Seismic Behavior of Rotating Shaft System at Start-up

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ABSTRACT

A rotating shaft system subjected to seismic motions has been investigated for the various operating modes at start-up. During an earthquake excitation, the rotor may hit the stator of machines due to the excessive deformation of shaft, and thus the response of rotating shaft system of which foundation is supported by the vibration isolation devices has been simulated. In order to examine the transient response of the rotating shaft system at the start-up to both the various operating conditions and the seismic excitation simultaneously, nonlinear equations of motion are derived and solved numerically using Runge-Kutta method. The response of the rotating shaft system is calculated according to the operating modes as recommended by the machine and the system parameters such as the spring stiffness of isolation devices.

Key words: rotating shaft system, operating mode, seismic isolation, seismic response

1. Introduction

The rotating shaft systems are widely used in a variety of machines ranging from the small precession motor to the turbine-generator in power plants. For the small rotating machines the critical speed is greater than the operating speed, and thus the stability at start-up is not important. However the critical speed of the large rotating machine are less than the operating speed, and the stability problem is occurred because the machine speed should pass the critical speed at start-up. In addition to that, if the machine foundation is installed on the vibration/seismic isolation device, the behavior of the rotating shaft system at start-up could be investigated corresponding to the operating mode and the seismic excitation. Also it is necessary that the isolation device should be designed by considering the characteristics of earthquake and of machines and that the response of supporting structure including isolation device be accurately predicted by a proper
method.

In order to predict the transient response of the rotating shaft system, a set of five nonlinear differential equations of motion have been derived from the kinetic and the potential energies of system. There are two coordinate systems as shown in Fig. 1. One is \( x_g \) and \( y_g \) coordinate of which origin is on the ground, the other is \( x \) and \( y \) originated at bearing or foundation. Actually the coordinate \( x_g - y_g \) stands for the seismic ground motion. As a result the rotating shaft system is considered to be five degree of freedom system and the governing equation of motions have five variables. Two represents the plane motion of rotating disk, two represent the plane motion of foundation, and one represents the rotation angle of rotor.

The transient motion of the rotating shaft system at start-up can be investigated according to the operating modes. An operating mode can be determined by the torque input. The reason is that the angular acceleration of rotor is proportional to torque. In the study, two-step operation mode is adopted to investigate the dynamic response of the rotating shaft system under both the imbalance, which is defined by the eccentricity(e) in Fig. 1, and the seismic ground motion. The unbalance is given by the eccentricity from the center of rotation. The first step is from zero to speed between 0.8 and 0.9 of the critical speed(\( w_{cr} \)), which is called the intermediate speed, and the second step is from the end of the first step to the normal operating speed of machine (Fig. 2). More torque may be required to pass the critical speed as fast as possible. If the shaft rotates near the critical speed, the amplitude of shaft vibration increases very fast and the system finally may be in danger.

Thus, the torque rate at the second step is greater than that of the first step. If the torque input at the second step were not enough, the rotating speed might not reach the operating speed and the shaft rotates near the critical speed, which is called the phenomena of limited power supply. Since the machine speed is near the critical speed, the amplitude of rotor is so large that the rotor hits the stator.

For the low-tuned foundation, the vertical fundamental frequency is much lower than the running speed of machine and the ratio of the machine speed to the fundamental frequency of foundation is greater than 3. But the fundamental frequencies of the low-tuned foundations generally coincide with the predominant frequency range of earthquake, and thus the fundamental frequency of foundation with seismic isolation device should be less than 1 Hz. Therefore in this study the spring stiffness of isolation has been treated as a parameter.

