
 (5)

where  $q_d$ ,  $\dot{q}_d$ ,  $\dot{q}_d \in R^n$  are respectively the position, velocity and acceleration vectors of the desired trajectory and the matrices  $K_V$ ,  $K_P$  are PD gains which are seleted such that the characteristic equation:

$$p^2 + K_V p + K_P = 0 ag{6}$$

has all its root in the left hand side of the complex plane. Let us define the sliding surface  $S^T = [s_1 \cdots s_n] = 0$  as

$$S = \dot{e}(t) + Ce(t) \tag{7}$$

where  $C = diag(c_1 \dots c_n)$  is positive definite gain matrix and  $e(t)=q(t)-q_m(t)$  is the tracking error between the reference model and manipulator angular position vector.

In order to derive the adaptive sliding mode law, the following assumption is required.

Assumption A1: The desired trajectory  $q_d(t)$  is chosen such that  $q_d$ ,  $\dot{q}_d$ ,  $\dot{q}_d$  are all bounded signals.

Assumption A2: The effects of input disturbance are assumed to satisfy the following:

$$\|z_{\mathbf{d}}\| \leq \sigma_1 + \sigma_2 \|\mathbf{S}\| \tag{8}$$

where  $\sigma_1 \ge 0$ ,  $\sigma_2 \ge 0$  are constants but unknown

Remark 1. The sliding variable S is a measurable signal vector for all time since it is only the function of positions and velocities of the actual joints and positions.

Remark 2. The assumption A2 is quite reasonable as far as the effects of actuators, friction forces or some other causes are concerned since it is assumed to be unbounded and fast-varying.

The adaptive sliding mode control law is now chosen as

$$z(t) = Y(q, \dot{q}, \dot{q}_m, \dot{q}_m) \hat{\theta} - \hat{\sigma}_2 S - \hat{\sigma}_1 sgn(S) - Ksgn(S) (9)$$

$$\hat{\theta} = - \Gamma \mathbf{Y}^{\mathsf{T}} (\mathbf{q}, \mathbf{q}, \mathbf{q}_{\mathsf{m}}, \mathbf{q}_{\mathsf{m}}) \mathbf{S}$$
 (10)

$$\hat{\sigma}_1 = \zeta_1 ||\mathbf{S}|| \tag{11}$$

$$\hat{\sigma}_2 = \left. \zeta_2 \|\mathbf{S}\|^2 \right. \tag{12}$$

where  $\theta \in R^m$  is the vector containing the parameters estimated on-line and  $\hat{\theta}$  is its estimate;  $\Gamma \in R^{mxm}$  is a symmetric positive definite matrix, usually diagonal,  $\zeta_1 > 0$ ,  $\zeta_2 > 0$  are arbitrary constants, and  $K = \operatorname{diag}(k_1 \cdots k_n)$  will be determined in the following.

#### Theorem:

For a mechanical manipulator governed by (1), with the sliding surface S=0 described by (7), if the adaptive sliding mode control laws given by (9) - (12) are used, the tracking error between the desired trajectory and manipulator angular position converges asymptotically to the zero, i.e,  $\lim_{t\to\infty} \left[ q_d(t) - q(t) \right] = 0$ .

#### Proof:

Define the Lyapunov function candidate as

$$V(t) = \frac{1}{2} S^{T} M(q) S + \frac{1}{2} \widetilde{\theta}^{T} \Gamma^{-1} \widetilde{\theta} + \frac{1}{2} \sum_{i=1}^{2} (\sigma_{i} - \overset{\wedge}{\sigma_{i}})^{2} / \zeta, \qquad (13)$$

where  $\tilde{\theta} = \hat{\theta} - \theta \in \mathbb{R}^m$  is the parameter etimation error vector.

