Design of Continuous Variable Structure Tracking Controller With Prescribed Performance for Brushless Direct Drive Servo Motor

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Abstract

A continuous, accrurate, and robust variable structure tracking controller(CVSTC) is designed for brushless direct drive servo motors(BLDDSM). Although conventional variable structure controls can give the desired tracking performances, there exists an inevitable chattering problems in control input which is undesirable for direct drive systems. With the presented algorithm, not only the chattering problems are removed by using the efficient compensation of the disturbance observer, but also the prescribed tracking trajectory can be obtained using the sliding dynamics when an initial of the desired trajcetory is different from that of a BLDDSM. The design of the sliding mode tracking controller for the prescribed, accurate, and robust tracking performance without the chattering problem is given based on the results of the detailed stability analysis. The usefulness of the suggested algorithm is demonstrated through the computer simulation for a BLDDSM under load variations.

I. Introduction

In servo control, two fundamental problems are the point to point control(regulation) problem and tracking problem(trajectory following). The point to point control problem is concerned with moving the control object from a point to another. In this problem, the controller is required to provide a small positioning error and superior regulation. In tracking control, the control object must be moved along the desired trajectory. The tracking control is extremely important in many mechanical system such as robot manipulators, machining system, and antenna etc.

In these mechanical systems, the use of direct drive servo motors ever increases[1]. Direct drive servo motors can yield very high torque at low to moderate speeds by directly coupling the load to the motor shaft without gears, belts, or any form of the mechanical leverages which possibly cause the backlash, cogging, compliance, friction, inertia multiplication, etc[2]. Since the load variations and external disturbances directly influence on a servo system, there are some unexpected gains and trade-off, besides the obvious advantages of eliminating the transmissions. The load variations and external disturbances are the dominant harmful factors for the control of brushless direct drive servo motors(BLDDSM). Thus, the robustness of the controllers is essentially required to meet the high performance servo specifications.

As a precise and robust algorithm with different level of the PID types, the variable structure system(VSS) with sliding mode control is considered both for brushless servo motors(BLSM)[3-5] or BLDDSMs[6-8]. The principal objective of this control technique is to force the trajectory of the system to follow a certain surface, known as the sliding surface. Once on the sliding surface, the control structure is changed discontinuously according to the predetermined switching rule to maintain the system on the switching function. At this stage, the system is in the sliding mode, which means that the controlled system is completely robust to the load variations and external disturbances[9,10]. Because of this feature, much attention has been paid to the VSS.

However. there exists an inevitable chattering, i. e. high frequency oscillation of the control input resulted from the

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switching of the control structure, which is undesirable for a direct drive servo system. It can cause the torque ripple and excite the unmodelled dynamics in the dynamics in the direct drive servo system[11]. Since the input of the physical system is limited by saturation of the driving current, the available dynamic range of the control is reduced due to the discontinuity of the control. Therefore, continuous approximation of the discontinuous variable structure system is basically investigated to replace the discontinuous parts in the input by the continuous saturation function in [11]and [12]or the bounded layer method in [14] and [18]. The uniform ultimately bounded stability of the continuous variable structure system with the saturation function us analyzed under the persistent disturbances[16]. In [13]and [15], the continuous VSS's are implemented under the assumption that the derivatives of all the states are known. The continuous VSS utilizing the sliding surface as the partial control gain in the state feedback has been suggested in [6]. In most these works, it is, however, difficult to obtain the information about the output performance of the tracking error as an important performance measure in the servo mechanism. And it is assumed that the initial of the planned trajectory only equals to that of the plant in most existing tracking controllers. When both are different for arbitrary initial of planned trajectory, the tracking problem is mixed with the point to point problem. Then the reaching phase problems arise, which may decrease the robustness of the controller because the sliding motion does not occur in this phase[5].

In this study, the design of a continuous variable structure controller(CVSTC) with prescribed performance is presented for the tracking control of a BLDDSM subjected to the load variations for arbitrary initial position of the desired trajectory. With proposed algorithm, the transient behavior is predetermined by the sliding dynamics in the situation that both initial conditions are different. The stability of the proposed algorithm is investigated in detail and the prescribed tracking performance without any chattering and reaching is guaranteed based on this stability analysis. The computer simulation studies are given to show the effectiveness of the algorithm.

II. Modeling of BLDDSM and Backgrounds

As one of the three commercial direct drive motors for servo applications[2], a brushless type direct drive servo motor physically has the same structure as the permanent magnet synchronous machine, but specially designed for the purpose of very high torque generation at low speed. The nonlinear mathematical equations of the BLDDSM can be expressed in the d-q model. The nonlinear mathematical motor equations of the BLDDSM can expressed in the d-q model as follows[20,21]:

$$\dot{i}_{qs}=-\frac{r_s}{L_q}i_{qs}-\frac{L_d}{L_q}\dot{\theta}i_{ds}+\frac{1}{L_q}V_{qs}-\frac{\lambda_m}{L_q}\dot{\theta} \tag{1a}$$

$$\dot{i}_{ds}=\frac{L_q}{L_d}\dot{\theta}i_{qs}-\frac{r_s}{L_d}i_{ds}+\frac{1}{L_d}V_{ds}$$

$$\dot{i}_{ds}=\frac{L_q}{L_d}\dot{\theta}i_{qs}-\frac{r_s}{L_d}i_{ds}+\frac{1}{L_d}V_{ds} \tag{1b}$$

$$T_e=\frac{3}{2}\left(\frac{p}{2}\right)[\lambda_m i_{qs}+(L_d-L_q)i_{qs}i_{ds}] \tag{1c}$$

$$=J\left(\frac{2}{p}\right)\ddot{\theta}+D\frac{2}{p}\dot{\theta}+T_L(t,\theta(t)) \tag{1d}$$

where r_s : stator resistance
L_q : q axis stator inductance
L_d : d axis stator inductance
λ_m : flux linkage of permanent magnet
k_t : torque constant
J : total moment of inertia
D : coefficient of the viscous friction term
p : number of poles
θ : angle displacement
$\dot{\theta}$: angular velocity or rotor
i_{qs} : q- axis current
$T_L(t,\theta(t))$: load variations