![Fig. 1 Coordinate system of rotating shaft:
\( \phi = \) precession angle, \( \theta = \) rotating angle
\( C = \) center of rotation, \( e = \) eccentricity](image-url)
2. Equations of Motion

The position vector to the center of rotor as shown in Fig. 1 can be expressed as

\[ \mathbf{r}^{m_1/F} = \mathbf{r}^{O/F} + \mathbf{r}^{C/O} + \mathbf{r}^{m_1/O} \]  

(1)

where \( \mathbf{r}^{O/F} \) is the position vector to bearing housing relative the fixed reference frame and can be expressed in terms of the seismic ground motion as

\[ \mathbf{r}^{O/F} = x_g \mathbf{n}_x + y_g \mathbf{n}_y \]  

(2)

where \( \mathbf{n}_x, \mathbf{n}_y \) are the unit vectors in the directions of \( x_g \) and \( y_g \), respectively.

\( \mathbf{r}^{C/O} \) is the position vector of rotating center relative the bearing housing or foundation,

\[ \mathbf{r}^{C/O} = x_1 \mathbf{n}_x + y_1 \mathbf{n}_y \]  

(3)

and the position vector from the elastic center to the mass center is given

\[ \mathbf{r}^{m_1/C} = e \cos \theta \mathbf{n}_x + e \sin \theta \mathbf{n}_y \]  

(4)

in which the rotation angle \( \theta \) is measured from the x-axis and \( e \) is the eccentricity of mass center to the rotation center.

Substitution of Eqs. (2), (3) and (4) into Eq.(1) may be expressed by

\[ \mathbf{r}^{m_1/F} = (x_g + x_1 + e \cos \theta) \mathbf{n}_x + (y_g + y_1 + e \sin \theta) \mathbf{n}_y \]  

(5)

Similarly the position vector of the foundation of machine can be expressed by

\[ \mathbf{r}^{m_2/F} = (x_g + x_2) \mathbf{n}_x + (y_g + y_2) \mathbf{n}_y \]  

(6)

The velocities of masses \( m_1 \) and \( m_2 \) can be obtained by differentiating Eq. (5) and (6) with respect to time and can be written as

\[ \mathbf{v}^{m_1/F} = (\dot{x}_g + \dot{x}_1 - e \dot{\theta} \sin \theta) \mathbf{n}_x + (\dot{y}_g + \dot{y}_1 + e \dot{\theta} \cos \theta) \mathbf{n}_y \]  

(7)

\[ \mathbf{v}^{m_2/F} = (\dot{x}_g + \dot{x}_2) \mathbf{n}_x + (\dot{y}_g + \dot{y}_2) \mathbf{n}_y \]  

(8)

The total kinetic energy of system is given

\[ T = \frac{1}{2} m_1 (\mathbf{v}^{m_1/F} \cdot \mathbf{v}^{m_1/F}) + \frac{1}{2} m_2 (\mathbf{v}^{m_2/F} \cdot \mathbf{v}^{m_2/F}) + \frac{1}{2} I \omega \cdot \omega \]  

(9)

where \( m_1 \) and \( m_2 \) are the masses of rotor and of foundation including machine, respectively.

The potential energy of system is composed of the strain energy due to the deformation of shaft, the vertical position of rotor center and the deformation of isolation device such as spring,

\[ V = \frac{1}{2} [k_{x_1} (x_1 - x_2)^2 + k_{y_1} (y_1 - y_2)^2] + \frac{1}{2} [k_{x_2} x_2^2 + k_{y_2} y_2^2] + m_1 g (y_1 + e \sin \theta) \]  

(10)

where \( k_{x_1} \) and \( k_{x_2} \) can be obtained from
the flexibility of shaft. The stiffness of shaft
is determined by assuming that the shaft is
simply supported at bearings and the rotor
is at the mid-point of shaft. If the cross-section of shaft is circle, the horizontal
stiffness $k_{x1}$ should be the same as the
vertical stiffness $k_{y1}$, $k_{x1} = k_{y1}$. And the
horizontal and vertical stiffnesses of isolation
devices beneath foundation bed are denoted by
$k_{x2}$ and $k_{y2}$, respectively. The
gravitational acceleration in the last term is
denoted by the symbol $g$.