Differentiating V(t) with respect to time yields

$$\dot{\mathbf{V}}(\mathbf{t}) = \frac{1}{2}\dot{\mathbf{S}}^{\mathsf{T}}\mathbf{M}(\mathbf{q})\mathbf{S} + \frac{1}{2}\mathbf{S}^{\mathsf{T}}\dot{\mathbf{M}}(\mathbf{q})\mathbf{S} + \frac{1}{2}\mathbf{S}^{\mathsf{T}}\mathbf{M}(\mathbf{q})\dot{\mathbf{S}} + \frac{1}{2}\dot{\widetilde{\boldsymbol{\theta}}}^{\mathsf{T}} \Gamma^{-1}\widetilde{\boldsymbol{\theta}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathsf{T}} \Gamma^{-1}\widetilde{\boldsymbol{\theta}} + \sum_{i=1}^{2}(\sigma_{i} - \overset{\wedge}{\sigma}_{i})(-\overset{\wedge}{\sigma}_{i})/\zeta,$$
(14)

Using the Property 2 that the matrix  $\dot{M}(q)$  -  $2B(q, \dot{q})$  is skew-symmetric, equation (14) can be represented as

$$\dot{\mathbf{V}}(t) = \mathbf{S}^{\mathsf{T}}(\mathbf{M}(\mathbf{q})\dot{\mathbf{S}} + \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{S}) + \tilde{\ell}^{\mathsf{T}} \Gamma^{-1} \dot{\tilde{\ell}}$$

$$+ \sum_{i=1}^{2} (\sigma_{i} - \hat{\sigma}_{i})(-\hat{\sigma}_{i})/\zeta_{i}$$
(15)

To derive the M(q)S term, differentiating S with respect to time yields

$$\dot{S} = \dot{q}(t) - \dot{q}_m(t) + C(\dot{q}(t) - \dot{q}_m(t))$$
 (16)

Multiplying the matrix M(q) to (16), inserting (1), and making use of Property 1 gives

$$M(q)\dot{S} = z(t) + z_{d} + ((M(q)C - B(q, q))\dot{q}(t) - M(q)C\dot{q}_{m}(t)$$

$$= G(q) - M(q)\dot{q}_{m}(t)$$

$$= z(t) + z_{d} - Y(q, q, q_{m}, q_{m}) \theta$$
(17)

Substituting (9) into (17) yields

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{S}} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{m}, \dot{\mathbf{q}}_{m}) \, \hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\sigma}}_{2}\mathbf{S} - \hat{\boldsymbol{\sigma}}_{1}\mathbf{sgn}(\mathbf{S}) - \mathbf{K}\mathbf{sgn}(\mathbf{S}) + \tau_{d}$$

- 
$$Y(q, q, q_m, q_m) \theta$$

= 
$$Y(q, \dot{q}, \dot{q}_m, \dot{q}_m) \tilde{\theta} - \overset{\wedge}{\sigma}_2 S - \overset{\wedge}{\sigma}_1 sgn(S) - Ksgn(S) + \tau_d$$
 (18)

where 
$$Y(q, \dot{q}, \dot{q}_m, \dot{q}_m) \tilde{\theta} = Y(q, \dot{q}, \dot{q}_m, \dot{q}_m) \hat{\theta} - Y(q, \dot{q}, \dot{q}_m, \dot{q}_m) \theta$$

Using the above result, equation (15) becomes

$$\dot{\mathbf{V}}(t) = \mathbf{S}^{\mathsf{T}}(\mathbf{Y}(\mathbf{q}, \mathbf{q}, \mathbf{q}_{\mathsf{m}}, \mathbf{q}_{\mathsf{m}}) \widetilde{\boldsymbol{\theta}} - \overset{\wedge}{\boldsymbol{\sigma}_{2}} \mathbf{S} - \overset{\wedge}{\boldsymbol{\sigma}_{1}} \mathrm{sgn}(\mathbf{S}) - \mathrm{Ksgn}(\mathbf{S}) + \tau_{\mathsf{d}} + \mathrm{B}(\mathbf{q}, \mathbf{q}) \mathbf{S})$$

$$+ \widetilde{\boldsymbol{\theta}}^{\mathsf{T}} \Gamma^{-1} \widetilde{\boldsymbol{\theta}} + \sum_{i=1}^{2} (\sigma_{i} - \overset{\wedge}{\boldsymbol{\sigma}_{i}})(-\overset{\wedge}{\boldsymbol{\sigma}_{i}})/\zeta_{i}$$