By means of the field-oriented vector for driving the BLDDSM, i_{ds} is set to be zero[21,22]. Thus, the linearized system equations of the BLDDSM can be described as

$$\dot{i}_{qs}=-\frac{r_s}{L_q}i_{qs}+\frac{1}{L_q}V_{qs}-\frac{\lambda_m}{L_q}\dot{\theta} \tag{2a}$$

$$\ddot{\theta}=\frac{3}{2}\frac{1}{J}\left(\frac{p}{2}\right)^2\lambda_m i_{qs}-\frac{D}{J}\dot{\theta}-\frac{p}{2J}T_L(t,\theta(t)) \tag{2b}$$

and the torque equation is reduced to

$$T_e=\frac{3}{2}\left(\frac{p}{2}\right)\lambda_m i_{qs}=k_t i_{qs} \tag{2c}$$

where

$$k_t=\frac{3}{2}\left(\frac{p}{2}\right)\lambda_m$$

For the realization of the field-orientation, each phase current command must be tracked by the current controller such as the current regulated PWM(CRPWM) scheme[22,23]. Hence, the current dynamics in (2a) can be negligible in modeling of the BLDDSM. Thus, from (2b) and (2c), the vector-controller linearized BLDDSM can be simply expressed as follows:

$$J\cdot \ddot{\theta}(t)+D\cdot \dot{\theta}(t)+T_L(t,\theta(t))=\frac{p}{2}k_t\cdot i_{qs}(t) \tag{3}$$

It is assumed that the left-hand side of (1) is smooth enough to ensure a local existence, a uniqueness of the solution for every initial condition (x, t_0), and also the continuity of the control[19]. Fig.1 shows the block diagram of linearized BLDDSM for sliding mode tracking control. The system parameters J, D and k_t in (3) are assumed to be bounded as

$$J\in[J_{\min}\quad J_{\max}] \tag{4a}$$

$$D\in[D_{\min}\quad D_{\max}] \tag{4b}$$

$$k_t\in[k_{t_{\min}}\quad k_{t_{\max}}] \tag{4c}$$

where subscripts 'min' and 'max' denote the each minimum and maximum values, respectively.

Let $X^0\in[X_{\min}\ X_{\max}]$, $X=J,D$ and K denote each nominal parameter possibly estimated from the manufacturer specifications. However, there exists the unavoidable estimation errors from the real values caused by the linearization and uncertain terms, etc. The robust variable structure tracking control of a BLDDSM, (1), is considered when the described trajectory, $\theta_d(t)$, its derivate, $\dot{\theta}_d(t)$, and its second derivative, $\ddot{\theta}_d(t)$, are given as follows:

$$\theta_d(t)=\theta_i+\frac{(\theta_f-\theta_i)}{T}\cdot t-\frac{(\theta_f-\theta_i)\cdot \sin(\pi t)}{\pi T}\ [^\circ] \tag{5a}$$

$$\dot{\theta}_d(t)=\frac{(\theta_f-\theta_i)}{T}-\frac{(\theta_f-\theta_i)\cdot\cos(\pi t)}{T}[^\circ/\mathrm{sec}] \tag{5b}$$

$$\ddot{\theta}_d(t)=\frac{\pi\cdot(\theta_f-\theta_i)\cdot\sin(\pi t)}{T}\ [^\circ/\mathrm{sec}^2] \tag{5c}$$

where θ_i : initial position of desired trajectory
θ_f : final position of desired trajectory
T : execution time from θ_i to θ_f

In this tracking problems, the initial position of the desired trajectory, i. e., θ_i may differ from $\theta(0)$ in (3). The main objective of the controller design is to drive the BLDDSM to exactly track the predetermined intermediate virtual trajectory with prescribed tracking performance from $\theta(0)$ to θ_f even in the presence of the parameter uncertainties.

III. Design of A Continuous Variable Structure Tracking Controller

First of all, define the state vector $X(t)\in R^2$ in the error coordinate system for a VSS as

$$X(t)\equiv[e_1(t)\quad e_2(t)]^T \tag{6}$$

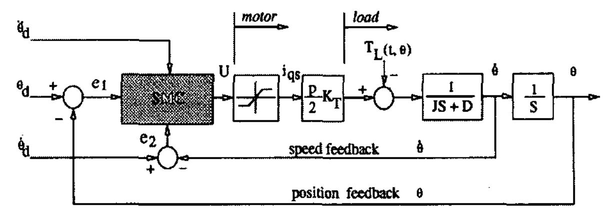


Fig. 1. Block diagram of linearized BLDDSM for tracking control.

where $e_1(t)$ and $e_2(t)\in R$ are the trajectory error and its derivative, respectively, and are expressed as

$$e_1(t)\equiv\theta_d(t)-\theta(t) \tag{6a}$$

$$e_2(t)\equiv\dot{\theta}_d(t)-\dot{\theta}(t) \tag{6b}$$

The the error state equation of a BLDDSM is expressed as

$$\dot{X}(t)=\begin{bmatrix}0 & 1\\ 0 & -\frac{D}{J}\end{bmatrix}\cdot X(t)+\begin{bmatrix}0\\ \frac{1}{J}\end{bmatrix}\cdot T_L(t,\theta(t)) +\begin{bmatrix}0\\1\end{bmatrix}\cdot w(t)-\begin{bmatrix}0\\ \frac{pk_t}{2J}\end{bmatrix}\cdot i_{qs}(t) \tag{7}$$

$$w(t)=\ddot{\theta}_d+\frac{D}{J}\dot{\theta}_d(t) \tag{7a}$$

To have the second order desired error dynamics and zero steady error and to predetermine the virtual desired trajectory, the conventional surface is augmented by an integral action with a certain initial in this study as follows:

$$s(t)\equiv e_2(t)+C_1\cdot e_1(t)+C_0\cdot e_0(t)=\sum_{i=0}^{2}C_i\cdot X_i\ ,\quad C_2=1 \tag{8}$$

where

$$e_0(t)=\int_0^t e_1(\tau)d\tau+e_0(t_0)\ ,\qquad e_0(t_0)=-\frac{1}{C_0}\cdot(e_2(t_0)+C_1\cdot e_1(t_0)) \tag{8a}$$

which is basically the linear combination of the states $e_0(t)$, $e_1(t)$ and $e_2(t)$. If $e_1(t_0)=\theta_i-\theta(t-0)\neq 0$ and without $e_0(t_0)$ in (8a), the reaching phase may occur since $s(t)\neq 0$ at $t=0$ as most of the previous researches on the VSS tracking controls. So it is assumed that both are equal until now. However, in this study, since there exists no reaching phase even in case of $e_1(0)\neq 0$ because of $s(t)=0$ at $t=0$ due to $e_0(0)$, the initial of the desired trajectory can be arbitrary.

