Analysis of a Building Structure with Added Viscoelastic Dampers

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ABSTRACT

Steel structures with added viscoelastic dampers are analysed to investigate their behavior under earthquake excitation. The direct integration method, which produces exact solution for the non-proportional or non-classical damping system, is used throughout the analysis. The results from modal strain energy method are also provided for comparison. Then a new analytical approach, based on the rigid floor diaphragm assumption and matrix condensation technique, is introduced, and the results are compared with those obtained from direct integration method and modal strain energy method. The well known phenomenon, that the effectiveness of the viscoelastic dampers depends greatly on the location of the dampers, is once again confirmed in the analysis. It is also found that the modal strain energy method generally underestimates the responses obtained from the direct integration method, especially when the dampers are placed in only a part of the building. The proposed method turns out to be very efficient with considerable saving in computation time and reasonably accurate considering the reduced degrees of freedom.

Key words : *vibration, viscoelastic damper, dynamic analysis, matrix condensation technique*

1. Introduction

For the last 30 years the viscoelastic dampers have been widely used as an effective energy dissipation device for buildings against lateral loads. The dampers are proven to work effectively not only to prevent motion sickness in case of strong winds but also to guarantee structural safety against large earthquake ground motion. They are especially useful for steel buildings which possess relatively low inherent damping compared with concrete structures. World Trade Center in New York and Columbia Center in Seattle, USA are the examples that viscoelastic dampers are applied successfully to enhance the structural performance against dynamic loads.

In the analysis of a structure installed with viscoelastic dampers the modal strain energy method has been generally applied to predict the equivalent damping ratios of the system (Chang et. al.[2]). The modal strain energy method derives the equivalent damping ratios based on the assumption of proportional (or classical) damping system, in which the damping is proportional to mass and stiffness of the system. However, this approach may not produce accurate results when the added damping changes abruptly along the building height, because the damping matrix may become no longer proportional as a results of the installation of discrete damping devices. In this case the most accurate way of obtaining precise results is to integrate the dynamic equation of motion directly after including the damping coefficients contributed from the added damping devices to the damping matrix of the structure.

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However, as the number of stories increases the direct integration requires a lot of computer memory space and computation time. This may be permissible for the analysis in the final design stage, but for preliminary analysis the direct integration method is too time consuming and uneconomical.

In this study a rigid diaphragm approximation and a matrix condensation technique are utilized for more economical and efficient preliminary analysis of a structure with added viscoelastic dampers. It is expected that this will decrease the degrees of freedom and therefore the computation time drastically while a reasonable accuracy is still maintained. A model structure is analysed for the comparison with direct integration method and for verifying the effectiveness of the proposed method.

2. Properties of viscoelastic dampers

The mechanical properties of viscoelastic materials can be represented by the storage modulus G' and the loss modulus G''. The loss factor, the energy dissipation capacity of the material, can be obtained from those two moduli:

$$\eta = \frac{G''}{G'} \tag{1}$$

These quantities decide the stiffness and damping coefficient of the damper once the dimension of the viscoelastic material is determined(Bergman and Hanson[1]):

$$K_d = \frac{G' A_S}{t}, \quad C_d = \frac{G'' A_S}{4 \pi f t} \tag{2}$$

where t is the thickness of the material and A_S is the area of the pad(Fig. 1).

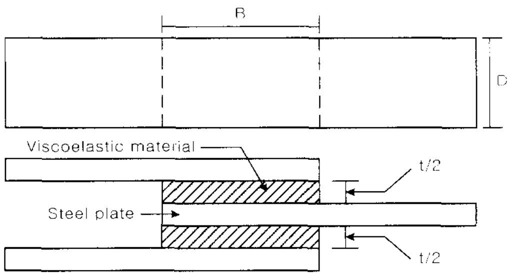


Fig. 1 A viscoelastic damper

3. Formulation of equation of motion

The general dynamic equation of motion of a structure is:

$$\mathbf{M}\ddot{\mathbf{D}} + \mathbf{C}\dot{\mathbf{D}} + \mathbf{KD} = \mathbf{A} \tag{3}$$

where **M**, **C** and **K** are the mass, damping and stiffness matrix of the structure, respectively, and **D** and **A** are the vectors for displacements and external loads.

