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Some model misspecification problems for time series: A Monte Carlo investigation

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Abstract

Recent work by Shin and Sarkar (1996) examines model misspecification problems for nonstationary time series. Shin and Sarkar introduce a general regression model with integrated errors and one system of integrated regressors and discuss the limiting distributions of the OLS estimators and the usual OLS statistics such as $\hat{\sigma}^2$, t, DW and R^2. We analyze three different model misspecification problems through a Monte Carlo study and investigate each model misspecification problem. Our Monte Carlo experiments show that DW and R^2 can be in general used as diagnostic tools to detect spurious regression, misspecification of nonstationary autoregressive and polynomial regression models.

1. Introduction

Shin and Sarkar (1996) considered a general regression model with integrated errors and one system of integrated regressors defined as follows:

$$y_t = \beta_0 + \beta_1 X_{t,1} + \dots + \beta_p X_{t,p} + U_{t,k}, t = 1, \dots, n, \tag{1.1}$$

where y_t is the regressand. The regressors are defined by

$$X_{t,j} = X_{1,j-1} + \dots + X_{t,j-1}, j = 1, 2, \dots, p, X_{t,0} = x_t, \tag{1.2}$$

for a sequence $\{x_t\}$. The regression error $U_{t,k}$ is defined by

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$$U_{t,j} = U_{1,j-1} + \dots + U_{t,j-1}, j = 1, \dots, k, U_{t,0} = u_t,$$

and p and k are the levels of integration and nonnegative integers. It is assumed that the two processes $\{x_t\}$ and $\{u_t\}$ have certain limiting behavior based on Assumption 1 given by Shin and Sarkar (1996, pp. 4-5).

If each of the functions g and f in Assumption 1 (Shin and Sarkar, 1996) is a standard Brownian motion, then this stochastic process holds with $w_t = x_t$ or $w_t = u_t$ under conditions (Herrndorf, 1984 and Phillips and Solo, 1992) such as

C1. $E(W_t) = 0$ for all t, $\mathrm{Sup}_t E|W_t|^{\xi+\nu} < \infty$ for some $\xi > 2$ and $\nu > 0$, $\sigma^2 = \lim_{n\to\infty} E(w_1 + + \dots + w_n)^2/n$ exists and $\sigma^2 > 0$, $\{w_t\}$ is strong-mixing with mixing coefficients Ψ_m satisfying $\sum \Psi_m^{1-2/\xi} < \infty$; or

C2. $w_t = \sum_{j=0}^{\infty} \phi_j e_{t-j}$ where $\{e_t\}$ is an iid sequence with $E(e_t) = 0$, $E(e_t^2) = \sigma_e^2 < \infty$, $\sum j^2 \phi_j^2 < \infty$ and $\sigma^2 = \sigma_e^2 (\sum_{j=0}^{\infty} \phi_j)^2 > 0$.

If $k = 0$ in model (1.1) and $x_t = u_{t-1}$ in (1.2), then model (1.1) becomes the autoregressive model of order p, AR(p),

$$y_t = \phi_0 + \sum_{j=1}^{p} \phi_j y_{t-j} + u_t, \tag{1.3}$$

where ϕ_j's are linear combination of β_j's whose characteristic roots are all one. Similarly, if $k = 0$ in (1.1) and $x_t = 1$ in (1.2), then (1.1) becomes the polynomial regression model

$$y_t = \delta_0 + \sum_{j=1}^{p} \delta_j t^j / j! + u_t, \tag{1.4}$$

where δ_j's are linear combinations of β_j's.