The total dissipation energy can be
written as

$$ D = D_1 + D_2.$$

The dissipation energy caused by the
isolation devices is given by

$$ D_1 = \frac{1}{2} [ c_{x2} \dot{x}_2 + c_{y2} \dot{y}_2 ]$$

(11)

where $c_{x2}$ and $c_{y2}$ are the damping
coefficients of isolation device in the
direction of $x$ and $y$, respectively. The
dissipation energy caused by the internal
rotor friction can be obtained from the force
relationship and is given by

$$ D_2 = c_1 \left[ \frac{(\dot{x}_1 - \dot{x}_2)^2 + (\dot{y}_1 - \dot{y}_2)^2}{2} + \dot{\theta}^2 \frac{d\phi}{dt} \right]$$

where $c_1$ is the damping coefficient
associated with the interface between shaft
and bearing, and $\phi$ is the rotor attitude
angle defined as

$$ \phi = \tan^{-1} \frac{x_1 - x_2}{y_1 - y_2} $$

$\phi$ is the rotor precession rate, and

$$ \rho = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} $$

Then the dissipation due to rotor friction
can be written as

$$ D_2 = c_1 \left[ \frac{(\dot{x}_1 - \dot{x}_2)^2 + (\dot{y}_1 - \dot{y}_2)^2}{2} + \dot{\theta} \left( (y_1 - y_2)(\dot{x}_1 - \dot{x}_2) - (x_1 - x_2)(\dot{y}_1 - \dot{y}_2) \right) \right] $$

(12)

Thus when the rotor precession rate is
zero, the internal friction dissipation energy
is assumed to be the conventional viscous
damping. It is very important to note that
only in the case when the dissipation
function has the special characteristic of
being dependent upon the rotor precession
rate, the self-excited whirl instability can be
developed.

The governing equations of motion of the
system can be obtained from Lagrange’s
equation, which states

$$ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_r} \right) - \frac{\partial L}{\partial q_r} + \frac{\partial D}{\partial q_r} = F_{sr} $$

(13)

where $L = T - V$, $q_r$ is the
generalized coordi-
ante, and $F_{sr}$ is the
generalized
force.

Application of the above equation (13) for
the five generalized coordinates, $x_1$, $y_1$, $x_2$,
$y_2$ and $\theta$, yields the following equations:

$$ m_1(x_1 - x_2 - e \sin \theta \dot{\theta} - e \cos \theta \dot{\theta} ) + c_1 [(\dot{x}_1 - x_2) + \dot{\theta}(y_1 - y_2)] + k_1(x_1 - x_2) = F_{x1} $$

$$ m_{l1} \dot{y}_1 + \ddot{y}_1 + e \cos \theta \dot{\theta} - e \sin \theta \ddot{\theta} + c_1 [(\dot{y}_1 - y_2) + \dot{\theta}(x_1 - x_2)] + k_1(y_1 - y_2) = F_{y1} $$

$$ m_{l2} (x_2 + \ddot{x}_2) + c_{x2x} \ddot{x}_2 + c_1 [(\dot{x}_2 - \dot{x}_1) + \dot{\theta}(\dot{x}_2 - \dot{x}_1)] + k_{x2} x_2 - k_1 x_1 = F_{x2} $$

$$ m_{l2} (\ddot{y}_2 + \dot{y}_2) + c_{y2y} \ddot{y}_2 + c_1 [(\dot{y}_2 - \dot{y}_1) + \dot{\theta}(\dot{y}_2 - \dot{y}_1)] + k_{y2} y_2 - k_1 y_1 = F_{y2} $$

(14)
\[(I + m_1 e^2) \ddot{\theta} - m_1 e (\dddot{x}_1 + \dddot{x}_2) \sin \theta + m_1 e (\dddot{y}_1 + \dddot{y}_2) \cos \theta - m_1 e (\dot{x}_1 + \dot{x}_2) \cos \theta - m_1 e (\dot{y}_1 + \dot{y}_2) \sin \theta - c_1 [(x_1 - x_2) (\dot{y}_1 - \dot{y}_2) - (y_1 - y_2) (\dot{x}_1 - \dot{x}_2)] + m_1 eg \cos \theta = F_0\]

Now the equations of motion are solved numerically by using Runge-Kutta method.