The dynamic behavior of viscoelastic dampers is considered to be well represented by the Kelvin-Voigt model, in which the spring and the dashpot are connected in parallel, and the dynamic equation of motion of the structure with added viscoelastic dampers can be obtained by superposing the damper properties to the stiffness and the damping matrices of the structure:

$$\mathbf{C} = \mathbf{C}_s + \mathbf{C}_d \tag{4}$$

$$\mathbf{K} = \mathbf{K}_s + \mathbf{K}_d \tag{5}$$

where $\mathbf{C}_s$ and $\mathbf{C}_d$ are the damping matrices of the structure and the added dampers, respectively, and $\mathbf{K}_s$ and $\mathbf{K}_d$ are stiffness matrices of the structure and the dampers, respectively. The damping matrix of a normal structure is generally constructed by a linear combination the mass and/or the stiffness matrix, so that the damping matrix can be uncoupled by the multiplication of the modal matrix. However, due to the addition of the discrete damping from the dampers, the combined dynamic system of buildings with viscoelastic dampers becomes the so called a nonproportional or a nonclassical damping system. This type of problem can be solved by direct integration method, which produces precise results but may not be applicable to problems with a lot of degrees of freedom due to the large computational time and computer memory space required.

3.1 Modal strain energy method

The modal strain energy method has been widely used in the analysis of a structure with added viscoelastic dampers due to its simplicity (Chang et. al.[3]). In this approach, the equivalent damping ratio for the ith mode of vibration of the structure with added viscoelastic dampers can be expressed as (Kanaan and Powell[4])

$$\xi_i = \frac{\eta}{2}\left(1 - \frac{\phi_i^T K \phi_i}{\phi_i^T K_s \phi_i}\right) \qquad (6)$$

where η is the loss factor of the viscoelastic damper, ϕ_i is the i-th mode vector of the viscoelastically damped structure, and K and K_s are the stiffness matrices of the structure without and with the dampers, respectively.

3.2 Development of efficient analysis procedure

To reduce the number of degrees of freedom and the computation time, two techniques are applied; rigid diaphragm assumption and matrix condensation method.

Generally floor slabs in a building have very large in-plane stiffness, and the assumption of rigid diaphragm greatly increase the efficiency of analysis without loss of accuracy (Weaver, W. and Johnston, P. R.[5]). Therefore in this study the in-plane DOF's of all the nodal points located on the floors are condensed to the three DOF's representing two translational and one rotational degrees of freedom as described in Fig. 2.

In addition to the rigid diaphragm simplification, the computational efficiency can be further increased by the matrix condensation technique. The condensed damping matrix of the viscoelastic dampers is added to that of the structure, which was constructed from the linear combination of the mass and stiffness matrices of the structure and condensed subsequently.

To condense the equation of motion shown in Eq. 3 the degrees of freedom are devided into two parts; the primary ones describing the x and y displacements and z rotation (denoted by the subscript F), and the secondary ones associated with the remaining degrees of freedom to be reduced (denoted by A):

$$\begin{bmatrix} \mathbf{M}_{AA} & \mathbf{M}_{AF} \\ \mathbf{M}_{FA} & \mathbf{M}_{FF} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{D}}_A \\ \ddot{\mathbf{D}}_F \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{AA} & \mathbf{C}_{AF} \\ \mathbf{C}_{FA} & \mathbf{C}_{FF} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{D}}_A \\ \dot{\mathbf{D}}_F \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{AA} & \mathbf{K}_{AF} \\ \mathbf{K}_{FA} & \mathbf{K}_{FF} \end{bmatrix} \begin{Bmatrix} \mathbf{D}_A \\ \mathbf{D}_F \end{Bmatrix} = \begin{Bmatrix} \mathbf{A}_A \\ \mathbf{A}_F \end{Bmatrix} \qquad (7)$$