Section 2 presents the basic asymptotic results of parameter estimates and the usual OLS statistics, which are applied to subsequent Sections 3-5. Spurious regression problem is discussed in Section 3 and misspecification of nonstationary AR and polynomial regression models is considered in Section 4. Section 5 discusses under-specification of orders in nonstationary AR and polynomial regression models. In each Section 3-5 we consider a Monte Carlo study regarding the model misspecification problems. We generate 10,000 samples of size n for n = 25, 50, 100 and 250. For a fixed sample size n, the corresponding empirical means of parameter estimates, $\widehat{\sigma^2}$ and other conventional regression statistics such as t-statistics, R^2 and DW are considered. The normal random numbers are generated by the subroutine DRNNOA of the IMSL package. Conclusive remarks are given in Section 6.

2. Asymptotic properties

Suppose $\beta = (\beta_0, \beta_1, \ldots, \beta_p)'$, $\mathbf{y} = (y_1, \ldots, y_n)'$, $\mathbf{X_t} = (1, X_{t,1}, \ldots, X_{t,p})'$, $\mathbf{X} = (\mathbf{X_1} \mid \mathbf{X_2} \mid \ldots \mid \mathbf{X_n})'$, $\mathrm{U} = (U_{1,k}, \ldots, U_{n,k})'$, and $\bar{y} = \sum_{t=1}^{n} y_t/n$ $\bar{U} = \sum_{t=1}^{n} U_{t,k}/n$. Then the OLS estimator $\hat{\beta}$ of β meets

$$(\hat{\beta} - \beta) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathrm{U}.$$

Now it is assumed that

$$\mathbf{A_n} = \mathrm{diag}[1, a_n(1, n, \ldots, n^{p-1})], \quad G(r) = [1, g_1(r), \ldots, g_p(r)]',$$

$$\mathbf{H} = \int_0^1 G(r)G(r)'dr, \quad \mathbf{V} = \int_0^1 G(r)f_k(r)dr, \quad z = \int_0^1 f_k^2 dr,$$

and suppose

$$\hat{\sigma}^2 = \sum_{t=1}^{n} \hat{U}_t^2/(n-p-1)$$

where $\hat{U}_t = y_t - \hat{\beta}_0 - \hat{\beta}_1 X_{t,1} - \ldots - \hat{\beta}_p X_{t,p}$. We now define the Durbin-Watson (DW) statistic as

$$\mathrm{DW} = \sum_{t=2}^{n} (\hat{U}_t - \widehat{U_{t-1}})^2 / \sum_{t=1}^{n} \hat{U}_t^2,$$

the coefficient of determination as

$$R^2 = [\mathbf{y}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} - n\overline{y^2})/(\mathbf{y}'\mathbf{y} - n\overline{y^2}),$$

and the t-statistic as

$$t_{\beta_j} = (\hat{\beta}_j - \beta_j)/s_{\hat{\beta}_j},$$

where $s_{\hat{\beta}_j}$ denotes the standard error of $\hat{\beta}_j$.

Now we state two theorems containing the limiting properties of the parameter estimators and other conventional regression statistics, which are given by Shin and Sarkar (1996).

Theorem 1. Let model (1.1) hold with Assumption 1. Suppose $k \geq 1$, $p \geq 1$ and that $\mathbf{H}$ is nonsingular. Then

(1) $A_n(\widehat{\beta}-\beta)/(n^{k-1}d_n)$ converges in distribution to $H^{-1}V$, i.e.,

$d_n^{-1}a_n n^{j-k}$ converges in distribution to $(H^{-1}V)_{j+1,1}$ for j = 1, ...,p;

(2) $(n-p-1)\widehat{\sigma}^2/(n^{2k-1}d_n^2)$ converges in distribution to $(z-V'H^{-1}V)$.