= 
$$S^{T}(\underline{B}(q, q)S + \tau_{d} - \overset{\wedge}{\sigma}_{2}S - \overset{\wedge}{\sigma}_{1}sgn(S) - Ksgn(S))$$
  
+  $\widetilde{\theta}^{T}(\Gamma^{-1}\overset{\sim}{\theta} + Y^{T}(q, q, q_{m}, q_{m})S) + \sum_{i=1}^{2}(\sigma_{i} - \overset{\wedge}{\sigma}_{i})(-\overset{\wedge}{\sigma}_{i})/\zeta_{i}$  (19)

Note that  $\widetilde{\theta} = \widehat{\theta}$ , since the unknown parameters  $\theta$  are constant. Substituting the adaptation laws (10)-(12) into (19) and arranging the function, equation (19) can be represented as

$$\dot{V}(t) = S^{T}(B(q, \dot{q})S + \tau_{d} - Ksgn(S)) - (\sigma_{1}||S|| + \sigma_{2}||S||^{2})$$
 (20)

Note that  $\|z_d\| \le \sigma_1 + \sigma_2 \|S\|$ . The resulting expression of  $\dot{V}(t)$  is

$$\dot{V}(t) = S^{T}(B(q, \dot{q})S - Ksgn(S))$$
 (21)

If K can be chosen to make  $\dot{V}(t)$  be a negative-semi definite function of S which vanishes only at S=0, so, by means of Lyapunov's theory, the sliding surface S=0 are asymptotically attractive. This can be achieved by supposing that there exist known functions  $f_{ij}$ , such that

$$| B_{ij}(q, \dot{q}) | < f_{ij}(q, \dot{q})$$
 (22)

which can be always be satisfied by choosing a suitable function, and selecting K of (21) as

$$k_i = \sum_{i=1}^{n} f_{ij}(q, q) | s_j | + \eta_i$$
, i=1, ..., n (23)

where  $\eta_i > 0$  is an arbitary constant [12]. We obtain

$$\dot{V}(t) = -\sum_{i=1}^{n} \eta_{i} |s_{i}| < 0$$
 (24)

From (13) and (24), it is well-known that S approaches zero asymptotically. This in turn shows that the tracking errors converge to the sliding surface S=0. Thus the resulting adaptive sliding mode control law is globally asymptotically stable and guarantees zero tracking errors.

Remark 3. The result of proof is the same as [12], while the scheme and the approach to the overall control of manipulators are different from those in [12]. These differences between the adaptive sliding mode controller in

[12] and the one presented in this paper are: 1) The adaptive sliding mode algorithm in [12] is based on the equivalent control method which consists of a low frequency component and a high frequency component. The design method presented in this paper is based on the model reference adaptive system technique. 2) In [12] the disturbances arising from the actuator or some other causes are neglected in the control design, while in this paper they are explicitly considered.