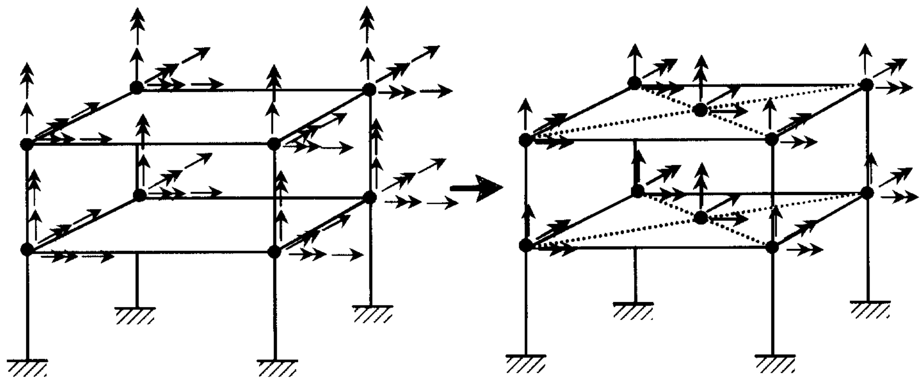


Fig. 2 Reduction of degrees of freedom using rigid floor diaphragm

Following the general procedure of matrix condensation technique, the condensed stiffness and mass matrices are obtained as follows (Weaver, W. and Johnston, P. R.[5]):

$$\mathbf{K}^{*}_{FF} = \mathbf{K}_{FF} - \mathbf{K}_{FA}\mathbf{K}^{-1}_{AA}\mathbf{K}_{AF} \tag{8}$$

$$\begin{aligned} \mathbf{M}^{*}_{FF} &= \mathbf{T}^{T}_{F}\mathbf{M}\mathbf{T}_{F} \\ &= \mathbf{M}_{FF} + \mathbf{T}^{T}_{AF}\mathbf{M}_{AF} + \mathbf{M}_{FA}\mathbf{T}_{AF} \\ &\quad + \boldsymbol{T}_{AF}\boldsymbol{M}_{AA}\boldsymbol{T}_{AF} \end{aligned} \tag{9}$$

where $\mathbf{T}_F$ and $\mathbf{T}_{AF}$ are

$$\mathbf{T}_F = \begin{bmatrix} \mathbf{T}_{AF} \\ \mathbf{I}_F \end{bmatrix} \tag{10}$$

$$\mathbf{T}_{AF} = -\mathbf{K}^{-1}_{AA}\mathbf{K}_{AF} \tag{11}$$

$\mathbf{I}_F$ is the identity matrix having the same size with $\mathbf{K}_{FF}$.

The load vector is condensed in the same way:

$$\mathbf{A}^{*}_{F} = \mathbf{A}_F - \mathbf{K}_{FA}\mathbf{K}^{-1}_{AA}\mathbf{A}_A \tag{12}$$

The condensed damping matrix of the structure is obtained from the condensed mass and stiffness matrices using the Rayleigh damping method:

$$\mathbf{C}^{*}_{S} = \alpha\,\mathbf{M}^{*}_{FF} + \beta\,\mathbf{K}^{*}_{FF} \tag{13}$$

The damping matrix of the added dampers are condensed in the same way that the mass matrix is condensed:

$$\begin{aligned} \mathbf{C}^{*}_{D} &= \mathbf{T}^{T}_{F}\mathbf{C}_{D}\mathbf{T}_{F} \\ &= \mathbf{C}_{DFF} + \mathbf{T}^{T}_{AF}\mathbf{C}_{DAF} + \mathbf{C}_{DFA}\mathbf{T}_{AF} \\ &\quad + \mathbf{T}_{AF}\mathbf{C}_{DAA}\mathbf{T}_{AF} \end{aligned} \tag{14}$$