### 3. Numerical Example

In order to verify the validity of Eq. (14) and to investigate the effectiveness of isolation spring for the rotating shaft system under seismic motion, the specifications of an example system are listed in Table 1. As mentioned before the spring constant of isolation device and the torque in Eq. (14) are treated as the system parameters.

At first the rotating speeds are calculated according to the constant torque rates. For convenience the operating mode at start-up is divided into two steps with respect to the ratio of machine speed(\(\omega\)) to the critical speed(\(\omega_{cr} = \sqrt{k_1/m_1}\)). The speed range of the first step is from 0 to the ‘intermediate speed’ which is between 0.8 and 0.9, and the second step starts from the intermediate speed to the operation speed. The intermediate speed in this study is selected to be 0.85(2). The torque rate of the first step in the simulation is usually less than that of the second step because the rotating speed should pass the critical speed as fast as possible.

Fig. 3 shows that the rotation speed and the stability are affected by the torque rate of the second step. If the torque rate is not sufficient(3), the machine speed reaches its operating speed after the several cycles of oscillations near the critical speed as shown in Fig. 3. The number of oscillation, which can be used as the measure of the system instability, decreases with increasing the torque rate on the second step. Although the speed reaches the operating speed, the displacement of rotor during oscillation increases so great that the rotor hit the stator and thus the rotating machine are not going to perform its function properly.

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
<th>numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_1)</td>
<td>mass of rotor disk</td>
<td>2,000 kg</td>
</tr>
<tr>
<td>(I)</td>
<td>mass moment of inertia</td>
<td>8,000 kg (-cm^2)</td>
</tr>
<tr>
<td>(k_1)</td>
<td>stiffness of shaft</td>
<td>5.0 (\times) (10^7) kg/cm</td>
</tr>
<tr>
<td>(m_2)</td>
<td>mass of foundation</td>
<td>10 (m_1)</td>
</tr>
<tr>
<td>(c_{r2}/m_2)</td>
<td>damping coefficient</td>
<td>0.075</td>
</tr>
<tr>
<td>(e)</td>
<td>eccentricity</td>
<td>0.01 cm</td>
</tr>
<tr>
<td>(\Delta t)</td>
<td>time increment</td>
<td>1.0 (\times) (10^{-6}) sec</td>
</tr>
<tr>
<td>(\omega_{cr})</td>
<td>critical speed</td>
<td>(\sqrt{k_1/m_1})</td>
</tr>
<tr>
<td>(\omega_{oper})</td>
<td>operation speed</td>
<td>3(\omega_{cr})</td>
</tr>
</tbody>
</table>
The isolated rotating system under Taft earthquake (Fig. 5) has been simulated with respect to the stiffness of isolation spring. The relations between the fundamental frequencies and the maximum horizontal displacements under the horizontal ground motion are shown in Fig. 6. As expected, the maximum displacements of rotor and foundation decrease with increasing the stiffness of isolation spring, but in Fig. 7 the relative displacement of rotor to foundation is not affected by the spring stiffness. The reason is that the ratio of the machine speed to the fundamental frequency is much large as the low-tuned spring foundation.
Fig. 7 Relative displacement of rotor to foundation with respect to isolating spring

Finally, the seismic response of rotating shaft system under the horizontal ground motion has been simulated according to the time lag between the arrival of the seismic motion and the start-up, and the maximum displacement of rotor disk with respect to the time lag is shown in Fig. 8. It can be determined that the time lag does not affect the seismic behavior of rotor system.

Fig. 8 Maximum horizontal displacement corresponding to time lag

4. Conclusion

The transient response of rotating shaft system having isolation device under an earthquake at start-up has been numerically simulated after deriving the equation of motion. Considering the simulated results, the seismic motion may not affect on the behavior of rotating system if the operating speed is much greater than the fundamental frequency of isolation device. However, the equation of motion derived can be appropriately used to simulate the transient response and to design the foundation of rotating machine.

References