Theorem 2. Let model (1.1) hold with Assumption 1. Assume that $k \geq 1$, $p \geq 1$ and H is nonsingular. also assume that f_k is not a linear combination of $g_1, g_2, \ldots, g_p$. Define $\tau = 0$ if $\beta = 0$, otherwise define $\tau = \lim_{n\to\infty}(n^{x}a_n)/(n^k d_n)$ where $x = \max\{0 \leq j \leq p: \beta_j \neq 0\}$ and $h(r) = \tau\beta_x g_x(r) + f_x(r)$. Then

(1) DW converges in probability to 0;

(2) if $\tau = \infty$, R^2 converges in probability to 1;

if $0 \leq \tau < \infty$, R^2 converges in distribution to

$$1-\left[\int_0^1 h^2(r)dr - \left(\int_0^1 h(r)dr\right)^2\right]^{-1}[z-V'H^{-1}V];$$

(3) for j=1, ...,p, $t_{\beta_j}/\sqrt{n}$ converges in distribution to

$$(H^{-1}V)_{j+1,1}[(H^{-1})_{j+1,j+1}(z-V'H^{-1}V)]^{-1/2}.$$

3. Spurious regression

A small value of DW together with a moderate R^2 value may be taken as an indication of possible misspecification in the sense that the error term $U_{t,k}$ in model (1.1) is nonstationary. Granger and Newbold (1974) showed, through simulation results, the danger of acceptance of spurious relationships if the traditional significant stest statistics are used. If autocorrelated errors in time series regression equations are ignored, problem arise involving inefficient parameter estimates, and invalid significant tests and sub-optimal results when the fitted equations are used to derive forecasts. Provided that a regression equation relating variables is found to have strongly autocorrelated residuals, equivalent to a low Durbin-Watson value, the only conclusion that can be reached is that the equation is misspecified, whatever the value of R^2 observed.

We discuss the general framework (1.1) in which $\nabla^k y_t = u_t$, $\nabla^p X_{t,p} = x_t$, where ∇ is the difference operator such that $\nabla X_{t,j} = X_{t,j} - X_{t-1,j}$. For simplicity of analysis and notation it is assumed that $u_t = 0$ and $x_t = 0$ for $t \leq 0$. Also suppose both $\{x_t\}$ and $\{u_t\}$ satisfy the above condition C1 or C2 for some $\sigma_x^2 > 0$ and $\sigma_u^2 > 0$. Then application of Theorems 1 and 2 gives the following (Shin and Sarkar, 1996, p. 11).

Corollary 1. Under the above assumptions of this section, $\widehat{\beta}_j$ diverges for $j \leq k$, while $\widehat{\beta}_j$ converges for $j > k$; $\widehat{\sigma^2}$ diverges; all the t-statistics diverges at the rate $n^{1/2}$; DW converges to 0; R^2 has a nondegenerate limit which is less than one.

An empirical study and summary. We consider the following special cases of the spurious regression based on $\nabla^k y_t = u_t$, $\nabla^p X_{t,p} = x_t$: (A) k = 1 and p = 2, (B) k = 2 and p = 1, and (C) k = 2 and p = 2. 10,000 samples of size n for n = 25, 50, 100 and 250 are generated. For a fixed sample size n, the corresponding empirical means of parameter estimates, $\widehat{\sigma^2}$ and other conventional regression statistics such as t-statistics, R^2 and DW are considered. The subroutine DRNNOA is used to generate the normal random numbers. And the following different algebraic equations are used in data generation.

$$\nabla y_t = u_t \Rightarrow y_t = y_{t-1} + u_t \Rightarrow y_t = \sum_{i=1}^{t} u_i$$

$$\nabla^2 y_t = u_t \Rightarrow y_t = 2y_{t-1} - y_{t-2} + u_t \Rightarrow y_t = 2\sum_{i=1}^{t-1} u_i - \sum_{i=1}^{t-2} u_i + u_t$$

$$\nabla X_{t,1} = x_t \Rightarrow X_{t,1} = X_{t-1,1} + x_t \Rightarrow X_{t,1} = \sum_{i=1}^{t} x_i$$

$$\nabla X_{t,2} = X_{t,1} \Rightarrow X_{t,2} = X_{t-1,2} + X_{t,1} \Rightarrow X_{t,2} = \sum_{i=1}^{t} X_{i,1}$$

$$\nabla^2 X_{t,2} = x_t \Rightarrow X_{t,2} = 2X_{t-1,2} - X_{t-2,2} + x_t \Rightarrow X_{t,2} = 2\sum_{i=1}^{t-1} X_{i,1} - \sum_{i=1}^{t-2} X_{i,1} + x_t$$

where $x_t = u_t = 0$, if $t \leq 0$.