The ideal sliding dynamics with respect to a given $\theta_d(t)$ can be obtained from $s(t)=0$ and expressed as

$$\dot{e}_{s2}(t)+C_1\cdot e_{s2}(t)+C_0\cdot e_{s1}(t)=0 \tag{9}$$

or in matrix form

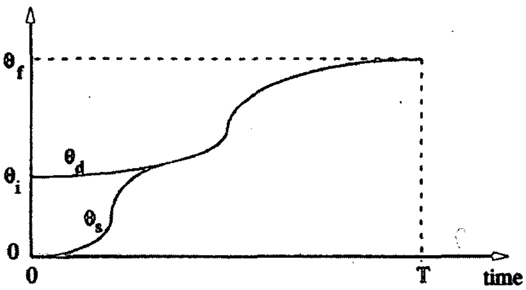


Fig. 2. Relationship between θs and θd.

$$\dot{X}_s(t) = \Lambda \cdot X_s(t) \quad X_s(t_0) = [e_1(t_0)\ e_2(t_0)]^T \tag{10}$$

where

$$\Lambda = \begin{bmatrix} 0 & 1 \\ -C_0 & -C_1 \end{bmatrix} \tag{10a}$$

and

$$\begin{aligned} X_s(t)^T &= [e_{s1}(t)\ e_{s2}(t)]^T \\ &\equiv [\theta_d(t) - \theta_s(t)\ \ \dot{\theta}_d(t) - \dot{\theta}_s(t)]^T \end{aligned}$$

The solution of (10), $\theta_s(t)$ predetermines the virtual desired trajectory from $\theta(t_0)$ to θ_f to be tracked by $\theta(t)$ with respect to $\theta_d(t)$. Its convergence rate in transient period depends on the choice of C_0 and C_1. The relationship between $\theta_d(t)$ and $\theta_s(t)$ is shown in Fig 2.

The coefficients C_0 and C_1 are chosen such that the sliding surface if Hurwitz polynomial which guarantees the exponential stability of (10). In other words, there exist positive constants K and x such that

$$\|\exp \Lambda t\| \le K \cdot \exp\{-xt\} \tag{11}$$

where $\|\exp \Lambda t\|$ denotes $(trace[\exp\{\Lambda t\}^T \exp\{\Lambda t\}])^{1/2}$ as the induced Euclidean matrix norm. The minimum value of (K/x) which satisfies (11) is defined as μ.

Instead of discontinuous input, the continuous input under consideration can result in the tracking error of $\theta(t)$ to $\theta_s(t)$. Let us this error vector be

$$\overline{X^T} \equiv [\theta_s(t) - \theta(t)\ \ \dot{\theta}_s(t) - \dot{\theta}_s(t)].$$

The relationship between the this maximum error and the bound of the maximum sliding surface will be stated in Lemma 1 in order to obtain the prescription of the tracking performance.

Lemma 1: *If the sliding surface defined in (8) satisfies* $|s(t)| \le \gamma$ *for any* $t \ge t_0$, *and* $|\overline{X(t_0)}| \le \gamma / x$ *is satisfied at initial time, then*

$$\overline{|X_1(t)|} \le \varepsilon_1 \quad \overline{|X_2(t)|} \le \varepsilon_2 \tag{12}$$

are satisfied for all $t \ge t_0$ *where* ε_1 *and* ε_2 *are the positive constants defined as follows:*

$$\varepsilon_1 \equiv \mu \cdot \gamma,\ \varepsilon_2 \equiv \mu \cdot (1 + \mu \cdot \gamma),\ Z \equiv \|[C_0\ C_1]\| \tag{12a}$$

Proof: The augmented sliding surface (8) can be expressed in the state space form using a new state vector $\overline{X(t)}$ defined by $[e_0(t), e_1(t)]^T$ as

$$\overline{X}(t) = \Lambda \cdot \overline{X}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(t) \tag{13}$$

In (13), $s(t)$ may be considered as the bounded disturbances, $|s(t)| \le \gamma$. The solution of (13) is expressed as [19]

$$\begin{aligned} \overline{X}(t) &= \exp\{\Lambda(t - t_0)\} \cdot batX(t_0) \\ &+ \int_{t_0}^{t} (\exp\{t - \tau)\} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(\tau))d\tau \end{aligned} \tag{14}$$

from (11) and the bound of $s(t)$, the Euclidean norm of vector $\overline{X}(t)$, i. e., $\|\overline{X}(t)\| = (e_0^2(t) + e_1^2(t))^{1/2}$ becomes

$$\begin{aligned} \|\overline{X}(t)\| \le\ & K \cdot \exp\{-x(t - t_0)\} \cdot \|\overline{X}(t_0)\| \|)d\tau \cdot \| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \| \cdot \gamma \\ &+ \int_{t_0}^{t} (\| \exp\{\Lambda(t - \tau) \end{aligned}$$

$$\begin{aligned} \|\overline{X}(t)\| &\le \frac{K}{x} + (\|\overline{X}(t_0) - \frac{\gamma}{x}) \cdot K \cdot \exp\{-x(t - t_0)\} \\ &\le \gamma \cdot \frac{K}{x} \end{aligned} \tag{15}$$

for all $t \ge t_0$, which drives $e_1(t)$ to satisfy the following inequality

$$|e_1(t)| \le \frac{K}{x} \cdot \gamma$$

The equation (12) can be simply obtained from (8) and (15) as

$$e_2(t) = s(t) - [C_0 C_1] \cdot \overline{X}(t) \tag{16}$$

if the norm is taken on both side, (15) becomes

$$|e_2(t)| \le \gamma \cdot (1 + Z \cdot \frac{K}{x}) \tag{17}$$

for all $t \ge t_0$, which proves Lemma 1 completelyThe above Lemma1 implies that the tracking error of the trajectory to $\theta_s(t)$ and its derivative are uniformly bounded, if the sliding surface if bounded by the continuous control for all time $t \ge 0$. Using the results of Lemma 1, the specifications of the tracking error can be given, which are dependent on the convergence rate and determined by the sliding surface.