# IV. Elimination of chattering

The control laws given above are discontinuous and it is well known that synthesis of such control laws give rise to chattering of trajectories about the surface S=0. Chattering is undesirable in pratice because it involves high control activity and further may excite high frequency dynamics neglected in the course of modeling(such as unmodelled structural modes, neglected time delays, and the like). This problem can be eliminated by smoothing out the discontinuous control law in the neighborhood of the sliding surface, as suggested by Slotine and Sastry[2]. To do this, we replace signim nonlinearity by a saturation nonlinearity, which is defined as

$$sat(S) = \begin{cases} 1 & \text{if } S/\phi \ge 1\\ S/\phi & \text{if } -1 \le S/\phi \le 1\\ -1 & \text{if } S/\phi \le -1 \end{cases}$$

where  $\phi$  is the boundary layer thickness[14]. With this boundary layer, the adaptive sliding mode control law, for example, given by (9) - (12), becomes

$$\dot{\tau}(t) = Y(q, \dot{q}, \dot{q}_m, \dot{q}_m) \stackrel{\wedge}{\theta} - \stackrel{\wedge}{\sigma_2} S_{\phi} - \stackrel{\wedge}{\sigma_1} sat(S) - Ksat(S)$$
 (25)

$$\stackrel{\cdot}{\hat{\theta}} = - \Gamma \mathbf{Y}^{\mathrm{T}} (\mathbf{q}, \stackrel{\cdot}{\mathbf{q}}, \stackrel{\cdot}{\mathbf{q}}_{\mathrm{m}}, \stackrel{\cdot}{\mathbf{q}}_{\mathrm{m}}) S_{\phi}$$
(26)

$$\hat{\sigma}_{1} = \zeta_{1} \| S_{\delta} \| \tag{27}$$

$$\dot{\hat{\sigma}}_2 = |\xi_2| ||S_\phi||^2 \tag{28}$$

$$k_i = \sum_{j=1}^{n} f_{ij}(q, \dot{q}) | s_{\phi j} | + \eta_i, i=1, \dots, n$$
 (29)

where  $S_{\phi} = (s_{\phi 1}, \dots, s_{\phi n})^{T}$  with  $s_{\phi i} = s_{i} - \phi_{i} sat(s_{i}/\phi_{i})$  is a measurement of the algebraic distance of the current state to the boundary layer. We can again demonstrate the attractiveness of the boundary layer by using the following Lyapunov function

$$V(t) = \frac{1}{2} S_{\phi}^{T} M(q) S_{\phi} + \frac{1}{2} \widetilde{\theta}^{T} \Gamma^{-1} \widetilde{\theta} + \frac{1}{2} \sum_{i=1}^{2} (\sigma_{i} - \overset{\wedge}{\sigma}_{i})^{2} / \zeta_{i}$$
 (30)

instead of (13), and noting that  $\dot{S}_{\phi} = \dot{S}$  outside the boundary layer, while  $S_{\phi} = 0$  inside the boundary layer, which yields

$$\dot{V}(t) = -\sum_{i=1}^{n} \eta_{i} |s_{\phi i}| < 0$$
 (31)

Definition (30) implies that  $\dot{V}(t) = 0$  inside the boundary layer, which shows that (31) is valid everywhere and further guarantees that trajectories eventually converge to the boundary layer. Thus it can be shown that closed-loop system is globally uniformly ultimately bounded[11].

## V. Simulation

A computer simulation is performed to evaluate the performance of control algorithm. Consider the two-link planar manipulator as shown in Fig.1, carrying a load of unknown mass[12].

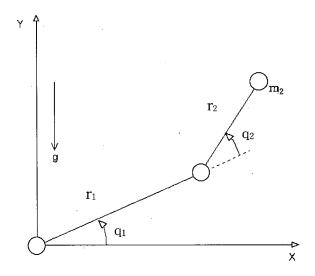


Fig. 1. Two-link manipulator model.