Tables 3.1-3.3 show the empirical results in spurious regression against the theoretical background given by Corollary 1. The following common empirical results from Tables 3.1-3.3 can be seen: all the DW's appear to converge in probability to 0; all $\widehat{\sigma^2}$'s diverge; all the R^2's seem to have a nondegenerate limit which is less than one (0.49, 0.37 and 0.78 in Tables 3.1-3.3, respectively); all the t-statistics seem to diverge slowly; all the parameter estimates 's may diverge very slowly for $j \leq k$ and converge very slowly for $j > k$. From these results we can observe the fact that OLS statistics such as R^2 and DW can be used as diagnostic tools to check the spurious regression in the sense that they can be used as remarkable symptoms of spurious regression when we obtain DW close to 0 and R^2 much less than one. Furthermore, as the orders of k and/or p are getting larger, the rate of divergence for $\widehat{\beta}_j$'s and t_{β_i} 's are getting faster.

4. Misspecification of nonstationary AR and polynomial regression models

We consider the effect of misspecification on the asymptotic behavior of the OLS estimate in which a polynomial regression model

$$y_t = \delta_0 + \delta_1 t + \delta_2(t^2/2!) + \dots + \delta_p(t^p/p!) + \varepsilon_t \tag{4.1}$$

is used to estimate a stochastic trend defined by the nonstationary AR model

$$y_t = \phi_0 + \phi_1 \nabla^{q-1} y_{t-1} + \dots + \phi_{q-1} \nabla y_{t-q+1} + \phi_q y_{t-q} + \eta_t \tag{4.2}$$

or vice versa. It is assumed that ε_t and η_t are the error components and satisfy condition C1 or C2 given in Section 1. For model (4.2) the s roots of the polynomial $A(L) = [1 - \phi_1(1 - L)^{q-1}L - \dots - \phi_q m^q]$ are assumed to lie on the unit circle and the remaining (q-s) roots to lie outside the unit circle.

4.1. Misspecification of a nonstationary AR as a polynomial regression model

Assume that the true y_t satisfies $\nabla^s y_t = \eta^*$ with η^* satisfying condition C1 or C2 of Section 1 for some $\sigma^2 > 0$, suppose model (4.1) is fit to the $\{y_t\}_{t=1}^{n}$ series. Then applying Theorem 1 and Theorem 2 with $x_t = 1$, $k = s$, and $U_{t,s} = y_t$ the next result (Shin and Sarkar, 1996, p. 14) can be obtained.

Corollary 2. Under the above assumptions of Section 4.1, for $j < s$, $\hat{\delta}_j$ diverges; all t-statistics diverges at the rate $n^{1/2}$; DW converges to zero; R^2 converges to a degenerate limit which is less than one.

An empirical study and summary. Table 4.1 displays the empirical results under misspecification of a nonstationary AR(2) as a polynomial regression model of order 2 against the theoretical backdrop provided by Corollary 2. From Table 4.1 the following can be observed: $\hat{\delta}_j$'s, for j = 0, 1, 2, diverge; all three t-statistics diverge; DW seems to converge to 0; R^2 appears to converge to 0.96; $\hat{\sigma}^2$ diverges. All these empirical values in Table 4.1 fully support the theoretical results of Corollary 2. Similarly, OLS statistics such as R^2 and DW may be exploited to check for misspecification of nonstationary AR(2) as a polynomial regression model of order 2 as follows: If DW is close to 0 and R^2 is close to 0.96, we can conclude that a nonstationary AR(2) model has been misspecified as a polynomial regression model of order 2.