Assuming the acceleration information is available, the following type continuous control input $i_{qt}(t)$ is proposed as follows:

$$i_{qs}(t) = i_{eqm}(t) + i_c(t) + i_s(t) \tag{18}$$

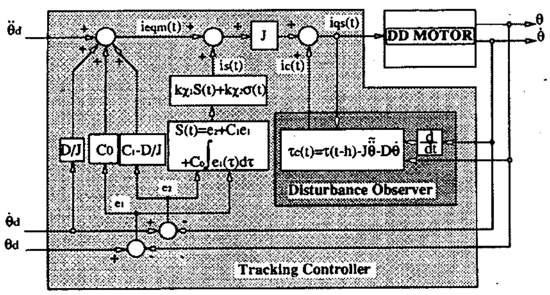


Fig. 3. Block diagram of CVSTC.

where $i_{eqm}(t)$ is a modified equivalent control; $i_c(t)$ is a compensation term by the disturbance observer; and $i_s(t)$ is a continuous feedback of the sliding surface. These components are as follows:

$$i_{eqm}(t) = (J^0 \cdot C_1 - D^0) \cdot e_2(t) + J^0 \cdot C_0 \cdot e_1(t) + J^0 \cdot w(t)$$

$$i_c(t) = i_{qs}(t-h) - \frac{2}{pK_t}(J^0 \cdot \hat{\ddot{\theta}}(t) + D^0 \cdot \dot{\theta}(t)) \quad \textbf{(18b)}$$

$$i_s(t) = J^0 \cdot (k_{\chi 1} \cdot s_1(t) + k_{\chi 2} \cdot \sigma(t)),\ \sigma(t) = \left(\frac{s(t)}{\|s(t)\| + \delta}\right) \quad \textbf{(18c)}$$

where $\hat{\ddot{\theta}}$ denotes the estimated value of the real acceleration and h is the sampling time. The (18a) is directly determined according to the design of the sliding surface. The 18(b) is the compensation term based on the disturbance observer[25] for effective estimating load torque $\hat{T}_L(t, \theta(t))$ in a BLDDSM using the available acceleration information and the previous control input. And (18c) is for the compensation of a small error in the estimation by the disturbance observer. The block diagram of the CVTSC algorithm is shown in Fig. 3. The control gains, $k_{\chi 1}$ and $k_{\chi 2}$ are to be designed to force the sliding surface to be bounded by γ so as to valid the assumption in Lemma 1.

By differentiating $s(t)$ with respect to time and substituting (18), the real dynamics of the sliding surface by the proposed control finally becomes

$$\dot{s}(t) = n_1(t) - \left(k_{\chi 1} \cdot s(t) + k_{\chi 2} \cdot \frac{s(t)}{|s(t)| + \delta}\right) \quad \textbf{(19)}$$

where $n_1(t)$ is a resultant disturbance vector given as

$$\begin{aligned} n_1(t) &= n(\Delta\ddot{\theta}(t), \Delta i_{qs}(t)) \\ &= J^0 \cdot \Delta\ddot{\theta} - \Delta i_{qs}(t) \end{aligned} \quad \textbf{(19a)}$$

where $\Delta\ddot{\theta}$ is the acceleration information error the real acceleration value and $\Delta i_{qs}(t)$ $\Delta i_{qs}(t)$ is the control input delay error resulting from the digital implementation. These two errors are defined as

$$\Delta\ddot{\theta}(t) = \ddot{\theta}(t) - \hat{\ddot{\theta}}(t),\ \Delta i_{qs}(t) = i_{qs(t)} - i_{qs}(t-h) \quad \textbf{(19b)}$$

As shown in (19), the original tracking problem is converted to the simple first order stabilization problem against the resultant disturbance by the proposed sliding mode tracking control algorithm.

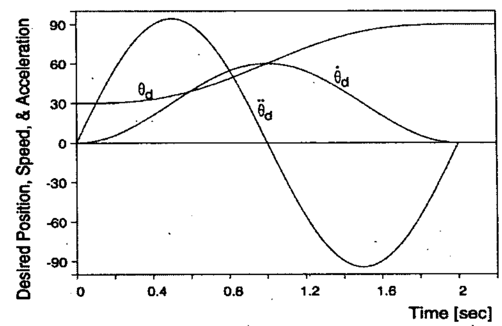


Fig. 4. Desied trajectory(θ_d), speed($\dot{\theta}_d$),acceleration($\ddot{\theta}_d$) of a BLDDSM.

Due to $k_{\chi 1}$ and $k_{\chi 2}$, there are two degree of freedoms to stabilize (19).

For the positive constants ε_1 and ε_2 defined in (12b), let the constant N be defined as follows:

$$\begin{aligned} N = \max\{&|n_1(\Delta\ddot{\theta}(t), \Delta i_{qs}(t))| \theta(t) \\ &\in B(\varepsilon_1; \theta_s(t)) \text{ and } \dot{\theta}(t) \in B(\varepsilon_1; \theta_s(t))\} \end{aligned} \quad \textbf{(20)}$$

The stability property of the controlled BLDDSM in (7) with the control laws in (18) is investigated in the next theorem.

Theorem 1: Consider the BLDDSM in (7) with the control given by (18). For some positive $\gamma, |s(t_0)| \le \gamma$ and $|X(t_0)| < \gamma/x$ are assumed to be satisfied at initial time $t = t_0$. If the gain $K_{\chi 2}$ satisfies

$$x_{\chi 2} \ge N - x_{\chi 1} \cdot \delta \quad \textbf{(21)}$$

for given $x_{\chi 1}$ and δ, then the global control system is uniformly bounded at origin in the state space for all $t \ge t_0$ until $|s(t)| \ge \eta$, where η is defined as

$$\begin{aligned} \mu &= \sqrt{\alpha_1^2 + \beta_1^2} - \alpha_1 \\ \alpha_1 &= \frac{\delta + \frac{(k_{\chi 2} - N)}{k_{\chi 1}}}{2}, \\ \beta_1 &= \frac{\delta \cdot N}{k_{\chi 1}} \end{aligned} \quad \textbf{(21a)}$$

proof: if $V(t) = 1/2s(t)$ is taken as a Lyapunov candidate and is differentiated with respect to time, it follows

$$\begin{aligned} \frac{dV(t)}{dt} &= s(t) \cdot \dot{s}(t) \\ &= s(t)\dot{n}_1(t) - s(t) \cdot \{k_{\chi 1} \cdot s(t) + k_{\chi 2} \cdot \frac{s(t)}{|s(t)| + \delta}\} \end{aligned} \quad \textbf{(22)}$$