The dynamics of the manipulator with payload can be written as

$$\begin{bmatrix} M_{11} M_{12} \\ M_{21} M_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} -B_{12} \dot{q}_2 & -B_{12} (\dot{q}_1 + \dot{q}_2) \\ B_{12} \dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

where

$$\begin{split} M_{11} &= (m_1 + m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2cos(q_2), \\ M_{12} &= m_2r_2^2 + m_2r_1r_2cos(q_2), \\ M_{21} &= M_{12}, \\ M_{22} &= m_2r_2^2, \\ B_{12} &= m_2r_1r_2sin(q_2), \\ G_1 &= (m_1 + m_2)r_1gcos(q_2) + m_2r_2gcos(q_1+q_2), \\ G_2 &= m_2r_2gcos(q_1+q_2), \end{split}$$

and g is the acceleration of gravity. The parameter values used are seleted as

$$m_1 = 0.5$$
kg,  $m_2 = 0.5$ kg,  $r_1 = 1$ m,  $r_2 = 0.8$ m

Let the equivalent parameter vector  $\theta$  be

$$\alpha = (m_1 + m_2)r_1^2$$
,  $\beta = m_2r_2^2$  and  $\gamma = m_2r_1r_2$ 

Thus the true values of unknown parameters are  $\alpha=1$ ,  $\beta=0.32$  and  $\gamma=0.4$ . The corresponding initial parameter estimates are selected as  $\hat{\alpha}=0.72$ ,  $\hat{\beta}=0.25$  and  $\hat{\gamma}=0.32$ . The constant parameters are chosen as C=4I, I=0.05I,  $K_V=10I$ ,  $K_P=25I$ ,  $\eta_1=\eta_2=2.0$ , and  $\zeta_1=\zeta_2=0.5$ , and  $\sigma_1=\sigma_2=0.5$ . The entries of the matrix B(q, q) can be upper -bounded

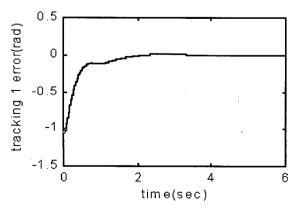
$$\begin{array}{l} \mid \ b_{11} \mid < \ \ \bar{\gamma} | \dot{q}_2 | < \ \bar{\gamma} | \dot{q}_2 | = \ f_{11} \\ \\ \mid \ b_{12} \mid < \ \ \bar{\gamma} | \dot{q}_1 + \ \dot{q}_2 | < \ \ \bar{\gamma} | \dot{q}_1 + \ \dot{q}_2 | = \ f_{12} \\ \\ \mid \ b_{21} \mid < \ \ \bar{\gamma} | \dot{q}_1 | < \ \ \bar{\gamma} | \dot{q}_1 | = \ f_{21} \\ \\ \mid \ b_{22} \mid = \ 0 = \ f_{22} \end{array}$$

and we select  $\bar{\gamma} = 1$ .

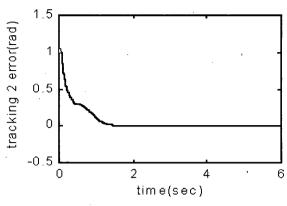
Example 1. The desired joint trajectories are chosen to be

$$q_{d}(t) = 0, \dot{q}_{d}(t) = 0$$

and the initial position of  $q_1(t)$  is  $60^0$  and  $q_2(t)$  is  $-60^0$ . The boundary layer thickness is chosen to be  $\phi_1 = 0.5d_1$  and  $\phi_2 = 0.5d_2$ . The selection of  $d_i$  depends on the strength of

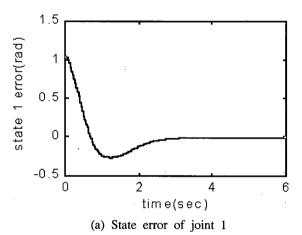


(a) Tracking error of joint 1



(b)Tracking error of joint 2

Fig. 2. The tracking errors in Example 1.



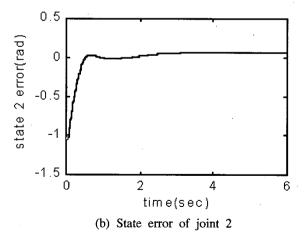


Fig. 3. The state errors in Example 1.

the discontinuities of control efforts. We choose  $d_1 = 1$  and  $d_2 = 1$  for this simulation. Figure 2 shows the tracking errors between the desired trajectory and manipulator angular position. Figure 3 shows the state errors between the reference model and manipulator angular position. Fugure 4 shows the values of  $s_1$  and  $s_2$  as functions of time. Figure 5

and figure 6 are the results of the parameter estimates and the torques exerted at manipulator joints respectively.