Finally the condensed damping matrix of the combined system of the structure and the added viscoelastic dampers $\mathbf{C}^{*}_{FF}$ is obtained by the addition of the two condensed matrices:

$$\mathbf{C}^{*}_{FF} = \mathbf{C}^{*}_{S} + \mathbf{C}^{*}_{D} \tag{15}$$

The final expression of the equation of motion of the condensed system is

$$\mathbf{M}^{*}_{FF}\,\ddot{\mathbf{D}}_F + \mathbf{C}^{*}_{FF}\,\dot{\mathbf{D}}_F + \mathbf{K}^{*}_{FF}\mathbf{D}_F = \mathbf{A}^{*}_{F} \tag{16}$$

The above equations are directly integrated in the time domain to obtain the responses. As the degrees of freedom are greatly reduced as a result of the rigid diaphragm assumption and the matrix condensation, the analysis is expected to be carried out more efficiently.

4. Responses of structures with viscoelastic dampers

The response of a building against earthquake ground excitations largely depends on the natural frequencies and mode shapes of the structure. As the installation of viscoelastic dampers increases both stiffness and damping of the structure, it changes the modal characteristics and therefore the response of the building structure. To investigate the behavior of a structure installed with viscoelastic dampers and to verify the efficiency and preciseness of the proposed analysis method, an eight-story steel building shown in Fig. 3 is taken for analysis. Mild steel rolled section H-400×400×13×21 is used for columns from 1st to 4th-story, and H-300×300×10×15 is used from 5th to 8th-story. The girders are made of H-300×200×10×15 throughout the building. The viscoelastic dampers are positioned as diagonal braces between adjacent floors as shown in the figure. The material properties of the viscoelastic materials used in the analysis, which are taken from Chang et al. (1995), are as follows: loss factor η=1.13, storage modulus G'=2.106kg/cm2, and the loss modulus G"=2.377kg/cm2. The dimension of the damper and the resultant stiffness and damping coefficient are described in Table 1. The frequency and temperature dependancy of the material are not considered in the analysis. The N-S components of El Centro earthquake ground excitation are used as input for the analysis.

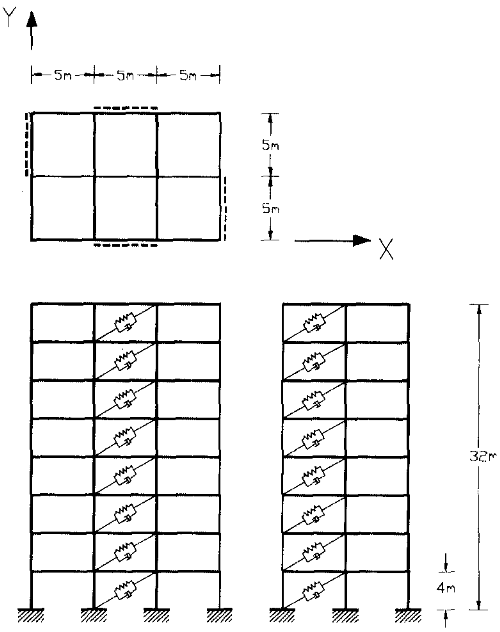
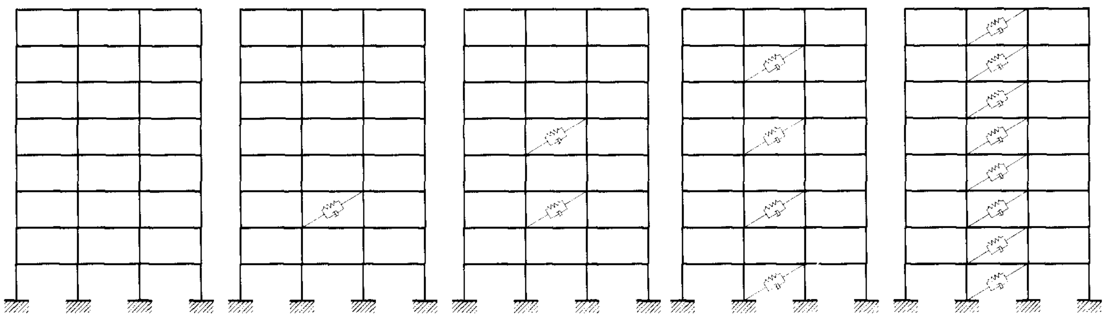