If the following norm inequality is employed as

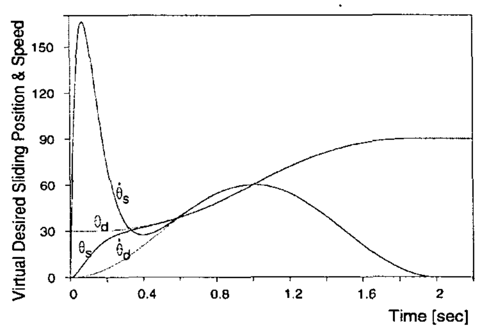


Fig. 5. Virtual desired trajectory and its speed by sliding dynamics.

$$|x \cdot y| \leq |x| \cdot |y| \tag{23}$$

if follows

$$\frac{dV(t)}{dt} \leq |s(t)| \cdot |n_1(t)| - k_{\chi 1} \cdot |s(t)|^2 - k_{\chi 2} \cdot \frac{|s(t)|^2}{|s(t)| + \delta} \tag{24}$$

Then $|n_1(t)| \leq N$ is satisfied from the definition of N, and the equation (24) can be rewritten for $t = t_0$ as

$$\begin{aligned} \frac{dV(t)}{dt} &\leq |s(t)| \cdot N - k_{\chi 1} \cdot |s(t)| - k_{\chi 2} \cdot \frac{|s(t)|}{|s(t)|_\delta} \\ &= -\frac{|s(t)| \cdot k_{\chi 1}}{|s(t)| + \delta} \cdot |s(t)|^2 + 2 \cdot \alpha_1 \cdot |s(t)| - \delta_2 \end{aligned} \tag{25}$$

If the gains and $k_{\chi 2}$ satisfy the condition (23), then (26) becomes

$$\frac{dV(t)}{dt} < 0, \qquad t \geq t_0 \tag{26}$$

until $|s(t)| \geq \eta$, which completes the proof of Theorem 1.

Using the results of Lemma 1 and Theorem 1, the desired tracking performance is guaranteed by the CVSTC algorithm. Thus, the CVSTC can be designed to be accurate and robust against the load variations. The first design procedure of the proposed sliding mode controller is to choose the desired sliding surface defining error dynamics. The second procedure is that the gains, $\chi_{\chi 1}$ and $\chi_{\chi 2}$, in (19) are selected based on Theorem 1 such that the predetermined tracking error is reserved. The detailed design procedures are summarized as follows:

(i) design the augmented sliding surface to predetermine the virtual trajectory, i. e., determine C_0 and C_1 in (8)

(ii) find the minimum of k/x satisfying (11)

(iii) choose γ for the desired tracking error based on the results of Lemma 1

(iv) find N from (20) for the desired position trajectory defined in (5a)

(v) design the controller gains, $\chi_{\chi 1}, \chi_{\chi 2}$ and δ for the boundedness of the sliding surface based on Theorem 1, which completes the design of the CVSTC algorithm.

The comparative studies between the proposed algorithm and the conventional VSS are carried out through the computer simulations.

IV. Simulation Studies

The specifications of the BLDDSM used in this simulations are listed in Table

The sample desired trajectories $\theta_d(t)$, $\dot{\theta}_d(t)$ and $\ddot{\theta}_d(t)$ of (2) are shown in Fig. 4. The initial and final position commands, θ_i and θ_f,are 30^0 and 90^0 apart from 0^0, respectively, and the execution time , T, is 2[sec] for a typical example. Based on the design guidelines, the coefficients of the proposed sliding surface, C_0 and C_1, are selected as 225 and 30 so that the matrix Λ has the double poles at -15. The initial condition f te integral, $e_0(t_0)$ in (8a) becomes 0.1333. Fig.5. shows the predetermined virtual desired trajectory, $\theta_s(t)$ and $\dot{\theta}_s(t)$ by the designed sliding surface with respect to Fig. 4. The K and κ are also found to be 11.2 and 7.5 respectively. Thus, the tracking error and its derivative become

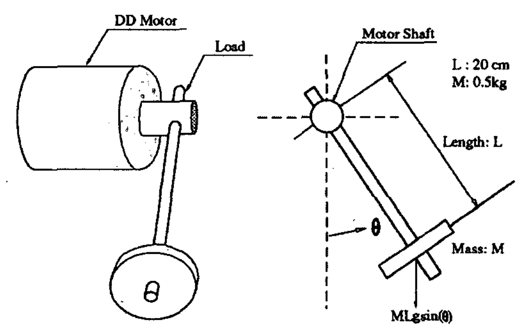
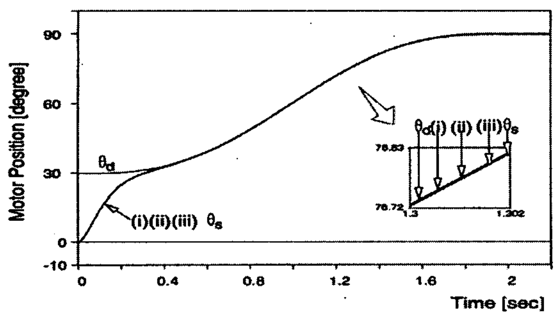


Fig. 6. DD motor and load.