4.2. Misspecification of a polynomial regression as a nonstationary AR model

It is assumed that the true y_t is generate by model (1.1) with errors $\{\varepsilon_t\}$ satisfying condition C1 or C2 with some $\sigma^2 > 0$, model (4.2) is fit to the $\{y_t\}_{t=1}^{n}$ series. Suppose $\delta_p \neq 0$ and $p \geq q \geq 1$. Let $X_{t,j} = \delta_p(t^{p-q+j}/(p-q+j)!)$, $j = 1, 2, \ldots, q$. Then $\nabla^{q-j} y_{t-j} = X_{t,j} + O_p(n^{p-q+j-1})$, uniformly in t. Hence, the asymptotics in the estimated model (4.2) in the same as those in

$$\hat{y}_t = \hat{\phi}_0 + \hat{\phi}_1 X_{t,1} + \ldots + \hat{\phi}_q X_{t,q}$$

and we have the following result (Shin and Sarkar, 1996, p. 15).

Corollary 3. Suppose that the above assumptions of Section 4.2 hold.

(1) If $\delta_0 = \delta_1 = \ldots = \delta_{p-q} = 0$ then $\hat{\phi}_j$ converges; $\hat{\sigma}^2$ converges to σ^2; t_{ϕ_j} converges; DW converges to $2(\sigma^2 - \psi)/\sigma^2$, where $\sum_{t=1}^{n} \varepsilon_t \varepsilon_{t-1}$ converges to ψ; R^2 converges to 1.

(2) If some of $\delta_0, \delta_1, \ldots, \delta_{p-q}$ are nonzero, then $\hat{\phi}_j$ converges; $\hat{\sigma}^2$ diverges; DW converges to 0; R^2 converges to 1; t_{γ_j} diverges for j such that $(\mathbf{H}^{-1}\mathbf{V})_{j+1,1} \neq 0$ where H and $\mathbf{V}$ are as defined in (5.2.1) with

$$G(r) = [1, r^{p-q+1}/(p-q+1)!, \ldots, r^p/p!]', \qquad f_k(r) = \delta_k r^k/k!$$

where $k = \varkappa = \max\{j: \delta_j \neq 0, 0 \leq j \leq p-q\}$.

An empirical study and summary. Table 4.2 depicts the empirical results under misspecification of a polynomial regression model of order 2 as a nonstationary AR(2) against the theoretical background provided by Corollary 3. From Table 4.2 the following can be seen: $\hat{\phi}_2$ appears to converge to 1.00; DW seems to converge in probability to 3.4; $\hat{\sigma}^2$ appears to converge in probability to 4.01; R^2 seems to converge in probability to 1. Each empirical value in Table 4.2 entirely defends the theoretical results of Corollary 3. Similarly, OLS statistics such as R^2 and DW may be exploited to check for misspecification of polynomial regression model of order 2 as a nonstationary AR(2) as follows: If DW is close to 3 and R^2 is close to 1.0, we can conclude that a polynomial regression model of order 2 has been misspecified as a nonstationary AR(2) model.

5. Underspecification of orders in nonstationary autoregressive and polynomial regression models

The order values p and q in the polynomial regression and nonstationary autoregressive models (4.1) and (4.2) and they might be underspecified are usually unknown.

5.1. Underspecification of the order in a nonstationary autoregressive model

If the data are underdifferenced, by which we mean that the AR model does in fact have some unit roots and/or cointegrating relationships but is nevertheless estimated completely in levels, so that no unit roots or cointegrating relationships are imposed on the data, then the unit roots and cointegrating relationships (if present) will nevertheless be satisfied asymptotically (although some efficiency is lost in finite samples), and moreover, convergence is typically at rates faster than $O(n^{1/2})$. The fact that the distribution theory for some of the estimated coefficients is nonstandard is of no consequence for construction of point forecasts. Thus, the costs of under- differncing are likely to be low. Overdifferencing, on the other hand, discards low-frequency information and destroys cointegrating relationships, and may cause difficulties for numerical estimation algorithm, due to the induced unit moving-average roots.