Fig. 3 Model structure for analysis

Table 1 Properties of a viscoelastic damper
(f=3.5Hz, ε=5%, temperature=25℃)

B	D	t	K_d	C_d
19.05cm	12.70cm	2.55cm	199.81kg/cm	5.13kg/cm

The model structures described in Fig. 4 are divided into 5 types according to the number and location of the dampers. In case (b) and (c) the dampers are placed where relatively large inter-story drift occurs. The dampers are analytically modeled as a spring and dashpot in parallel. The damping matrix of the structure is constructed using the Rayleigh damping method with 3% of modal damping ratios for the first and the second modes.



(a) Case-A (b) Case-B (c) Case-C (d) Case-D (e) Case-E

Fig. 4 Location of viscoelastic dampers in analysis models

Fig. 5 shows the time history of the top story displacement along the X-axis obtained from the direct integration of the equations of motion without condensation. As expected the displacement decreases as the number of damper installed increases. It is interesting to note that the placement of the damper only on the third floor (case-B) reduces the top story drift to almost half the amount of the value that occurs in the structure with no dampers (case-A).

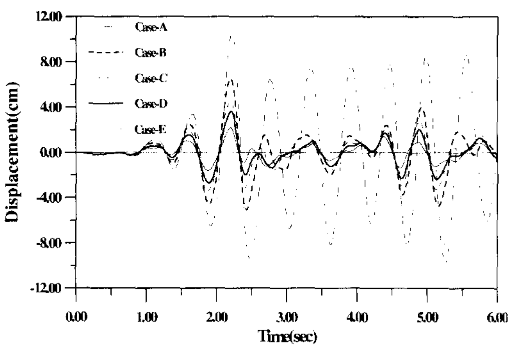


Fig. 5 Displacement time histories at top story

The inter-story drifts, plotted in Fig. 6 show that the dampers installed on the third floor, where the maximum inter-story drift occurs when no dampers are installed, also contribute significantly to the reduction of the drift of the adjacent floors. In case-C, where dampers are placed on the third and fifth floors, the displacement is further reduced. When the viscoelastic dampers are installed on all the floors (case-E) the story drifts become minimum. However, it can be noticed that as the number of floors installed with dampers increases, the efficiency of each damper decreases. This observation leads to the conclusion that an economic solution for the excessive dynamic response of a building can be achieved if the minimum number of viscoelastic dampers are installed in appropriate places.