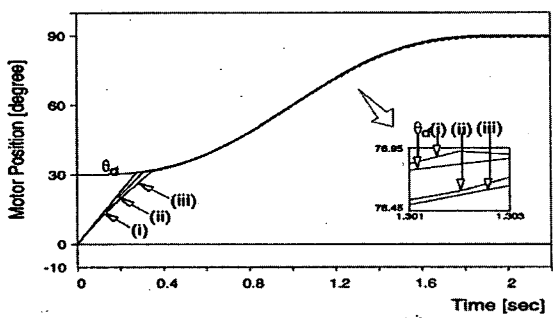
$$|e_1(t)| < 1.492 \cdot \gamma \tag{27a}$$

$$|\dot{e}_1(t)| < 227 \cdot \gamma \tag{27b}$$

based on Lemma 1. For the 0.2^0 maximum tracking error, γ is selected as 0.134. Also N defined in (22) becomes 2. Using the results of Theorem 1, the controller gains, $k_{\chi 1}, k_{\chi 2}$ and δ are designed as 100, 20, 0.05, respectively. The available acceleration information needed in the disturbance observer of the CVSTC is simply obtained by Euler's method as follows:

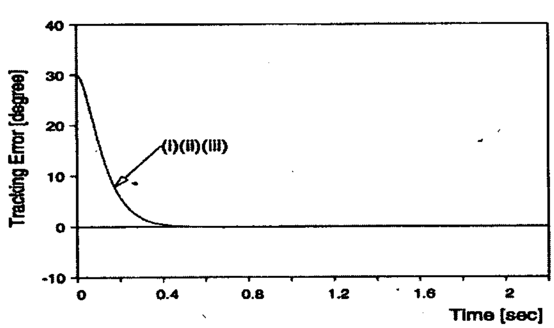


(a) CVSTC

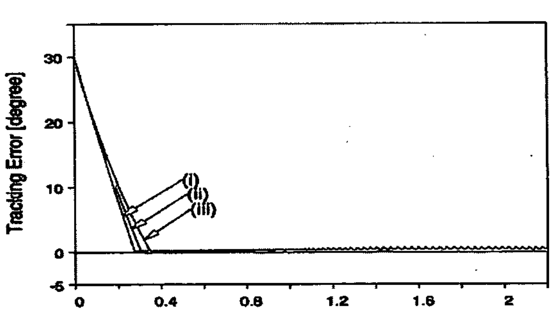


(c) conventional VSS

Fig. 7. Positions of BLDDSM for three cases (i) M= 0[kg](no load), (ii) M= 0.5[kg], (iii) M= 1.0[Kg].

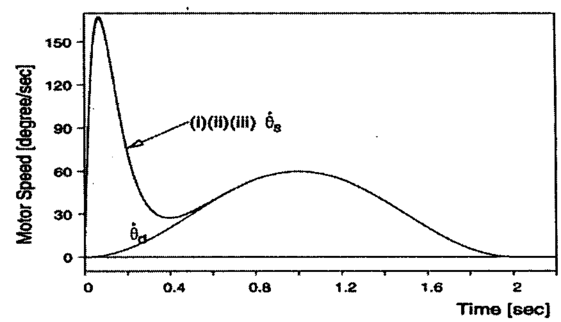


(a) CVSTC

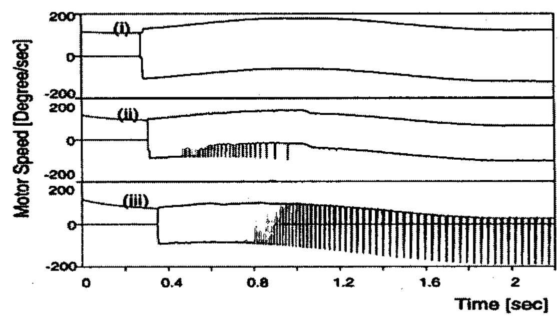


(c) conventional VSS

Fig. 8. Tracking Error.

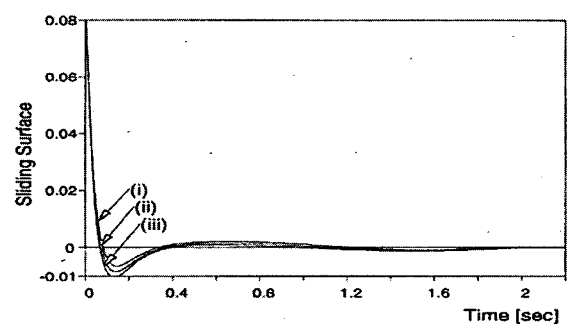


(a) CVSTC

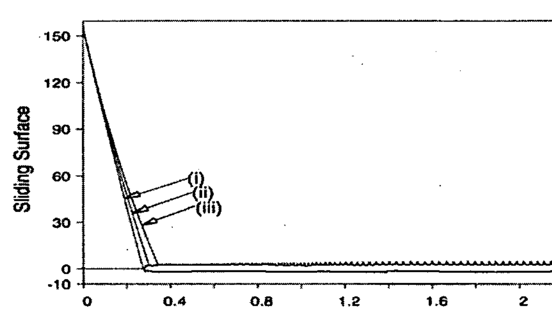


(c) conventional VSS

Fig. 9. Speeds of BLDDSM.

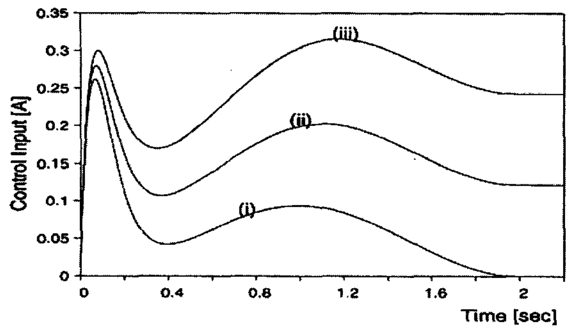


(a) CVSTC

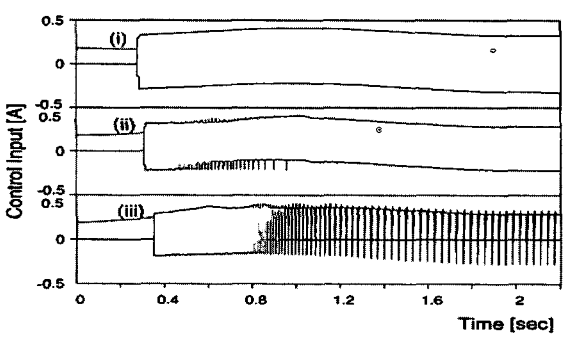


(c) conventional VSS

Fig. 10. Sliding of BLDDSM.