Suppose the true y_t satisfies $\nabla^s y_t = \eta^*$ with η^* satisfying condition C1 or C2 for some $\sigma^2 > 0$ and is estimated by model (4.2) with $q < s$. In this case the model is partially misspecified. Using Theorem 1 and 2 we get the following (Shin and Sarkar, 1996, p. 17).

Corollary 4. Under the above assumptions of section 5.1, $\widehat{\phi}_j$ converges; $\widehat{\sigma^2}$ diverges at the rate $n^{2(s-q)}$; R^2 converges to 1; DW converges to 0; t_{ϕ_i} diverges at the rate $n^{1/2}$.

An empirical study and summary. Table 5.1 gives the simulation results under underspecification of the order in a nonstationary AR model. In this case Corollary 4 provides the theoretical background. From Table 5.1 the following can be observed: $\widehat{\phi}_0$ seems to converge to 0.01 and $\widehat{\phi}_1$ to 1.00; $\widehat{\sigma^2}$ appears to diverge; R^2 seems to converge in probability to 1; DW seems to converge in probability to 0; two t-statistics appear to diverge. All these empirical values in Tables 5.1 fully support the results of Corollary 4.

5.2. Underspecification of the order in a polynomial regression model

Let the true model for y_t be a *q*th order polynomial regression $y_t = \delta_0 + \delta_1 t + \delta_2(t^2/2!) + \ldots + \delta_q(t^q/q!) + \varepsilon_t$, $\delta_q \neq 0$, and is estimated by the *p*th order polynomial regression $\widehat{y}_t = \widehat{\delta}_0 + \widehat{\delta}_2(t^2/2!) + \ldots + \widehat{\delta}_p(t^p/p!)$, where $p < q$. By Theorem 1 and Theorem 2 the following is obtained (Shin and Sarkar, 1996, p. 19):

Corollary 5. Under the above assumptions of Section 5.2, all the $\hat{\delta}_j$'s and $\hat{\sigma}^2$ diverges; the t-statistics diverge at the rate $n^{1/2}$; DW converges to 0; R^2 converges to a constant lying strictly between zero and one.

An empirical study and summary. Table 5.2 give the simulation results under under-specification of the order in a polynomial regression model. The following can be seen from Table 5.2: all the parameter estimates, $\hat{\sigma}^2$ and two t-statistics seem to diverge; R^2 seems to converge in probability to 0.94; DW seems to converge in probability to 0. All these empirical values in Table 5.2 totally support the results of Corollary 5.

6. Conclusion

It is observed from the simulation results that DW and R^2 and $\hat{\sigma}^2$ can be in general used as diagnostic tools to detect spurious regression, misspecification of nonstationary AR and polynomial regression models.

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APPENDIX

Table 3.1. Spurious regression (k = 1, p = 2)

statistics \ n	25	50	100	250
$\hat{\beta}_0$	-0.0000	0.0075	0.0292	0.0673
$\hat{\beta}_1$	0.0041	0.0001	-0.0118	-0.1915
$\hat{\beta}_2$	-0.0004	-0.0002	-0.0001	-0.0001
t_{β_0}	-0.008	0.006	-0.022	0.092
t_{β_1}	0.002.	-0.003	-0.118	-0.249
t_{β_2}	0.082	0.063	0.017	-0.271
R^2	0.479	0.496	0.490	0.489
DW	0.907	0.486	0.253	0.104
$\hat{\sigma}^2$	1.768	3.339	6.572	16.218