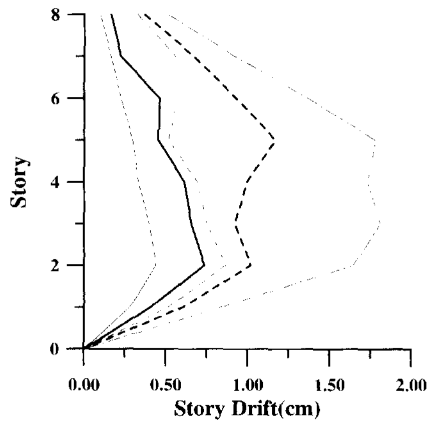


Fig. 6 Story drift in X-dir

Fig. 7 represents the Fourier transform of the El Centro earthquake records. It can be seen in the Fig. that main components of vibration appear around 1.7 Hz of the frequency contents, which is very close to the second natural frequency of the model structures. Therefore it can be predicted that the dynamic response of the model structures will be greatly influenced by the second mode of vibration. Fig. 8 is the Fourier transform of the displacement time history obtained from the analysis of the 5 model structures. As expected the largest amplitudes occur near the second natural frequency, and as in the time history response the installation of a viscoelastic dampers on a single story (case-B) greatly reduces the maximum amplitude. Further reduction in the amplitude can be achieved with additional damper installment (case-C to E), but not as efficiently as in case-B.

Fig. 9 compares the displacement time histories for case-B and case-E obtained from the modal strain energy method, and the direct integration procedures with and without matrix condensation. The results obtained from the proposed procedure generally coincide well with the exact solution within a reasonable accuracy, confirming the possibility of employing the matrix condensation method combined with the rigid diaphragm approximation. The small discrepancy with the exact solution may have resulted from the loss of some vibration modes in the process of the matrix condensation. The comparison of the overall computation time further certifies the efficiency of the proposed method as shown in Table 2. Due to the great reduction of the degrees of freedom, the computation time required for the proposed method is hundreds of times less than those required for the other two approaches. The analyses were carried out using a personal computer with a Pentium 150MHz CPU and 32MB RAM.

5. Concluding remarks

In this study steel buildings with added viscoelastic dampers in various locations are analysed by the two conventional methods (modal strain energy method and direct integration method) and a proposed matrix condensation method combined with the assumption of rigid diaphragm of floors to observe their behavior under earthquake ground excitation and to verify the effectiveness of the proposed method. The results obtained from this study are summarized as follows:

Table 2 Degrees of freedom and computation time for each method

	Modal Strain Energy Method	Direct Intgration without Condensation	Direct Intgration with Condensation
Degree of freedom	576	576	24
Time for eigen-value analysis(sec)	476	478	2
Time for time history analysis(sec)	13253	13258	22
Total computation time(sec)	13731	13738	24