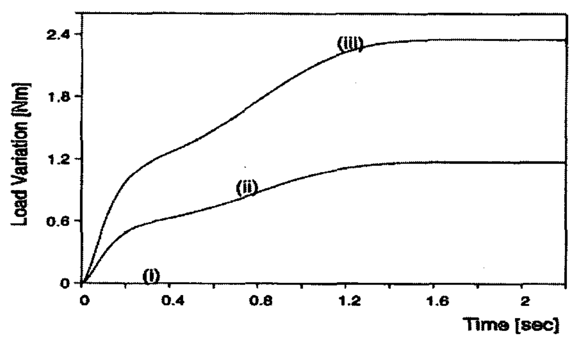


(a) CVSTC

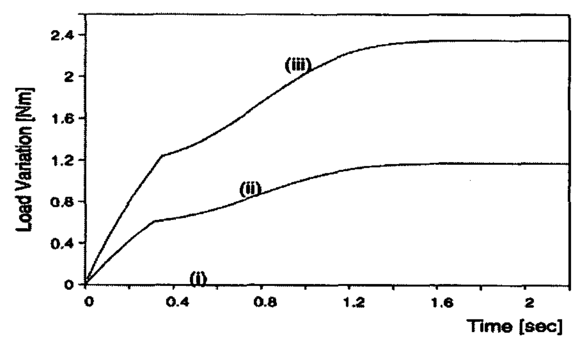


(c) conventional VSS

Fig. 11. Control inputs.



(a) CVSTC



(c) conventional VSS

Fig. 12. Load variations.

$$\dot{\theta} = \{\theta(t) - \theta(t-h)\}/h \tag{28}$$

for the relatively small sampling time h

On the other hand, in the conventional VSS, the coefficients of the sliding surface, C_1 and C_0 are selected as 990 and 0 to have a simple pole. The control input is as follows:

$$\begin{aligned} i_{qs}(t) &= g_1 \cdot sgn(e_1 \cdot s) \cdot e_1 \\ &+ g_2 \cdot sgn(e_2 \cdot s) \cdot e_2 + g_3 \cdot sgn(s) + u \end{aligned} \tag{29}$$

where the gains are $g_1 = 0.09, g_2 = 0.32$ and $g_3 = 0.32$. The sampling time is 1[msec] for the both algorithm. The load disturbance is a function of the rotor position, and it expressed as

$$T_L(t, \theta(t)) = 1.601 \cdot M \cdot L \cdot \sin(\theta(t)) \tag{30}$$

as can be seen in Fig. 6.

The simulations are carried out under three different conditions of load disturbance, i. e.,

M=0.0, 0.5 and 1.0[kg]. The simulation results of the conventional VSS and the CVSTC are shown in Fig.7. through Fig.14. Fig. 7. shows the motor positions trajectories, i. e., Fig. 7(a) for the CVSTC, Fig. 7(b) for the conventional VSS, respectively. As can be seen in these figures, while the output responses in Fig 7(b) are disturbed by the load variations due to the reaching phase, the motor positions by the CVSTC follow the predetermined $\theta_s(t)$ with robustness for all load variations as designed. Fig. 8 shows the tracking errors of the both algorithms are depicted in Fig. 10. The corresponding control inputs are depicted in Fig. 11. While the control inputs of the conventional VSS show the three different reaching to $s(t) = 0$ and chattering can be seen in Fig. 11(b), those of the CVSTC are continuous without the chattering and reaching. The control in Fig. 11(a) much smaller than that in Fig. 11(b). The continuous load variations applied to a BLDDSM are shown in Fig. 12. Each component of the proposed control input for M = 1.0 [kg]is shown in Fig. 13. In addition, the output response of the CVSTC are shown in Fig. 14. for the three different sliding surfaces, i. e., (iv) C_0=400 and C_1=40,(v), C_0=625 and C_1=50 and C_0 =900 and C_1=60, which illustrates that the transient dynamics of the output is changeable as desired with CVSTC algorithm.

As can be seen in comparative simulation results, the tracking performance of the CVSTC algorithm is better than those of the conventional VSS in view of tracking error and continuity and cost of control, which is the desirable feature of the CVSTC. the CVSTC algorithm guarantees the predetermined tracking performance without any chattering and reaching problem and with relatively low control cost.

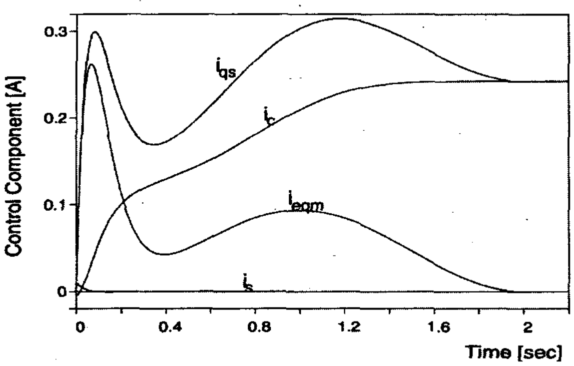


Fig. 13. Components of control inputs by CVSTC case(iii).

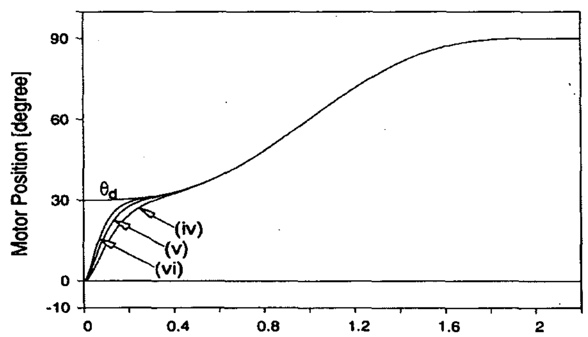


Fig. 14. Output response of CVSTC for different sliding surfaces, (iv) $C_0=400$ $C_1=400$ (v) $C_0=625$ $C_1=50$ (vi) $C_0=900$ $C_1=60$

V. Conclusions

In this paper, the design of a continuous variable structure tracking controller is presented for the accurate and robust control of a BLDDSM under the load variations. This algorithm solves the limitation in most of the existing tracking controllers that the initial of the desired trajectory only equals to that of the motor position. As an intermediate trajectory, the virtual desired trajectory from the initial of the motor position to the desired trajectory is predetermined by the modified sliding surface. The maximum tracking error to the virtual desired trajectory as a function of the maximum switching function bound is derived in Lemma 1 provided that the sliding surface is bounded for all time. The continuous control input with compensation using the disturbance observer is suggested in order to constrain the sliding surface within a given bound, which satisfies the assumption of Lemma 1. The uniformly bounded stability properties are analyzed using Theorem 1 in detail. With this analysis, the prescribed tracking performance without any chattering problems can be guaranteed under the load variations using the proposed CVSTC, whereas the information on the tracking error can not be obtained from the conventional VSS. The systematic design guideline is also suggested and the comparative simulation studies are carried out to show the superior performance of the proposed algorithm to the conventional VSS's.