Table 3.2. Spurious regression (k = 2, p = 1)

statistics \ n	25	50	100	250
$\hat{\beta}_0$	0.185	-00281	-1.569	-9.702
$\hat{\beta}_1$	-0.063	-0.135	0.058	-0.090
t_{β_0}	0.039	0.026	-0.044	-0.142
t_{β_1}	-0.057.	-0.024	-0.013	-0.001
R^2	0.376	0.377	0.372	0.374
DW	0.447	0.177	0.093	0.015
$\hat{\sigma}^2$	360.4	2630.9	20221.2	307268.1

Table 3.3. Spurious regression (k = 2, p = 2)

statistics \ n	25	50	100	250
$\widehat{\beta}_0$	-0.049	0.093	0.111	6.951
$\widehat{\beta}_1$	-0.014	-0.002	0.001	0.149
$\widehat{\beta}_2$	-0.011	-0.042	-0.090	-0.230
t_{β_0}	0.013	-0.003	-0171	-0.246
t_{β_1}	-0.018.	0.005	0.067	-0.165
t_{β_2}	0.038	-0.152	-0.536	0.740
R^2	0.774	0.786	0.781	0.780
DW	1.001	0.318	0.103	0.033
$\widehat{\sigma}^2$	90.3	664.3	5020.1	74303.8

Table 4.1. Misspecification of a nonstationary AR(2) as
a polynomial regression model of order 2

statistics \ n	25	50	100	250
$\widehat{\delta}_0$	-0.028	-0.109	-0.147	-1.498
$\widehat{\delta}_1$	0.026	0.010	0.025	0.121
$\widehat{\delta}_2$	-0.001	0.001	0.000	-0.001
t_{δ_0}	-0.017	-0.022	-0.053	-0.189
t_{δ_1}	-0.123.	0.005	0.147	0.857
t_{δ_2}	-0.015	0.027	0.182	0.308
R^2	0.965	0.965	0.965	0.964
DW	0.414	0.113	0.028	0.004
$\widehat{\sigma}^2$	3.304	24.774	200.348	3070.343

Table 4.2. Misspecification of a polynomial regression model of order 2 as a nonstationary AR(2) model

statistics \ n	25	50	100	250
$\hat{\phi}_0$	2.988	2.354	1.748	1.285
$\hat{\phi}_1$	1.604	1.860	1.961	1.994
$\hat{\phi}_2$	1.030	1.005	1.000	1.000
t_{ϕ_0}	3.092	2.354	1.748	1.701
t_{ϕ_1}	10.240.	9.658	9.585	9.526
t_{ϕ_2}	5.264	5.658	5.783	5.789
R^2	0.998	0.998	0.998	0.999
DW	2.759	3.222	3.411	3.415
$\hat{\sigma}^2$	3.487	3.845	4.009	4.019

Table 5.1. Underspecification of a nonstationary AR(2) as a nonstationary AR(1) model

statistics \ n	25	50	100	250
$\hat{\phi}_0$	0.019	0.014	0.017	0.013
$\hat{\phi}_1$	1.019	1.012	1.007	1.003
t_{ϕ_0}	0.058	0.042	0.042	0.167
t_{ϕ_1}	93.065.	257.153	704.187	2722.667
R^2	0.982	0.994	0.997	0.998
DW	0.706	0.369	0.187	0.075
$\hat{\sigma}^2$	1.186	2.273	4.618	11.459

Table 5.2. Underspecification of a polynomial regression model of order 2 as a polynomial regression model of order 2

statistics \ n	25	50	100	250
$\hat{\delta}_0$	-58.497	-220.999	-858.501	-5271.000
$\hat{\delta}_1$	14.000	26.500	51.500	126.500
t_{δ_0}	-5.861	-8.094	-11.315	-17.756
t_{δ_1}	20.838.	28.441	39.470	61.691
R^2	0.949	0.943	0.939	0.938
DW	0.087	0.023	0.006	0.001
$\hat{\sigma}^2$	585.196	9018.000	141570.278	5468059.648