As we expected the tracking errors converge to zero. However, it is obvious in Fig. 5 that the parameter estimates do not converge to the true value. It should be noted that the stability only guarantees the output error to converge to zero, and that it does not guarantee nor require the parameter error to converge to zero.

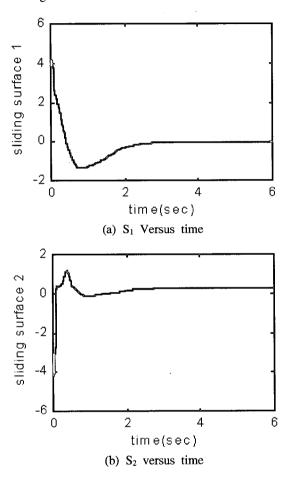
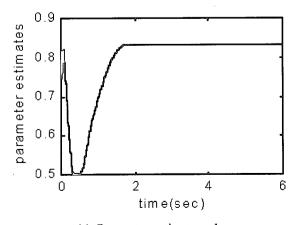
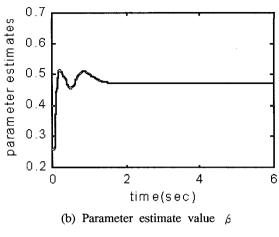
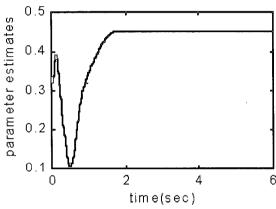


Fig. 4. The sliding surfaces as function of time in Example 1.



(a) Parameter estimate value a



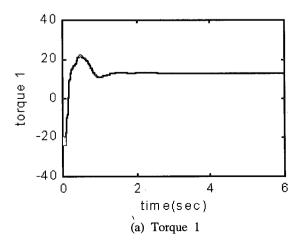


(c) Parameter estimate value  $\gamma$ 

Fig. 5. The Parameter estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$  in Example 1.

**Example 2.** The parameter error converges to zero when the input signal is sufficiently rich in its frequency content[17]. To see this relationship, the desired joint trajectories are chosen to be

$$q_{d1}(t) = -90^{\circ} - 52.5^{\circ}(1 - \cos(1.26t))$$
  
 $q_{d2}(t) = 170^{\circ} - 60^{\circ}(1 - \cos(1.26t))$ 



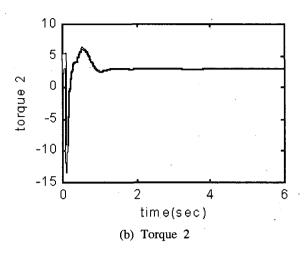


Fig. 6. The Control torques in Example 1.

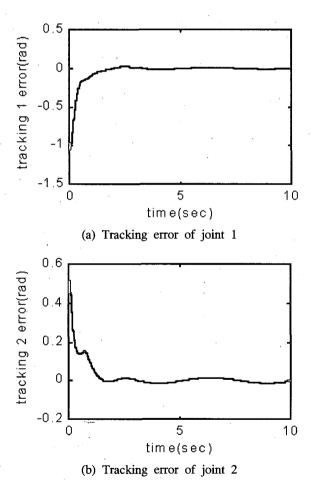


Fig. 7. The tracking errors in Example 2.

and the initial position of  $q_1(t)$  is  $-30^0$  and  $q_2(t)$  is  $140^0$ . In this case, all the conditions are the same as Example 1. Figure 7 - 11 are respectively the simulation results of tracking errors, state errors, sliding surfaces, parameter estimates, and torques. From the simulation results, we see

that the tracking errors are converge to zero and the parameters estimates are converge to near the true values after an initial adaptation process. On the other hand, figure 8 shows state errors of  $0.045 \, \mathrm{rad}$ ,  $0.034 \, \mathrm{rad}$  for  $q_1$  and  $q_2$ . These state errors may be reduced by reconstructing a suitable reference model instead of a simple double integrator used in this papaer.