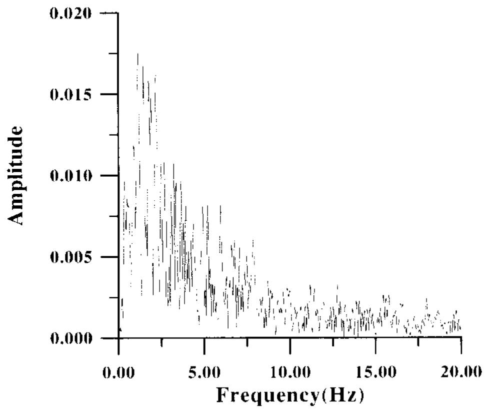


Fig. 7 FFT of ground accelerations

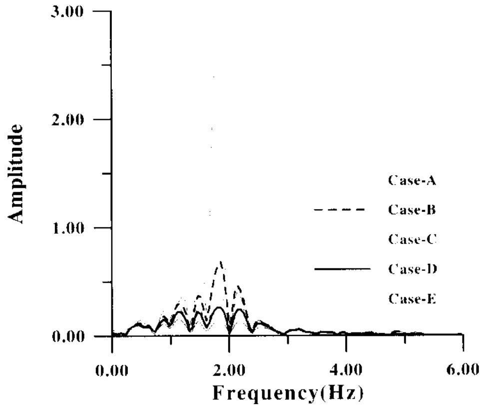
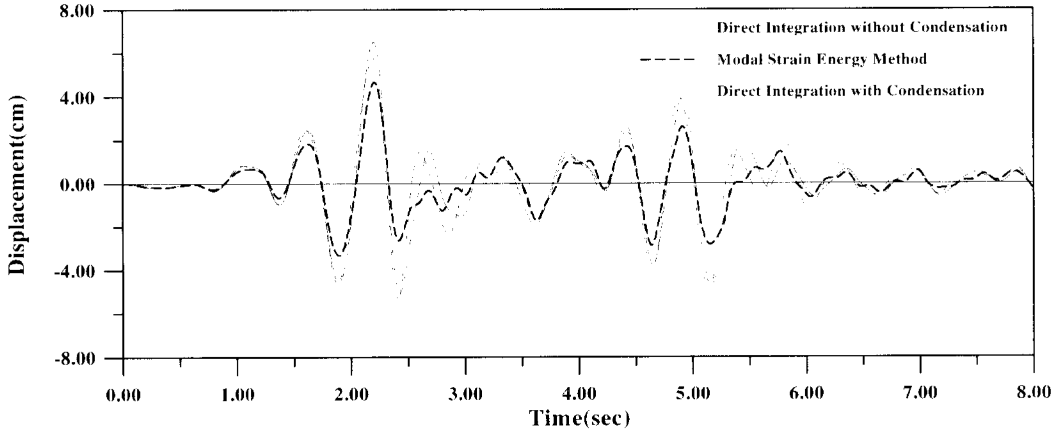
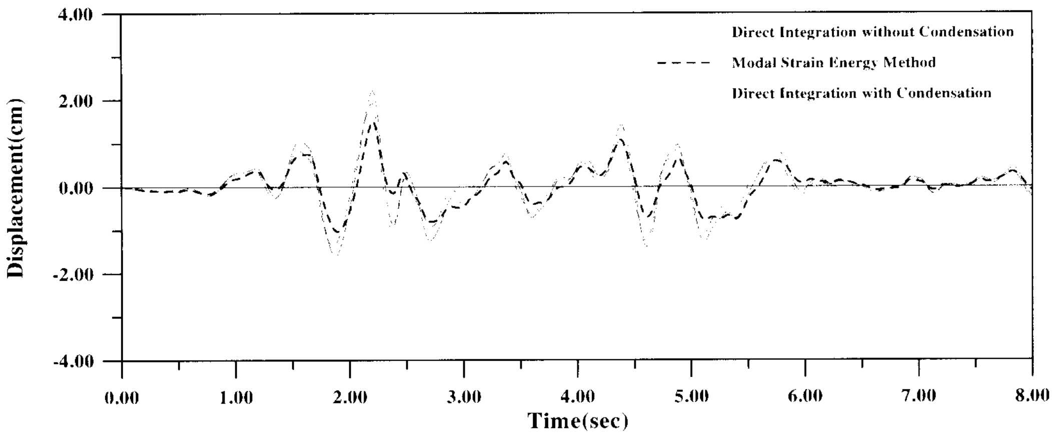


Fig. 8 FFT of displacements at top story



(a) Case-B



(b) Case-E

Fig. 9 Displacement time histories at the top story

- The installation of only a few viscoelastic dampers may be satisfactory in reducing the dynamic responses if they are placed in proper locations. With further installation of the dampers, the efficiency of energy dissipation per damper gradually decreases.
- The top story displacement obtained from modal strain energy method slightly underestimates but generally coincides well with the exact solution obtained from the direct integration method, but overestimates them in some higher modes because of the fact that the modal strain energy method underestimates the damping ratios in higher modes especially when the dampers are placed localized in the building.
- The proposed method also provides good agreement with the exact solution, with a slight failure of realizing some higher mode components.
- The efficiency of computation for the proposed method is greatly enhanced due to the reduction of degrees of freedom.

Although the direct integration method obviously provides most reliable results, the required computer memory space and analysis time are tremendously higher, consequently confining its application to the final analysis of medium-rise building structures having limited number of degrees of freedom. Therefore the proposed method can be a valuable tool for the preliminary analysis of a tall building structure with added viscoelastic dampers, in which the degree of accuracy provided by the proposed method may be enough.

Acknowledgement

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