References

[1] K.Jomiya. "Direct Drive Motors and Those Applications," *Monthly Journal of Automation system* Vol.10, No. 10, pp 40-45, 1994.

[2] H. Asada and K.Youcef-Toumi, "*Direct Drive Robots: Theory and Practice,*". Cambridge, MA: MIT Press, 1987.

[3] H.Hashimoto, "Brushless DC Motor Drive Control Using Variable Structure Approach," *IEEE Trans. on Ind. Appli*, Vol **IA-34**, No.1 Jan. 1988.

[4] K..W. Li, T.S. Low, M.F. Rahman, and L. B. Wee, "A Discrete Time Variable Structure Controller for a Brushless DC Motor Drive," *IEEE Trans. on Industrial Electronics*, Vol. **IE-38**, No.2 Jan. 1991.

[5] C.Rossi, A. Tonoelli, and R.Zanazi, "Variable Structure Control of a Brushless Motor," *EPE Conference (Firenze) Record*, pp.1-007-0012, 1991.

[6] K.Furuta and S. Kobayashi, "Bang-Bang Control of Direct Drive Motor," *Proc. Of IECON'90*, pp.148-153, 1990

[7] K.Furuta, K. Kosuge, and K. Kobayashi, "VSS-Type Self-Tuning Control of Direct-Drive Motor," *Proc. of IECON'89*. pp.281-286, 1989.

[8] K.Furuta, K. Kosuge, and K. Kobayashi, "Self-tuning, Sliding Mode and Adaptive Control for Direct Drive Motor," *Proc. of IECON'88*. pp.459-465, 1988.

[9] V.I.Utkin, "Variable Structure Systems with Sliding Mode," *IEEE Trans. Automat. Control*, Vol. **AC-22,** No.2, pp.212-222, 1977.

[10] R.A. Decarlo,S.H. Zak, and G. P. Mattews, "Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial," *Int. J. Control* **Vol. 38**, No. 2, pp.212-232, 1988.

[11] J.J.Slotine and S. S. Sastry, "Tracking control of Nonlinear Systems Usung Sliding Surface, with Application to Robot Manipulators" *Int. J. Control* **Vol. 38,** No. 2, pp.465-492, 1983.

[12] J. A. Burton and A. S. I. Zinober, "Continuous approximation of Variable Structure Control" *Int. J. System Sci.* **Vol.17**, No.6, pp. 875-885, 1986.

[13] G. Bartolini, "Chattering Phenomena in Discontinuous Control System" *Int. J. System Sci.* **Vol. 20,** No. 12,

pp.2471-2481, 1989.

[14] J. X. Xu, H. Hashimoto, J. J. Slotine, Y. Arai, and F. Harashima, "Implementation of VSS Control to Robotic Manipulators-Smoothing Modifications" *IEEE Trans. on Industrial Electronics*, **Vol I.E.-36**, No.3, Aug. 1989.

[15] F. J. Chang, S. H. Twu, and S. Chang, "Adaptive Chattering Alleviation of Variable Structure System Control," *IEE Proc.-D*, **Vol .137**, No.1, Jan. 1990.

[16] F. Esfandiari and H. K. Khali, "Stability Analysis of a Continuous Implementation of Variable Structure Control," *IEEE Trans. Automat. Control*, **Vol. AC-36**, No. 5 pp.616-620, May 1991

[17] F.Zhou and D. G. Fisher, "Continuous Sliding Mode Control," *Int. J. Control* **Vol.55**, No. 2, pp.313-327, 1992

[18] F. J. Chang, S. H. Twu and S. Chang, "Tracking Control of DC Motors via An Improved Chattering Alleviation Control," *IEEE Trans. on Industrial Electronics*, **Vol. I.E.-39**, No.1, Feb. 1992.

[19] W. A. Coppel, *Stability and Asymptotic Behavior of Differential Equations*(Doston, D.C.HEATH AND COMPANY, 1965).

[20] Y.Dote and S. Kinoshita, *Brushless Servomotors: Fundamental and Applications* (Oxford Science Publications, 1990).

[21] B. K. Bose, *Power Electronics and AC DRives*, (New Jersey, Prentice-Hall, 1986).

[22] W. Leonhar, "Microcomputer Control of High Dynamic Performance AC-Drives - A Survey," *Automatica*, **Vol. 22**, No. 1, pp.1-99,1986

[23] P. C. Sen, "Electric Motor Drivers and Control - Past, Present and Future," *IEEE Trans. on Industrial Electronics*, **Vol. I.E.-37**, No. 6, Dec. 1990.

[24] T.L. Chern and Y. C. Wu, "Design of Intergral Variable Structure Controller and Application to Electro-hydraulic Velocity Servosystems," *IEE Proc.-D*, **Vol. 138**, No.5, Sept.1991.

[25] S. Komoda and K. Ohnishi, "Force Feedback Control of Robot Manipulator by the Acceleration Tracking Orientation," *IEEE Trans. on Industrial Electronics*, **Vol. I.E-38**, No. 2, 1991.

[26] J. H. Lee, S. K. Chung, G. M. Woo, I. S. M. J. Youn, and T. Y. Ahn, "Design of a Variable Structure Controller Using an Integral-Augmented Sliding Surface for Position Control of BLDDSM with Prescribe Output Dynamics," *Journal of KIEE*, Vol. 8, no.1, pp.16-16, 1995.

[27] J. H. Lee, J. S. Ko, S. K. Chung, D. S. Lee, J. J. Lee, and M. J. Youn, "Continuous Variable Structure Controller for BLDDSM Position Control with Prescribed Tracking Performance," *IEEE Transactions on Industrial Electronics*, Vol.IE-41, no.5, pp.1-9, 1994.

[28] J. H. Lee and M. J. Youn, "An Integral- Augmented Optimal Variable Structure Control for Uncertain Dynamical SISO Systems," *Journal of KIEE*, Vol.43, no 8. pp.1332-1351, 1994.

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