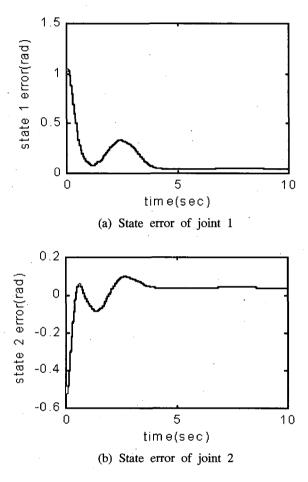
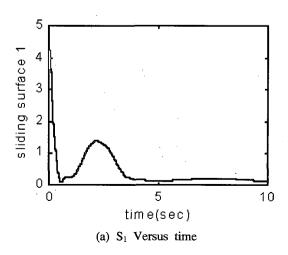


Fig. 8. The state errors in Example 2.



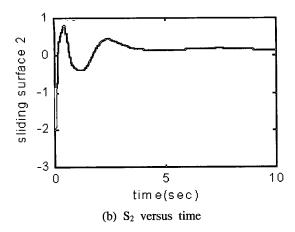
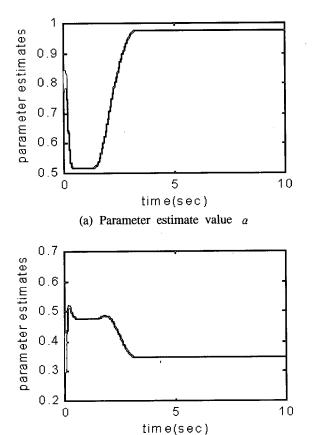


Fig. 9. The sliding surfaces as function of time in Example 2.

## VI. Conclusion

In this paper, a robust adaptive sliding mode control scheme for accurate tracking of robot manipulators is derived making use of the fundamental properties of the manipulator equations, with unknown manipulator and payload parameters being estimated on-line. Unlike usual adaptive sliding mode control algorithms, the proposed controller is developed based



(b) Parameter estimate value  $\beta$ 

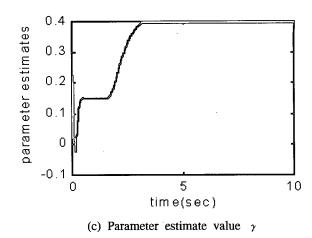


Fig. 10. The Parameter estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$  in Example 2.

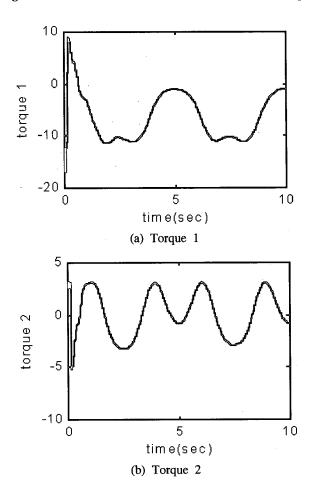


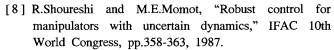
Fig. 11. The Control torques in Example 2.

on the model reference adaptive systems technique. From simulation results, despite the existence of the parameter uncertainties and the disturbances arising from the actuator or some other causes, we see that the proposed controller is globally asymptotically stable and guarantees zero tracking errors. In addition, although it may not be necessary to have

two frequency components for the desired trajectory to be persistently exciting, simulations show that sufficiently rich desired trajectories yield convergence of the parameter estimation. Ongoing research will be on extending the algorithm to combine the learning control scheme.

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